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A DEMONSTRATION BENCH FOR REPRESENTING THE CHARACTER OF PHASE TRANSITIONS OF THE FIRST AND SECOND KIND[†]

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The paper presents the description of a demonstration bench, which includes a mathematical model and analysis tools for understanding the features of phase transitions of the first and second kind. The advantage of this demonstration bench is the rejection of all phenomenology and the obvious limitation of the application of various approximations and hypotheses. The description is formed on the well-known equations of hydrodynamics, which are well-tested and are a reliable basis for the construction of realistic models. The Proctor-Sivashinsky model, which was used to describe the process of convection development in a thin layer of liquid with poorly conductive heat boundaries, is the basis for the demonstration bench. Exactly this model allows to observe phase transitions of the first and second kind. The feature of the model is that it allocates one spatial scale of interaction, leaving for the evolution of the system the possibility to choose the nature of symmetry. All spatial disturbances of the same size but of different orientation interact with each other. This allows us not to distract from the main task of this work, which is to demonstrate the process of structure formation as a result of a cascade of phase transitions. The mechanism of phase transitions associated with the presence of minimums of the interaction coefficients of modes of the spectrum of the instability. There are a large number of structural defects, which appear as attributes of phase transition. The instability spectrum modes interference is the reason of the high rate of correlations in the propagation of a new phase.
Keywords: demonstration, phase transitions of the first and second kind, Proctor-Sivashinsky equations
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INTRODUCTION

The process of convection development in a thin fluid layer has been considered in [1,2]. In this case a system of toroidal convective structures is formed, providing a significantly stronger heat transfer in the system. The authors of [3] noticed instabilities in this system, which were described by the Proctor Sivashinsky equations obtained in [1,2]. The Swift - Hohenberg equation, which was a simplification of the Proctor - Sivashinsky equations (instead of vector nonlinearity, the scalar one was used [4]) discussed the first kind phase transition in this description model. In this work and in work [5] the process of phase transition was discussed when from amorphous state of disorderly convection, the structure of convective rolls was formed, which in turns turned out to be unstable with formation of hexagonal convective cells.

By the way, the equations of the Proctor Sivashinsky model, after the instability of the primary structure - convective rolls, formed a system of square convective cells The question of the appearance of a phase transition of the first kind did not need to be discussed, but it remained unclear what the instability of the primary structure of convective shafts is and whether this process is a phase transition of the second kind. This is exactly what was found out later in [6-13], where the state function whose values determined the type of topology of emerging spatial structures was found and verified, the process of their formation was shown and thus it was proved that the that transition is nothing but a phase transition of the second kind (see also [14], which results in the formation of a more stable field of convective square cells [15-19]).

The aim of the work is to create a mathematical model and means of describing the bench to demonstrate the features of phase transitions of the first and second kind The advantage of this bench is the rejection of all phenomenology and the obvious limitation of the application of various approximations and hypotheses. The description is formed on the well-known equations of hydrodynamics, which are well-tested and are a reliable basis for the construction of realistic models.

1. MATHEMATICAL MODEL OF CONVECTION DEVELOPMENT IN A THIN LAYER OF LIQUID OR GAS WITH POORLY CONDUCTIVE HEAT BOUNDARIES

Below we consider the Proctor-Sivashinsky model [1-2], which was used to describe the process of convection development in a thin layer of liquid with poorly conductive heat boundaries. The feature of the model is that it allocates one spatial scale of interaction, leaving for the evolution of the system the possibility to choose the nature of symmetry. All spatial disturbances of the same size but of different orientation interact with each other. This allows us not to distract from the main task of this work, which is to study the process of structure formation as a result of a cascade of phase transitions Consider the simplest and most convenient representation of the Proctor-Sivashinsky model to describe this convection:

$$\frac{\partial A_j}{\partial t} = A_j - \sum_{i=1}^{N} V_{ij} \, | A_i |^2 \, A_j + f \, , \tag{1}$$

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where the interaction coefficients are defined by the relations

$$V_{jj}=1, \qquad V_{ij}=(2/3)\left(1-2\left(\vec{k}_i\vec{k}_j\right)^2\right)=(2/3)\left(1+2\cos^2\vartheta_{ij}\right), \tag{2}$$

and ϑ_{ij} – angle between the vectors $\vec{k}_i$ and $\vec{k}_j$. If we require zero values at the boundaries, the spatial dependence of each n - th mode will be $A_{n,m}\ \sin(2\pi nx)\sin(2\pi my)$ where *n, m* (they can be represented as $n = N\cdot\cos\vartheta_s$, $m = N\cdot\sin\vartheta_s$) – are integers, and $N^2 = n^2 + m^2$.

Expressions (1) - (2) must be supplemented with the initial values of the spectrum amplitudes A_j. That is $A_j\big|_{t=0} = A_{j_0}$. The width of the instability interval in k-space represents a ring – whose average radius is equal to one, and whose width is of the order of the value of the relative overthreshold $\mathcal{M}$, i.e., much less than one. From the results of preliminary studies [3] it became clear that in the system, besides the initial amorphous state, the existence of at least two long-lived solutions in the form of a roller structure (rolls) (see Fig. 1a), and in the form of a field of square cells (see Fig. 1b) is possible.

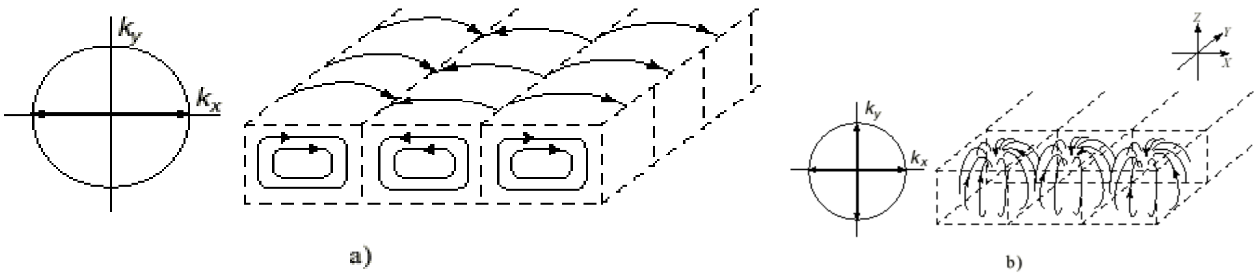

Figure 1. Convective structures: rolls (a) and square cells (b)

2. TOOLS TO DESCRIBE PHASE TRANSITIONS

Numerical analysis of the model allowed us to confirm the presence of structural-phase transitions. The state function turned out to be the quadratic form of the spectrum $I = \frac{1}{N}\sum_j A_j^2$.

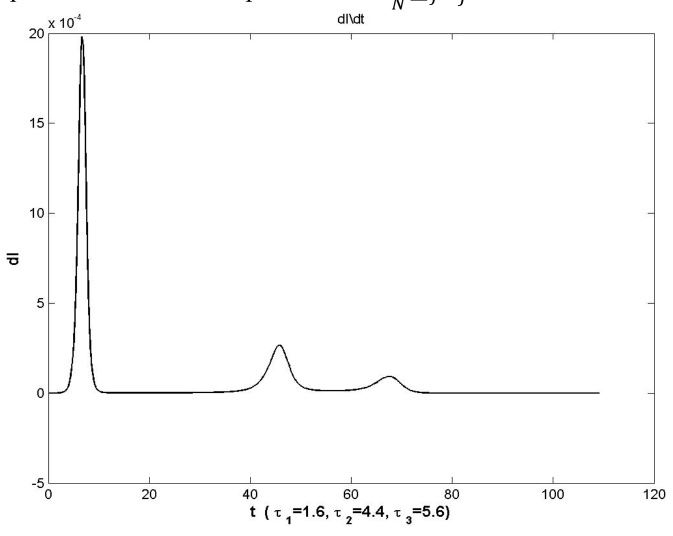

Figure 2. Behavior of the derivative state function $\partial I/\partial t$

After the first burst of the derivative $\partial I/\partial t$ an amorphous structure is formed - a system of convective rolls, and up to the second burst the value of $I\approx 0.75$ changes little. State function $I\ \to 1$ in the formation of a quasi-stable roller structure, later $I = 1.2$ - corresponds to the formed field of square cells. Characteristic transient times $\tau_1 = 1.6$ – the time of occurrence of the "amorphous" state $\tau_2 = 4.4$ – time of formation of pronounced roller-shaped structures $\tau_3 = 5.6$ – the time of cell system formation for one of the realizations of the process of establishing convective motion.

It can be seen that the formation times of the states τ_n is inversely proportional to the difference between the values of $I = \sum_i A_i^2$ after the structural-phase transition $I_n^{(+)} = (\sum_i A_i^2)_n^{(+)}$ and before this transition $I_n^{(-)} = (\sum_i A_i^2)_n^{(-)}$. That is,

$$\tau_n \propto \left\{\left(\sum A_i^2\right)_n^{(+)} - \left(\sum A_i^2\right)_n^{(-)}\right\}^{-1} = \Delta I_n^{-1}. \tag{3}$$

It is easy to see that $\tau_3/\tau_2\ \approx\ \Delta I_2/\Delta I_3$, faster phase transitions precede slower ones. By tracking the values of the state function, we can see at what stage of the process development the convective layer is.

3. VISUALIZATION OF PHASE TRANSITIONS PROCESSES

The demonstration model must form a physical intuition, so the role of visualization of the phase transition process is very important. Consider the view of large fragments of the field of convective structures, calculated [20] using the mathematical model outlined in Section 1.

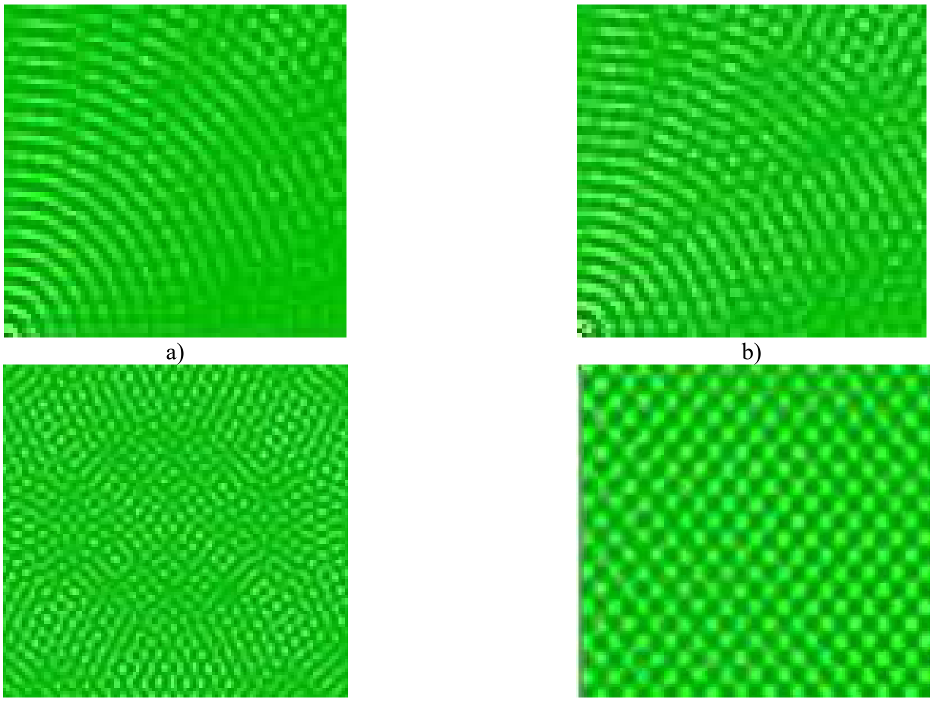

Figure 3. Temperature field distribution of convection on the layer surface

Segment (a) shows what happens after the first-order phase transition with the formation of convective rolls. The next segment (b) shows the dynamics of transverse modulation of the rolls. This is already the initial stage of the phase transition of the second kind. Segment (c) shows the process of nucleation of a new phase - formation of domains - a metastable spatial structure, after destruction of the roll system, Segment (d) demonstrates formation of a stable convective structure - square convective cells.

4. CONCLUSIONS

Thus, there are three states in the Proctor-Sivashinsky model of description of convection. The times of phase transitions between metastable states are much shorter than the times of their existence. Characteristic size of convective structures in the regime of extended instability in accepted order units $2\pi/k \propto 2\pi$ and the length of the wave vectors of the order of one. Interaction potential of spatial modes $V_{ij} = (2/3)\left(1 + 2\cos^2\vartheta_{ij}\right)$ has a deep minimum for corners $\vartheta_{ij} = \vartheta_i - \vartheta_j$ between the vectors $\vec{k}_i$ and $\vec{k}_j$ of two spatial modes $\vartheta_{ij} = \pm\pi/2$. Exactly these minima generate the unstable structure of rolls. For the existence of a minimum V_{ij} allows modes with relatively small amplitudes to continue their growth, while suppressing the neighboring disturbances.

When approaching a stable or metastable state, the spatial structure gets rid of many defects.

Defects occur mostly on the boundaries of homogeneous areas - domains. There is a correlation between the relative proportion of visually observed (geometrically) structure defects and the defectiveness value, defined as the ratio of the squares of the amplitudes of the spectrum modes that do not correspond to the square cell system to the total sum of the squares of amplitudes (see [12, 13]).

Thus, in the model of convection, Proctor-Sivashinsky it is possible to observe both the process of phase transition of the first kind and the process of phase transition of the second kind. This description of phase transitions did not use phenomenological approaches and various speculative considerations, which allows us to consider in detail the nature of the transition processes by the example of this model.

It is important to note that the correlation speed of spatial perturbations in this case is extremely large. The process of phase transformations covers not separate local regions, but at once the entire convection zone, the spatial structure in different places of the convective layer being a consequence of interference of a set of eigenfunctions of the task. That is, the observed significant rate of these correlations is due to phase changes and is not related to the energy-momentum transfer.

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REFERENCES

[1] J. Chapman, and M.R.E. Proctor "Nonlinear Rayleigh-Benard convection between the poorly conducting boundaries", J. Fluid Mech. **101**, 759 (1980). https://doi.org/10.1017/S0022112080001917

[2] V. Gertsberg, and G.E. Sivashinsky, "Large cells in nonlinear Rayleigh-Benard convection", Prog. Theor. Phys. **66**, 1219 (1981). https://doi.org/10.1143/PTP.66.1219

[3] B.A. Malomed, A.A. Nepomnyashchy, and M.P. Tribelsky, "Twodimensional quasiperiodic structures in nonequilibrium systems", ZhETF, **96**, 684 (1989).

[4] J.V. Swift, and P.C. Hohenberg, "Hydrodynamic fluctuations at the convective instability", Phys. Rev. A, **15**, 319 (1977). https://doi.org/10.1103/PhysRevA.15.319

[5] M.I. Rabinovich, A.L. Fabrikant, and L.S. Tsimring, "Finite Dimensional Disorder", UFN, **162**(8), 1–42 (1992).

[6] E.V. Belkin, I.V. Gushchin, A.V. Kirichok, and V.M. Kuklin, "Structural transitions in the Proctor-Sivashinsky model", PAST, **68**, 296 (2010). (in Russian). https://vant.kipt.kharkov.ua/ARTICLE/VANT_2010_4/article_2010_4_296.pdf

[7] I.V. Gushchin, A.V. Kirichok, and V.M. Kuklin, "Pattern Transitions in Unstable Viscous Convective Medium", https://doi.org/10.48550/arXiv.1311.3884

[8] I.V. Gushchin, A.V. Kirichok, and V.M. Kuklin "Pattern formation in convective media", «Journal of Kharkiv National University», physical series «Nuclei, Particles, Fields», **1040**(1), 4-27 (2013). http://nuclear.univer.kharkov.ua/lib/1040_1(57)_13_p04-27.pdf

[9] I.V. Gushchin, A.V. Kirichok, and V.M. Kuklin, "Pattern formation in unstable viscous convective medium", PAST, **4**, 251 (2013). https://vant.kipt.kharkov.ua/ARTICLE/VANT_2013_4/article_2013_4_251.pdf

[10] I.V. Gushchin, A.V. Kirichok, and V.M. Kuklin, "Structural-phase transitions and state function in unstable convective medium", PAST, **4**, 252 (2015). https://vant.kipt.kharkov.ua/ARTICLE/VANT_2015_4/article_2015_4_252.pdf

[11] I.V. Gushchin, A.V. Kirichok, and V.M. Kuklin, "State function in unstable convective medium", East Eur. J. Phys. **2**(1), 32 (2015). https://periodicals.karazin.ua/eejp/article/view/2811/2550

[12] I.V. Gushchin, and V.M. Kuklin, "Structural phase transition in thin convection at dependence of viscosity on temperature", PAST, **4**, 256 (2018). https://vant.kipt.kharkov.ua/ARTICLE/VANT_2018_4/article_2018_4_256.pdf

[13] I.V. Gushchin, V.M. Kuklin, and E.V. Poklonskiy, "Phase transitions in convection", East Eur. J. Phys. **4**, 34-40 (2019). https://doi.org/10.26565/2312-4334-2019-4-03

[14] V.M. Kuklin, *Selected chapters (theoretical physics)*, (V.N. Karazin KhNU, Kharkiv, 2021). pp. 244. http://dspace.univer.kharkov.ua/handle/123456789/16359

[15] L. Pismen, "Inertial effects in long-scale thermal convection", Phys. Lett. A, **116**, 241–243 (1986). https://doi.org/10.1016/0375-9601(86)90141-6

[16] A.V. Kirichok, V.M. Kuklin, I.P. Panchenko, S.S. Moiseev, and L.M. Pismen, "Dynamics of large-scale vortices formation in regime of convective instability", in: *International Conference "Physics in Ukraine"*, (Bogolyubov Institute of Theor. Phys., Kiev, 1993), pp. 130-135. https://inis.iaea.org/collection/NCLCollectionStore/_Public/25/076/25076395.pdf

[17] A.V. Kirichok, V.M. Kuklin, and I.P. Panchenko, "On the possibility of dynamo mechanism in a nonequilibrium convective medium", Reports of NASU, **4**, 87–92 (1997). (in Russian)

[18] A.V. Kirichok, and V.M. Kuklin, "Allocated Imperfections of Developed Convective Structures", Physics and Chemistry of the Earth Part A, **6**, 533–538 (1999). https://www.sciencedirect.com/science/article/abs/pii/S1464189599000678

[19] S.S. Moiseev, K.R. Oganesyan, P.B. Rutkevich, A.V. Tur, G.A. Khomenko, and V.V. Yanovskiy, "Vortex dynamo in spiral turbulence", in: *Integrability and kinetic equations for solitons*, edited by V.G. Bar'yakhtar, V.E. Zakharova, and V.M. Chernousenko. Sat. scientific. (in Russian), http://smiswww.iki.rssi.ru/files/publications/rutkevich/jetf_1988_2_94.pdf

[20] I.V. Gushchin, V.M. Kuklin, O.V. Mishin, and O.V. Priymak, "Model of physical processes due of CUDA technology", (V.N. Karazin KhNU, Kharkiv, 2017), pp. 116. (in Ukrainan). https://cutt.ly/UMH5G8f

ДЕМОНСТРАЦІЙНИЙ СТЕНД ДЛЯ ПРЕДСТАВЛЕННЯ ХАРАКТЕРУ ФАЗОВИХ ПЕРЕХОДІВ ПЕРШОГО І ДРУГОГО РОДУ

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В роботі представлено опис демонстраційного стенду, що включає математичну модель і засоби аналізу для розуміння особливостей фазових переходів першого і другого роду. Перевагою цього демонстраційного стенду є відмова від будь-якої феноменології та очевидна обмеженість застосування різних наближень та гіпотез. Опис будується на відомих рівняннях гідродинаміки, які добре апробовані та є надійною основою для побудови реалістичних моделей. В основу демонстраційного стенду покладено модель Проктора-Сивашинського, яка використовувалася для опису процесу розвитку конвекції в тонкому шарі рідини з межами, що погано проводять тепло. Саме ця модель дозволяє спостерігати фазові переходи першого та другого роду. Особливість моделі у цьому, що вона виділяє один просторовий масштаб взаємодії, залишаючи для еволюції системи можливість вибору характеру симетрії. Усі просторові обурення одного розміру, але різної орієнтації взаємодіють друг з одним. Це дозволяє не відволікатися від основного завдання даної роботи – демонстрації процесу структуроутворення внаслідок каскаду фазових переходів. Механізм фазових переходів пов'язаний із наявністю мінімумів коефіцієнтів взаємодії мод спектру нестійкості. Є велика кількість структурних дефектів, які виявляються ознаки фазового переходу. Інтерференція мод спектру нестабільності є причиною високої швидкості кореляцій поширення нової фази.

Ключові слова: демонстрація, фазові переходи першого та другого роду, рівняння Проктора-Сивашинського