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NUCLEAR ENERGY LEVELS IN ^{44}Ca USING FPD6PN INTERACTION†

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Nuclear energy levels; Inelastic electron scattering C4 form factors for nucleons that were present outside closed core for the isobars ^{44}Ca nuclei, which occupied low levels fp-LS shell ($1f_{7/2}$,$1f_{5/2}$, $2p_{3/2}$,$2p_{1/2}$), within shell model calculations had been studied. The interaction has been used to calculate the nuclear energy levels which is fpd6pn with fp shell model space. The results are compared with each other and with available experimental data its agreement with some results is clear. All inscriptions are given in diagrammatic notation., the wave vectors and analysis are modeled in the so-called diagrammatic notation. The potential of oscillator is utilized to construct single particle vector, considering $^{40}_{20}\mathrm{Ca}_{20}$ as a core. For the form factors the residual interaction M3Y has been adopted to include the inert core to the calculations. The OXFORD BEUNES AIRES SHELL MODEL CODE is utilized to accomplish the results for all tested nuclei.
Keywords: *Nuclear energy levels; Calcium isotope 44; Nuclear reaction; Diagrammatic notations FPD6pn; OXBASH*
PACS: 21.10.-k, 21.60.Cs

1. INTRODUCTION

Many studies had been performed to understand the nuclear properties and the internal structure of nuclei. Due to the complex nature of nuclei, there is no unified theory to describe the nuclear behaviors, properties and structures [1]. The shell theory has many benefits and properties such as the model independence of suggested, the applied physical N-N potential, beside the traditional Hamiltonian related to different categories of eigenvectors, and for plenty of nuclei. The shell theory stays valid to supplies the main theoretical methods for realizing all measurable of nuclei [1].

Excitation energies, binding energies, and spectroscopic factors were calculated in the LS shell ($1f_{5/2}$,$2p_{3/2}$, $2p_{1/2}$) space so acquired effective N-N matrix elements [2]. Interactions between PN had been inspired to measure for the presence of a orbits distance at N=32 in isotopes rich neutron localized in the nearby of magic nucleus ^{48}Ca [3]. Filled pf-LS shell model inspections of A=48 nuclei were executed [4], modified Kuo-Brown (KB) [10] to KB1 and KB3G. The isobaric chains A=50, A=51 and A=52 studied [5] using KB3G and FPD6 and their released version KB3G [6].

The shell theory introduced an important method for such research. In this hypothesis, realistic potentials are founded and the basis vectors are denoted by exact quantum numbers of angular momentum (J), isospin (T) and parity (π) [7]. A plenty of researches [8] were done to detect the distribution of eigen functions constructs the framework of the shell model [9]. Independently by Maria Mayer, and by Jensen, Haxel, and Suess) in the 1950s, the nuclear shell theory has regarded a major theory in the understanding of nuclear structure [10]. Extreme single-particle motion in spherical symmetry, only the addition of strong spin-orbit term was invoked to permit redesign of a wide range of results for isotopes near the nuclear magic numbers [11].

Calculations had been accomplished in model space of full fp- *LS* shell contains $1f_{7/2}$,$1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$ subshell and considering^{40}Ca as a core. The number of particles which can be excited to higher configurations is not restricted. Thus, apart from testing the suitability of GXPF1A interaction in explaining the experimental data, a comparison of results with that of him results would also throw light on the role of intruder $g_{9/2}$ orbital, appropriate choice of core, and the effect of truncation on the particles to be excited [12]. Nuclear energy levels; total angular momenta and even-even parity for nucleons that were present outside closed and no core for (^{42}Ca, ^{44}Ca, ^{46}Ca and^{48}Ca), which occupied fp-shell ($1f_{7/2}$,$1f_{5/2}$,$2p_{3/2}$,$2p_{1/2}$), within shell model calculations had been interested.

Four interactions had been assigned to calculate the nuclear energy spectrum of ^{42}Ca, ^{44}Ca, ^{46}Ca and^{48}Ca. The results of the FPD6, GXPF1 and KB3G interactions are compared with each other and with available experimental data, its agreement with some results is clear, the results are compared with GOGNY-P2 (fp, fpg and fpgd model space) interaction. The technique of frozen orbitals and restricted occupations were adopted (applied) in the framework of full space calculation, when GOGNY-P2 interaction had been used as an effective full space two body interaction [13]. Code OXBASH had been utilized to generate model space wave vectors and in the same time receive the comparable model space effective interaction that are selected for this study. The aim of this thesis is to reproduce the nuclear energy levels of (^{44}Ca) isotope, utilizing FPD6pn as a model space effective interaction to generate model space vectors, the calculations is performed by using OXBASH code [14]. The calculated energy levels for the isotopes under study with different set of effective interactions will be compared with the available experimental data.

2. THEORY

2.1. Interacting Particles in One and Two Active Orbits

The two particles wave function can be written as a product of a spin and an isospin dependent part as [15]:

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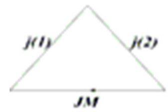
$$\Phi_{JMTTz}(1,2) = \Phi_{JM}(j(1)j(2))\Theta_{TTz}(t(1)t(2)). \quad (1)$$

where $j + j = J$ and $t + t = T$ with $T = 0$ or 1 since t = 1/2.

A diagrammatic notation for the spin part of Eq. (1) has been introduced and one can write as [15]:

$$\Phi_{JM}\big(j(1)j(2)\big) = \sum_{mm'} \langle jmjm'|JM\rangle \phi_{jm}(1)\phi_{jm'}(2) \equiv \quad (2)$$

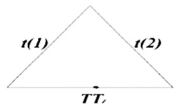


where $\Phi_{jm}(1)$ and $\Phi_{jm'}(2)$ are the single-particle States for particles 1 and 2 with their angular momenta *j* has been coupled to a total *J.* The coupling yields:

$$P_{12}\Phi_{JM}\big(j(1)j(2)\big) = (-1)^{J-2j}\Phi_{JM}\big(j(1)j(2)\big) = -(-1)^{J}\Phi_{JM}(j(1)j(2)) \quad (3)$$

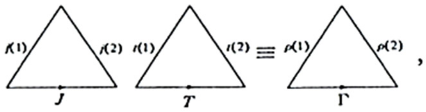
when P_{12}: interchanges operator. So, the isospin dependent part as [15]:

$$\Theta_{TT_z}(t(1)t(2)) = \sum_{t_z t'_z} \langle tt_z tt'_z|TT_z\rangle \theta_{tt_z}(1)\theta_{tt'_z}(2) \equiv \quad (4)$$



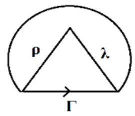
Condensed the notation to include spin and isospin as, $\rho \equiv (j,t)$ and $\Gamma \equiv (J,T)$. So, Eq. (1) can be rewritten as:

$$\Phi_{\Gamma}(1,2) = \quad (5)$$



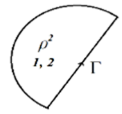
Anti-symmetry of a wave function is referred to by a circular arc and one obtains for two particles in two different orbits ρ and λ

$$\Phi_{\Gamma}^{as}(1,2) \equiv \quad (6)$$



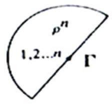
For two particles in the same orbit the notation can be further formed as

$$\Phi_{\Gamma}^{as}(1,2) \equiv \quad (7)$$



One can be extended easily to wave functions of more than two particles in one Orbit *ρ* as

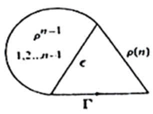
$$\Phi_{\Gamma}^{as}(1,2,\dots,n) \equiv \quad (8)$$



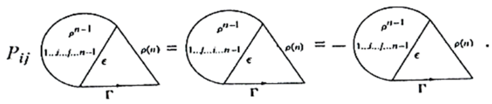
2.2. Coefficients of Fractional Parentage

The *n*-particle function with all particles in one orbit *ρ* is given as [14]:

$$\Phi_{\Gamma}(1,2,\dots,n) = \quad (9)$$



The group ρ^{n-1} is coupled to $J_\epsilon, T_\epsilon, x_\epsilon$ with x_ϵ denoting all further quantum numbers needed to specify the state $|\rho^{n-1}\rangle_\epsilon$ uniquely. When the operator P_{ij} interchanges all coordinates of particles *i* and *j*, then one obtains for $i, j \leq n-1$ due to the anti-symmetry:

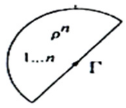
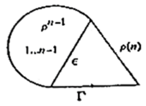


(10)

The result of the permutation P_{ij} for *i* or *j* equal to *n*, however, cannot in general be represented by a simple expression in terms of the original function as in Eq. (10).

$$\Phi_{\Gamma}^{as}(1,2,\dots,n) \equiv$$  (11)

The wave function of eq. (9) due to anti symmetrization. Also, one can write:

$$= \sum \langle \rho^n \Gamma | \} \rho^{n-1} \epsilon \rangle \quad (12)$$

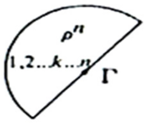
where $\langle \rho^n \Gamma | \} \rho^{n-1} \epsilon \rangle$ represented "coefficients of fractional parentage" or c.f.p. The normalization and orthogonality lead to the states $|\rho^n\rangle_{\Gamma x}$ be denoted by x as:

$$\sum_{\Gamma' x'} \langle \rho^n \Gamma x | \} \rho^{n-1} \Gamma' x' \rangle \langle \rho^n \Gamma x'' | \} \rho^{n-1} \Gamma' x' \rangle = \delta_{xx''}. \quad (13)$$

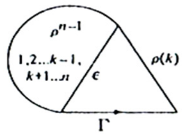
The simple reordering depending on equation (12), if the particle numbered k is willing to decouple, then:

$$\Phi_{\Gamma}^{as}(1,2,\dots,k,\dots,n) = (-1)^{n-k} \Phi_{\Gamma}^{as}(1,2,\dots,n,k). \quad (14)$$

In the completely antisymmetric wave function leads to the expansion [15].



$$= (-1)^{n-k} \sum_{\epsilon} \langle \rho^n \Gamma | \} \rho^{n-1} \epsilon \rangle$$

 (15)

It is beneficial to discuss in detail the derivation of c.f.p. for the relatively simple case of three identical particles (maximum isospin) in one orbit with $j \le 7/2$. It is only for $j \le 7/2$ that three particles couple in an unique way to a given total spin J. Thecouplingofthreesingle-particlewavefunctionstoanon-antisymmetrized function of total spin J can be obtained by using the same diagrammatic representation as given in details in [15].

2.3. The Reduced Matrix Elements of the Longitudinal Operator ($\eta = Co$)

The longitudinal form factor describes the spatial distribution of the charge (the transition charge densities), so the longitudinal scattering might be considered as a result of the interactions of the incident electrons with the charge distribution of the nucleus [18]. The longitudinal form factor operator is defined as [18]:

$$\hat{T}_{JMt_z}^{Co}(q) = \int d\vec{r} j_J(qr) Y_{JM}(\Omega_r) \hat{\rho}(\vec{r}, t_z) \quad (16)$$

where $j_J(qr)$ is the spherical Bessel basis, $Y_{JM}(\Omega_r)$ is the spherical harmonic and $\hat{\rho} = (\vec{r}, t_z)$ is the nucleon charge density operator. From equations (2-29) and (2-30), one obtains:

$$\hat{T}_{JMt_z}^{L}(q) = e(t_z) j_J(qr) Y_{JM}(\Omega_r) \quad (17)$$

2.4. Core polarization effects

Microscopic theory will include the discarded space as a first order perturbation that is particle hole state (p-h), and using mixing interaction in order to calculate these effects as a residual interaction, For Nuclei of A>40, Z, N≥ 20, the fpLS shell model space is the suitable space [18], with a core of ^{40}Ca is assumed. The electron scattering operator $\hat{T}_{\Lambda}^{\eta}$ reduced matrix elements is formed by two parts, the former "Model space" matrix elements, and the latter is for the "Core-polarization" matrix elements [18].

$$\left\langle \Gamma_f \left\| \hat{T}_{\Lambda}^{\eta} \right\| \Gamma_i \right\rangle = \left\langle \Gamma_f \left\| \hat{T}_{\Lambda}^{\eta} \right\| \Gamma_i \right\rangle_{MS} + \left\langle \Gamma_f \left\| \delta \hat{T}_{\Lambda}^{\eta} \right\| \Gamma_i \right\rangle_{CP}. \quad (18)$$

3. RESULTS AND DISCUSSION

Microscopic models have been introduced to constitute nuclear energy states. The model with mixed multi-nucleon conformations is one of the most important models. In the adopted method, the systems 40Ca and 32S are considered a non-active core with extractive baryons (neutrons only) that are named the LS shell. Calculations of the shell model are carried out within a model-space in which the nucleons are free to occupy a few orbits and are able to reproduce the measured static moments and transition strengths [16,19].

The shell theory is a major part of the nuclear theory and an essential theoretical topic for the micro scale calculations of nucleus build-up. The essential assumption in the shell model is that every particle plays separately in a potential average, including a dominant non-central spin-orbit part, and consists of the baryons themselves [19]. After this, the baryons allied into classes, the "shells, "distant from each other. By this approach, the nucleus is divided into an inert core made up of filled LS shells plus a certain number of valence nucleons called the valence bodies [16,20]. Energy level values in this work are calculated by the shell model calculations that are performed via the computer code OXBASH [14].

The calcium elements that existed in the human's bone structure are very important. It's very important to study the calcium isotopes. Some of the experimental properties of^{42}Ca, ^{44}Ca, ^{46}Ca and^{48}Caisotopes shown in Table.

Table. Experimental properties of some Ca isotopes [17]

	^{42}Ca	^{44}Ca	^{46}Ca	^{48}Ca
Jπ	0^+	0^+	0^+	0^+
M (micro-u)	41958617.828	43955481.543	45953687.988	47952522.904
M_{ex} (MeV)	-38.547245	-41.468675	-43.139361	-44.224629
B/A (MeV)	8.616563	8.658175	8.668979	8.666686
S_p (MeV)	10.27667	12.18226	13.81269	15.80162
S_{2p} (MeV)	18.08529	21.62394	25.04405	29.02964
S_n (MeV)	11.48067	11.13117	10.3985	9.95153
S_{2n} (MeV)	19.84349	19.06406	17.81332	17.2279
Q_α (MeV)	-6.25734	-8.8537	-11.1416	-13.97629
E_β (MeV)	-6.426092	-3.65269	-1.378143	0.279213
$Q_{\beta\text{-}n}$ (MeV)	-17.97615	-13.35189	-10.13878	-7.95935
$Q_{2\beta}$ (MeV)	-13.44257	-3.92011	0.98844	4.26808
$Q_{4\beta}$ (MeV)	-45.277	-28.108	-13.66779	-1.40261

4. INELASTIC LONGITUDINAL FORM FACTOR IN ^{44}Ca FOR THE TRANSITION ($0^+2 \rightarrow 4^+2$) AT E_X=1.561 MEV

Inelastic longitudinal (C4) form factors were calculated by using M3Y-P1. Fig. 1 represents the calculated form factors using (M3Y-P1) as a residual interaction in the first peak the results overestimate the data at all q ≥ 1.2 till 2.2 fm^{-1} while the calculations underestimate the data at the second peak. The form factor for C4 transition in ^{44}Ca with an excitation energy Ex=1.561 MeV is displayed in Fig. 1, where the total contributions are due to the core polarization effect. The data are well explained for the first lobe, and also up to q =3 fm^{-1}. Higher q values are estimated. The values of $|F(q)|^2$ are in between 10^{-4} and downward with the maximum at q = 1.8 fm^{-1}.

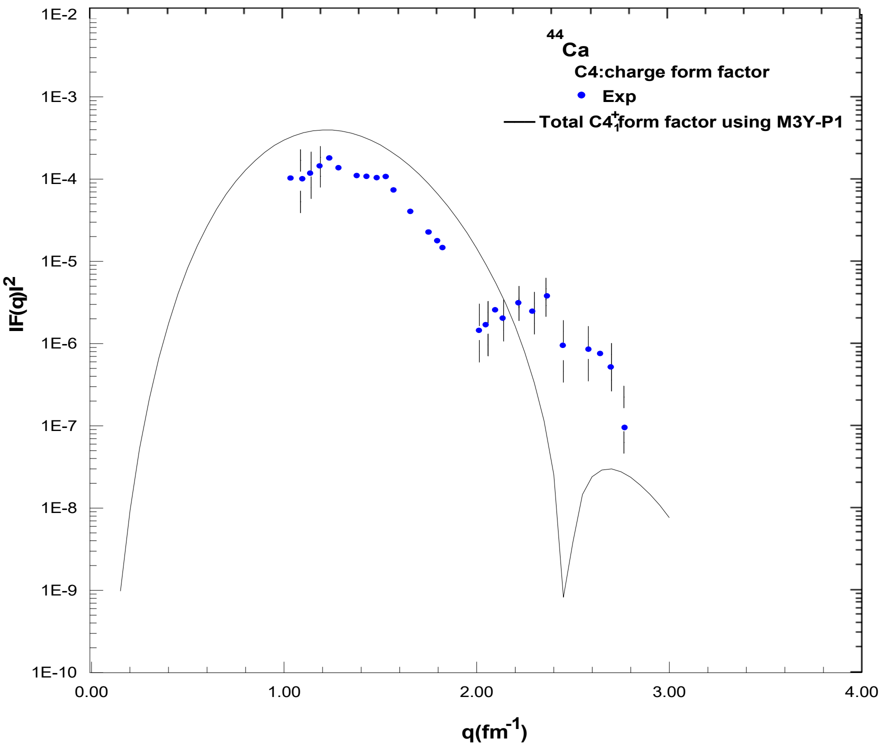


Figure 1. Inelastic longitudinal C4 form factors for the 4_1^+ (E_x=1.561 MeV) (The value of Ex is theoretical) state in ^{44}Ca using M3Y-P1 as a residual interaction.

5. CONCLUSIONS

From Figure 2 which represents the energy level scheme for ^{44}Ca, it is clear that there are clear differences between the calculated and experimental results in general. The calculated results reveals that there are an energy gap between $J_k^\pi = 0_1^+$ and $J_k^\pi = 4_2^+$ by the value of $\Delta E = 3MeV$ but this state has a well defined value as compared with the experimental one and the state $J_k^\pi = 2_2^+$ has a fair agreement with the experiment but the higher the states the wider the difference with the laboratory states. The function of energy levels density will be very useful to identify the energy spectrum and study the distribution of states between 1 to 10 MeV besides the number of every state and their values, nuclear shell theory is based on some dependable not sure realistic and a wide range of fitting parameters are not well reproduced to generate the static and dynamic nuclear properties and they need to be readjusted to meet the experimental results table 1 tabulates some nuclear properties for some even Ca isotopes.

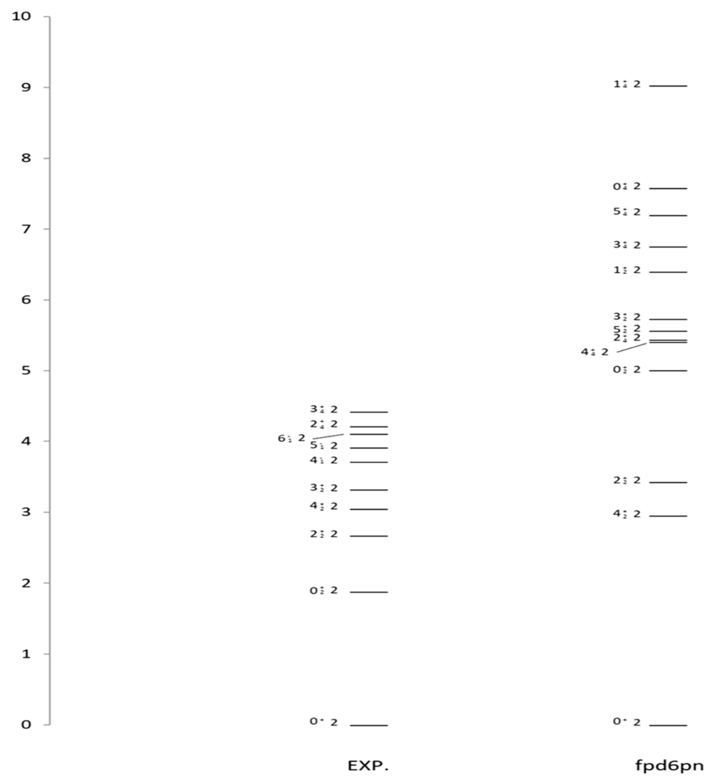


Figure 2. The energy levels scheme of ^{44}Ca by using fpd6pn interactions with closed core ^{40}Ca for **(J_K^πT),** positive parity, ten orders

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REFERENCES

[1] M. Honma, T. Otsuka, B.A. Brown and T. Minzusaki, "Effective interaction for pf-shell nuclei", Phys. Rev. C, **65**, 061301(R) (2002). https://doi.org/10.1103/PhysRevC.65.061301

[2] H. Crannell, R. Helm, H. Kendall, J. Oeser, and M. Yearian, "Electron-Scattering Study of Nuclear Levels in Cobalt, Nickel, Lead, and Bismuth", Phys. Rev. **123**(3), 923 (1961). https://doi.org/10.1103/PhysRev.123.923

[3] J.I. Prisciandaro, P.F. Mantica, B.A. Brown, D.W. Anthony, M.W. Cooper, A. Garcia, D.E. Groh, A. Komives, W. Kumarasiri, P.A. Lofy, A.M. Oros-Peusquens, S.L. Tabor, and M. Wiedeking, Phys. Lett. B, **510**, 17-23 (2001). https://doi.org/10.1016/S0370-2693(01)00565-2

[4] E. Caurier, and A. P. Zuker, Phys. Rev. C, **50**, 225 (1994). https://doi.org/10.1103/PhysRevC.50.225

[5] A. Poves, J. Sánchez-Solano, E. Caurier, and F. Nowacki, Nucl. Phys. A, **694**, 157 (2001). https://doi.org/10.1016/S0375-9474(01)00967-8

[6] W.A. Richter, M.G. Van Der Merwe, R.E. Julies, and B.A. Brown, Nucl. Phys. A, **532**, 325 (1991). https://doi.org/10.1016/0375-9474(91)90007-S

[7] V. Zelevinsky, B.A. Brown, N. Frazier, and M. Horoi, Phys. Rep. **276**, 8 (1996). https://doi.org/10.1016/S0370-1573(96)00007-5

[8] R.R. Whitehead, A. Watt, D. Kelvin, and A. Conkie, Phys. Lett. B, **76**, 149 (1978). https://doi.org/10.1016/0370-2693(78)90262-9

[9] B.A. Brown, and G. Bertsch, Phys. Lett. B, **148**, 5 (1984). https://doi.org/10.1016/0370-2693(84)91598-3

[10] J. Suhonen, *From Nucleons to Nucleus Concepts of Microscopic Nuclear Theory*, (Springer, Finland, 2006).

[11] R.D. Lawson, *Theory of the Nuclear Shell Model*, (Clarendon Press, Oxford, New York, 1980).
[12] L. Coraggio, A. Covello, N. Itaco, and T.T.S. Kuo, Prog. Part. Nucl. Phys. **62**, 135 (2009). https://doi.org/10.1016/j.ppnp.2008.06.001
[13] F.Z. Majeed, and S.S. Mashaan, Indian Journal of Natural Sciences, **9**, 50 (2018).
[14] B.A. Brown et aI, OXBASH code, MSUNSCL Report 524 (1988).
[15] P.J. Brussaard, and P.W.M. Glademans, *Shell-model Applications in Nuclear Spectroscopy*, (North-Holland Publishing Company, Amsterdam, 1977).
[16] H. Sagawa, X.R. Zhou, and X.Z. Zhang, "Evolution of Deformations in Medium-Mass Nuclei", Phys. Rev. C, **72**, 054311 (2005). https://doi.org/10.1103/PhysRevC.72.054311
[17] G. Audi, F.G. Kondev, Meng Wang, W.J. Huang, and S. Naimi, "The NUBASE2016 Evaluation of Nuclear Properties", Chinese Phys. C, **41**, 030001 (2017). https://doi.org/10.1088/1674-1137/41/3/030001
[18] A.A. Al-Rahmani, and S.F. Kazem, "Elastic Electron-Nucleus Scattering Form Factors of Selected Medium Mass Nuclei", NeuroQuantology, **18**(8), 49-58 (2020). https://www.neuroquantology.com/data-cms/articles/20201001121608pmNQ20204.pdf
[19] A.H. Ali, "Investigation of the Quadrupole Moment and Form Factors of Some Ca Isotopes", Baghdad Science Journal, **17**(2), 0502 (2020). https://orcid.org/0000-0002-1525-1406
[20] L. Wang, J. Liu, R. Wang, M. Lyu, C. Xu, and Z. Ren, "Global analysis of nuclear cluster structure from the elastic and inclusive electron scattering", Phys. Rev. C, **103**, 054307 (2021). https://doi.org/10.1103/PhysRevC.103.054307

РІВНІ ЯДЕРНОЇ ЕНЕРГІЇ У ^{44}Ca ЗА ВИКОРИСТАННЯ FPD6PN ВЗАЄМОДІЇ

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У рамках розрахунків оболонки моделі були вивчені рівні ядерної енергії, формфактори C4 непружного розсіювання електронів для нуклонів, які були присутні за межами закритого ядра для ізобар ядер ^{44}Ca, і які займали низькі рівні оболонки fp-LS (1f7/2,1f5/2, 2p3/2,2p1/2). Для розрахунку рівнів ядерної енергії була використана взаємодія, яка є fpd6pn з простором моделі оболонки fp. Результати порівнюються один з одним, і з наявними експериментальними даними чітко збігається з деякими результатами. Хвильові вектори та аналіз моделюються у так званому діаграмному записі. Для побудови одно часткового вектора використовується потенціал осцилятора, розглядаючи $^{40}_{20}\mathrm{Ca}_{20}$ у якості ядра. Для форм-факторів було прийнято залишкову взаємодію M3Y, щоб включити до розрахунків інертне ядро. Для отримання результатів для усіх перевірених ядер використовується код моделі оболонки OXFORD BEUNES AIRES.

Ключові слова: *ядерні енергетичні рівні; ізотоп кальцію 44; ядерна реакція; діаграмний запис FPD6pn; OXBASH*