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SHELL MODEL INVESTIGATION OF SOME *p* AND *sd*-SHELL NUCLEI WITH HARMONIC OSCILLATOR AND SKYRME INTERACTIONS[†]

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In this study, the longitudinal charge $|F_{ch}(q)|^2$ and $|F(C2,q)|^2$form factors for the nuclei ^{9}Be and ^{28}Si lying in the *p* and *sd* shells are studied by employing the Harmonic Oscillator potential (HO) and Skyrme effective interaction (Sk35−Skzs*). The C0 and C2 from factors calculated for the ground state 3/2$^-$, the 5/2$^-$ (2.429 MeV) and 7/2$^-$ (6.380 MeV) for ^{9}Be, while the ground state 0$^+$ and 2$^+$ (1.779 MeV) state for ^{28}Si nucleus. Calculations of microscopic perturbations that involve intermediate one-particle, one-hole excitation from the core and MS orbits into all upper orbits with $n\hbar\omega$excitations are utilized to generate the effective charges necessary to account for the "core polarization effect". The shell model calculations are utilized on the extended model space to include all 1s, 1p, 2s–1d, 2p-1f orbits with $(0\ +2)\hbar\omega$ truncation. Bohr-Mottelson collective model and Tassie model with properly estimated effective neutron and proton charges are taken into account to consider the effect of the core contribution. The estimated form factors were compared with the measured available data and they were in good agreement for most of the studied states. A conclusion can be drawn that $(0\ +2)\hbar\omega$ truncation is very good choice to study the longitudinal form factors.

- The choice of Harmonic Oscillator potential (HO) and Skyrme effective interaction (Sk35−Skzs*) is adequate for form estimation of longitudinal form factors.
- The estimation of the effective charges based on microscopic perturbations that involve intermediate one-particle, one-hole excitation from the core and MS orbits into all upper orbits with $n\hbar\omega$ excitations is adequate.
- The $(0\ +2)\hbar\omega$ truncation proves to be very successful to perform the study.

Keywords: *Shell Model; Charge form factor; Longitudinal Form Factors; Harmonic Oscillator; Skyrme Interactions*
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INTRODUCTION

The investigation of nuclei has more access neutrons, than what is known as "neutron-rich nuclei" far from the line of stability is important for understanding nuclear structure. Exotic features differ from those of stable nuclei and deserve experimental and theoretical exploration. One of the most striking characteristics of neutron-rich nuclei is nuclear deformation, which may be studied both theoretically and empirically using electromagnetic properties like electric quadrupole (Q) moments and magnetic dipole (μ) moments. Using a microscopic particle vibration model, Sagawa and Asahi [1] investigated the N/Z dependence of the quadrupole polarization charges of C isotopes. The single-particle wave functions and gigantic quadrupole resonances are approximated using the Hartree-Fock and random-phase approximations. The polarization charges of nuclei with a high N/Z ratio experienced a significant quenching. Cohen and Kurath [2] model properly explain the features of low energy p shell nuclei, but it fails to represent the form factors of higher momentum transfer. Radhi et al. [3-6] have previously stated that the CP effects must be taken into account for nuclei in the *p* shell and *sd* shell to improve form factor calculations. Taihua Heng et al. [7] used the ab initio no-core full configuration NCFC technique to explore the characteristics of ^{7}Li with the NNLO$_{opt}$ chiral nucleon-nucleon and JISP16 interactions, as well as ^{7}Be with the JISP16 interaction. They calculated observables like energy spectra, proton point radii at the root mean value, transitions, and electromagnetic moments. Zheng et al. performed calculations based on large-basis shell model without core calculations for *p-shell* nuclei using six main shells (from 1s to 3p-2f-1h) [8, 9]. All nucleons are active in these calculations, according to Zheng et al., If computer resources are restricted, we have to adopt a truncated calculation without core with some freezing orbits, in which only a few $\hbar\omega$ excitations of the lowest unperturbed configurations are evaluated can be used. The result will converge and approximate that of the full no-core calculations as the number of $\hbar\omega$ increases. In the work of Navratil *et al.* [10] it is found that the predicted rate of the E2 transition with $4\hbar\omega$ space for ^{6}Li is weaker than $6\hbar\omega$ space prediction. Majeed *et al.* [11-13] revealing that the form factor calculation on a large basis of the shell model was used to analyze nuclei in the *p*, *sd*, and *Fp* shells, including the contribution of high-energy configurations beyond the *p*, *sd*, and *Fp* shell space model space is essential to be considered to consider the effect of the core polarization contribution arises from the closed core.

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In this study, the longitudinal charge and C2 form factors for ^{9}Be and ^{28}S exotic nuclei utilizing the shell model calculations by considering the major shells 1s, 1p, 2s–1d, 2p–1f, with partially inert core using the model space *spsdpf* with *wbt* effective interaction using the code NushellX@MSU [14]. Since the space configuration is a very large dimension, the shell model calculations in the whole *spsdpf* space are not possible, therefore a truncation is required in the valence space. We will consider the $(0+2)\hbar\omega$ truncated *spsdpf* model space (ms) calculation. The form factors will be calculated by considering the residual interactions, between harmonic oscillator (HO) and Skyrme effective interaction (Sk35−Skzs*) [15] with using the Tassie and Bohr-Mottelson collective models. The core polarization (CP) will be considered by evaluating the suitable effective charges at zero photon point. Theoretical results of C0 and C2 were compared with the available measured data for each studied case for the studied nuclei.

METHOD DETAILS

Tassie and Bohr-Mottelson models were adopted along with Harmonic Oscillator potential and Skyrme effective interaction (Sk35−Skzs*) with suitable parametrization were adopted. As mentioned in Ref. [16], the neutron and proton effective charges for the model of Tassie are predicted by using microscopic perturbation calculations that include intermediate (1p-1h) excitations from the orbit contribution MS and core to include higher orbits with $n\hbar\omega$ excitations. The effective proton and neutron charges for Bohr-Mottelson were estimated using the equations [17]

$$e^{eff}(t_z) = e(t_z) + e\delta e(t_z)$$

$$e\delta e(t_z) = Z/A - 0.32(N-Z)/A - 2t_z[0.32 - 0.3(N-Z)/A]\,, \tag{1}$$

where $t_z(p) = 1/2$ and $t_z(n) = -1/2$

In terms of transition charge density, the element matrix of Coulomb can be represented as the sum of the MS and CP elements [18]

$$O(C\lambda, q) = q\int_0^\infty dr r^2 j_\lambda(qr)\rho_{\lambda,p}^{MS}(r) + \int_0^\infty dr r^2 j_\lambda(qr)\Delta\rho_\lambda(r) \tag{2}$$

where the momentum transfer is q and $j_\lambda(qr)$ is spherical Bessel function. For the initial (i) and final (f) nuclear states, the nucleons charge density F of the transition is described using the one-body density matrix [18]

$$\rho_{\lambda,u}^{MS}(r) = \sum_{k_a,k_b}^{MS} F(i,f,k_a,k_b,\lambda,u)\langle j_a||Y_\lambda||j_b\rangle R_{n_a l_a}(r)R_{n_b l_b}(r), \tag{3}$$

where k stands for $(n\,l\,j)$ the s.p. states and (u) is the index which refers to either neutrons or protons.

The transition density for the CP valence model is given by [18]

$$\Delta\rho_\lambda^V(r) = \delta e_p\rho_{\lambda,p}^{MS}(r) + \delta e_n\rho_{\lambda,n}^{MS}(r) \tag{4}$$

δe_p , δe_n are the charges associated with the neutron and protons to account for polarization.

The CP for Tassie model transition density is provided by

$$\Delta\rho_{\lambda,p}^T(r) \propto r^{\lambda-1}\frac{d\rho_{0,p}^{core+MS}(r)}{dr} = Nr^{\lambda-1}\frac{d\rho_{0,p}^{core+MS}(r)}{dr} \tag{5}$$

The charge density of the ground state is [18]

$$d\rho_{0,p}^{core+MS}(r) = \sum_{k_a,k_b}^{Core+MS} F(i,f,k_a,k_b,0,p)\langle j_a||Y_0||j_b\rangle R_{n_a l_a}(r)R_{n_b l_b}(r) \tag{6}$$

At the photon point, the proportionality constant N is given by the matrix elements of gamma transitions $M(E\lambda)$, $q = E_\gamma/\hbar c$, where E_γ is the energy due to excitation [18]

$$M(E\lambda) = \left\{e\int_0^\infty dr r^2\, r^\lambda(qr)\rho_{\lambda,p}^{MS}(r) + N\int_0^\infty dr r^2 r^{2\lambda+1}\frac{d\rho_{0,p}^{core+MS}(r)}{dr}\right\} \tag{7}$$

The matrix of gamma transition elements can be described as MS matrix elements with effective charges.

$$M(E\lambda) = e_p^{eff}\int_0^\infty dr r^2\, r^\lambda(qr)\rho_{\lambda,p}^{MS}(r) + e_n^{eff}\int_0^\infty dr r^2\, r^\lambda(qr)\rho_{\lambda,n}^{MS}(r) \tag{8}$$

Equating Eq. (7) with Eq. (8) yields the constant of proportionality N by means of the effective charges. A detailed discussion of these above-mentioned models for effective neutrons and protons [18]

$$|F_{L\lambda}(q)|^2 = \frac{4\pi}{Z^2}\frac{1}{2j_i+1}|O(C\lambda,q)|^2\left|F_{cm}(q)F_{fs}(q)\right|^2 \tag{9}$$

Skyrme energy is produced through Skyrme interaction. According to E_{Skyrme} is a two-body density-dependent interaction that represents central spin-orbit and tensor components in coordinate space and replicates the strong force in the particle-hole channel [19]

$$v_{12} = t_0\left(1 + x_0\hat{P}_\sigma\right)\delta(\vec{r}_1 - \vec{r}_2) + \frac{t_1}{2}\left(1 + x_1\hat{P}_\sigma\right)\left(\hat{k}^2\delta(\vec{r}_1 - \vec{r}_2) + \delta(\vec{r}_1 - \vec{r}_2)\hat{k}'^2\right) + t_2\left(1 + x_2\hat{P}_\sigma\right)\hat{k}'.\delta(\vec{r}_1 - \vec{r}_2)\hat{k} + \frac{t_3}{6}\left(1 + x_3\hat{P}_\sigma\right)\rho^\alpha(\vec{R})\delta(\vec{r}_1 - \vec{r}_2) + iW_0\hat{k}'(\hat{\sigma}_1 + \hat{\sigma}_2) \times \hat{k}\delta(\vec{r}_1 - \vec{r}_2) \quad (10)$$

here $\vec{R} = \frac{\vec{r}_1+\vec{r}_2}{2}$ and α is the force of Skyrme parameters. The operators $\hat{k} = (\vec{\nabla}_1 - \vec{\nabla}_2)/2i$ and $\hat{k}' = -\left(\overleftarrow{\nabla}_1 - \overleftarrow{\nabla}_2\right)/2i$ are the relative momentum wavevectors for the two nucleons acting to the right and left, with coordinate $\acute{r}$ respectively. When the nuclear structure data is fitted, the variables t_0, t_1, t_2, t_3, x_1, x_2, x_3, and W_0 are free parameters taken to describe the components of various interactions. The Pauli matrices ($\hat{\sigma}$), the spin-exchange operator, and the delta function of Dirac $\delta(\vec{r}_1 + \vec{r}_2)$ are all mathematical terms. The energy of the total density of the nucleus is expressed as follows in the (SHF) model [19]:

$$E = E_{kin} + E_{Sky}\left(\rho_u + \tau_u + \vec{s}_u + \vec{j}_u + \vec{\mathfrak{H}}_u\right) + E_{Coul} + E_{pair} - E_{cm} \quad (11)$$

Here E_{kin} refers to energy of kinetic motion, E_{Sky} refers to Skyrme energy which include time even (nucleon ρ_u, kinetic energy τ_u and orbital–spin $\vec{\mathfrak{H}}_u$ and the odd-time (current $\vec{j}_u$, spin $\vec{s}_u$, and the vector of the kinetic energy $\vec{T}_u$) both densities and the pairing energy E_{pair} and the mass at the center E_{cm}. The u label refers to neutrons or protons. The Skyrme type of parametrization is taken to perform the calculations is (Sk35−Skzs*) [16].

The "mean square charge radius" is expressed by the formula [16]

$$\langle r_c^2 \rangle = \langle r_p^2 \rangle + \langle R_p^2 \rangle + \frac{N}{Z}\langle r_n^2 \rangle + \frac{3}{4}\left(\frac{\hbar}{M_p c}\right)^2 \quad (12)$$

$$\langle r_c^2 \rangle = \langle r_p^2 \rangle + 0.769 - \frac{N}{Z}0.1161 + 0.033 \quad (13)$$

Here r_p is the radius comes from the distribution of the proton point of the nucleus, R_p and r_n are the charge radii of the free proton and neutron, and the final term is known as the Darwin-Foldy term (0.033 fm^2).

RESULTS AND DISCUSSION

The longitudinal Coulomb charge $|F_{ch}(q)|^2$ and $|F(C2,q)|^2$ form factors for the ^{9}Be and ^{28}Si nuclei have been calculated by considering a truncated *spsdpf* model space with *wbt* effective interaction [20] with $(0 + 2)\hbar\omega$. The C0 and C2 from factors calculated for the ground state 3/2$^-$, the 5/2$^-$ (2.429 MeV) and 7/2$^-$ (6.380 MeV) for ^{9}Be, while the ground state 0$^+$ and 2$^+$ (1.779 MeV) state for ^{28}Si nucleus. In all proceeding figures (see Fig.1, panels a, b, c, and d), the dotted grass green and magenta curves display the results of the calculations of the Tassie and Bohr-Mottelson models using the valence model (MS) calculations only, while the solid red and blue curves show the calculations of Tassie and Bohr-Mottelson models including the core polarization effect by means of effective charge of protons and neutrons.

^{9}Be nucleus

The nucleus ^{9}Be is a neutron-halo with 4 protons and 5 neutrons considered as (4α+n) and it is stable with the ground state is 3/2$^-$. The effective charge at the zero-photon point considered in this work is taken from Ref. [21]. The longitudinal Coulomb charge $|F_{ch}(q)|^2$ and $|F(C2,q)|^2$ Form factors for the ground state have been calculated $\left(J_f^\pi = 3/2^-, T = 1/2\right)$ of the ^{9}Be as displayed in Figure 1 panels (a, b, c and d) by utilizing the Tassie and Bohr-Mottelson collective models. Due to large number of dimensions to be used in the model space *spsdpf,* a truncation has to be used, therefore the truncation is taken as $(0 + 2)\hbar\omega$ following the restriction adopted in Ref. [16].

The state (3/2$^-$ 1/2) at 0.000 MeV (g.s)

The C0 and C2 form factors and their sum (C0+C2) for the ground state 3/2$^-$ are depicted in Fig.1 , b, c, and d. The calculations were performed by using harmonic oscillator (HO) and Skyrme effective interactions (Sk35−Skzs*) for both Tassie and Bohr-Mottelson models. Panels a and b represent the Tassie model with the Harmonic Oscillator potential (HO) and Skyrme effective interaction (Sk35−Skzs*), while in c and d in Fig.1, represent Bohr-Mottelson model Tassie model with the Harmonic Oscillator potential (HO) and Skyrme effective interaction (Sk35−Skzs*). The effective charges for neutron and proton are 0.70e and 1.075e, respectively [21], while for Bohr-Mottelson estimated using Eq.1 as 0.17 and 0.4 for the proton and neutron respectively. The observed data from Ref. [22] for this state form factor. The data could not be reproduced in all regions of the momentum transfer using bare model space computations. Introducing the effective charge in both Tassie and Bohr Mottelson with account for the core polarization effect makes a remarkable improvement especially for Bohr Mottelson calculations shown in panel (a) where the solid bule line which is (C0+C2) form factor matches the measured data in all momentum transfer regions using HO as the residual effective interaction.

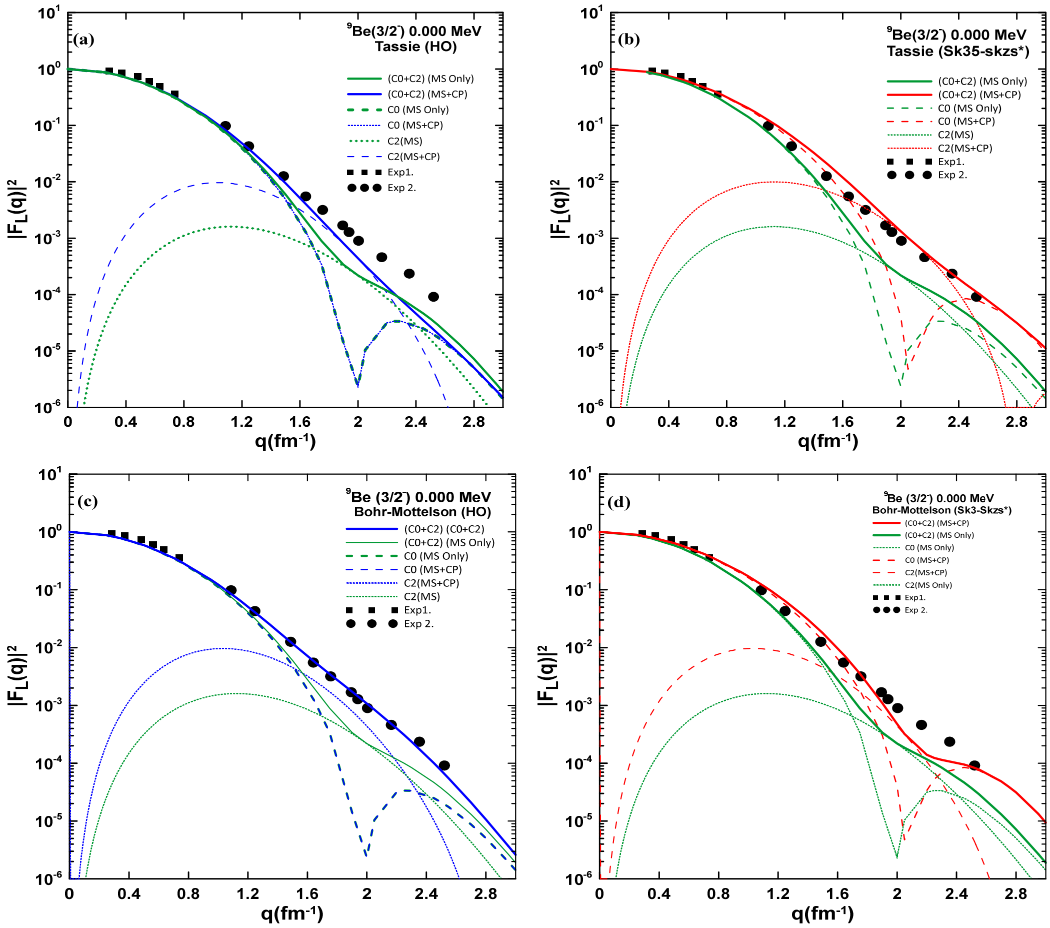

Figure 1. Longitudinal C0 and C2 form factors and their sum C0+C2 for the Tassie and Bohr-Mottelson models by using the Harmonic Oscillator potential (HO) and Skyrme effective interaction (Sk35−Skzs*) in comparison to the measured data [22].

The state ($5/2^-$ 1/2) 2.429 MeV

Figure 2 shows the calculation of the form factor for C2 for the state ($5/2^-$ 1/2) at E_x=2.429 MeV. The bare model space calculation for both Bohr-Mottelson collective model and Tassie model by using the Harmonic Oscillator potential (HO) and Skyrme effective interaction (Sk35−Skzs*) underestimates the measured data in all momentum transfer regions. The measured data for this state are taken from [22-24]. The form factor of Bohr-Mottelson model reproduces the high q values of the effective charge to consider the CP effects, explains the data up $q \geq 1.5fm^{-2}$.

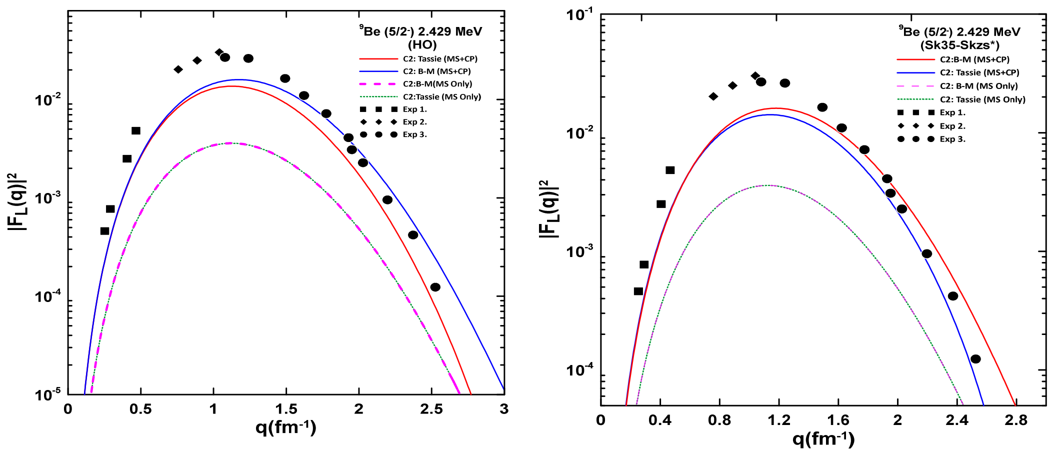

Figure 2. The C2 longitudinal form factors for the Tassie and Bohr-Mottelson models by using the Harmonic Oscillator potential (HO) and Skyrme effective interaction (Sk35−Skzs*) in comparison to the measured data [23, 24].

The state ($7/2^-$ 1/2) 2.429 MeV

The longitudinal C2 form factor for ($7/2^-$ 1/2) at E_x=6.380 MeV. The calculation of the model space for the Tassie and Bohr-Mottelson models at the peak value, it underpredicts the data by nearly a factor of three. The measured data are scattered therefore Introducing the effective charge in both Tassie and Bohr Mottelson to account for the core polarization effect aren't able to reproduce the measured data for these states. The measured data are taken from [22] (Fig. 3).

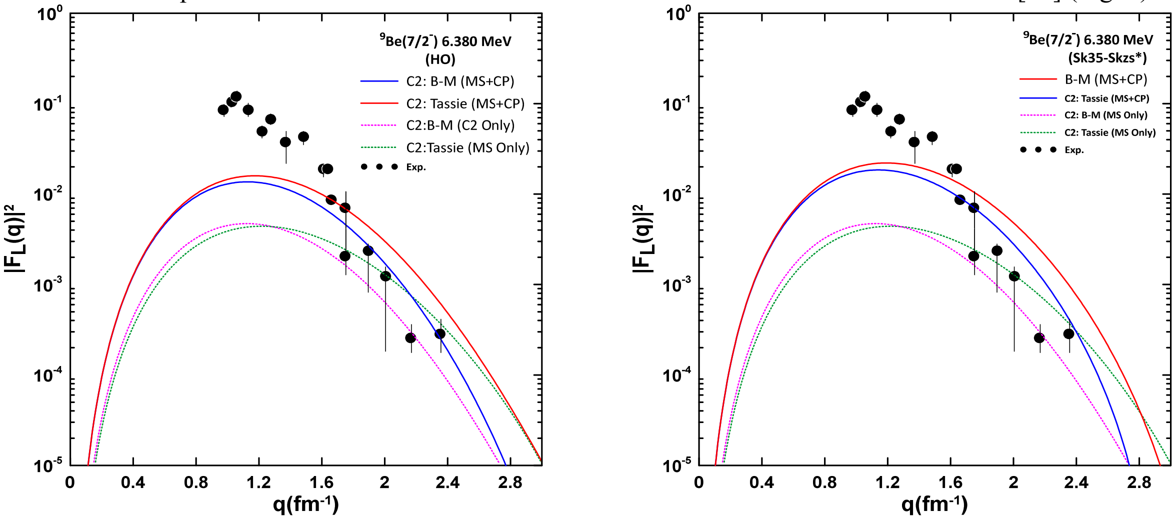

Figure 3. The C2 longitudinal form factors for the Tassie and Bohr-Mottelson models by using the Harmonic Oscillator potential (HO) and Skyrme effective interaction (Sk35−Skzs*) in comparison to the measured data [23, 24].

^{28}Si nucleus

The nucleus ^{28}Si is even-even nucleus with 14 protons and 14 neutrons and it is stable with the ground state is 0^+. The longitudinal Coulomb charge $|F_{ch}(q)|^2$ and $|F(C2,q)|^2$ form factors are calculated for the transitions to the ground state $\left(J_f^\pi = 0^+, T = 0\right)$ of the ^{28}Si as demonstrated in Figure 4 by utilizing the Tassie and Bohr-Mottelson collective models. Due to large number of dimensions to be used in the model space *spsdpf,* a truncation has to be used, therefore the truncation is taken as $(0+2)\hbar\omega$ following the restriction model from Ref. [16].

The state (0^+ 0) 0.000 MeV (g.s)

The C0 form factor for the ground state $0^+$is displayed in Fig.4. The calculations were performed by using Harmonic oscillator (HO) and Skyrme effective interactions (Sk35−Skzs*) for both Bohr-Mottelson collective model and Tassie model. In both Tassie and Bohr Mottelson models, the effective charge is included to account for the core polarization effect. The measured data is taken from [25]. The theoretical calculations of the model space only don't reproduce the measured data for all q values. The inclusion of the core polarization in the Bohr Mottelson model harmonic oscillator (HO) describes the experimental data very well up to momentum transfer $q \geq 1.5fm^{-2}$ and is able to reproduce the measured data very well and agrees with previous theoretical work in Ref. [6].

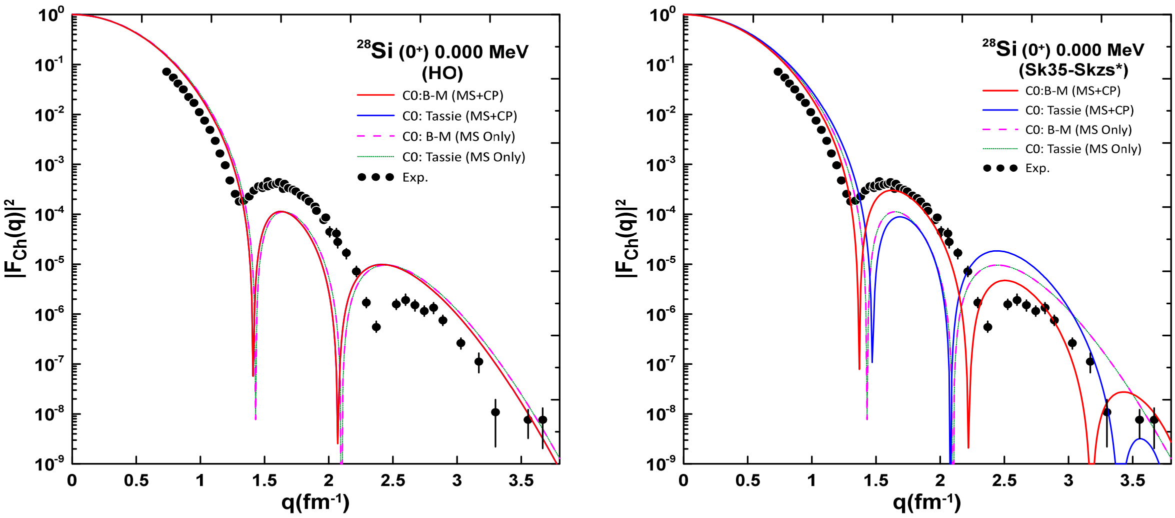

Figure 4. Longitudinal C0 form factors for the Tassie and Bohr-Mottelson models by using the Harmonic Oscillator potential (HO) and Skyrme effective interaction (Sk35−Skzs*) in comparison to the measured data [25].

The state (2^+ 0) 1.779 MeV

Figure 5 displays the longitudinal C2 calculations with their comparison to the measured data for the state (2^+ 0) at 1.779 MeV to the ground state (0^+ 0). The measured data is taken from [25] The model space $(0 + 2)$ only with Harmonic oscillator (HO) and Skyrme effective interactions (Sk35−Skzs*) for Bohr-Mottelson collective model and Tassie model underestimate the experimental data in the first maxima and second maxima. The inclusion of the core polarization in the Bohr Mottelson model harmonic oscillator (HO) and Skyrme effective interactions (Sk35−Skzs*) describe the experimental data very well up to momentum transfer $q \geq 1\ fm^{-2}$. The effects of core polarization enhance the C2 form factors at the first and second maximums, bringing the calculated values extremely near to the experimental data.

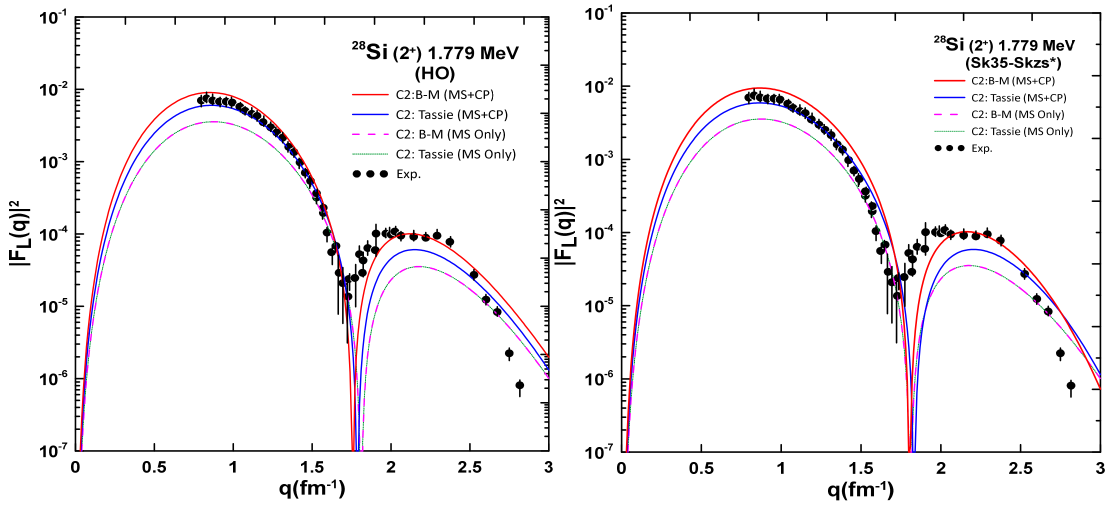

Figure 4. Longitudinal C2 form factors for the Tassie and Bohr-Mottelson models by using the Harmonic Oscillator potential (HO) and Skyrme effective interaction (Sk35−Skzs*) in comparison to the measured data [25].

CONCLUSION

The longitudinal charge $|F_{ch}(q)|^2$ and $|F(C2, q)|^2$ form factors the nuclei (^{9}Be and ^{28}Si) in the *p*-and *sd*-shells by utilizing the shell model calculations by considering the major shells 1s, 1p, 2s–1d, 2p–1f, including a partially inert core using the model space *spsdpf* with *wbt* effective interaction using the code NushellX@MSU were conducted. The residual interactions used in the calculation of the form factors are harmonic oscillator (HO) and Skyrme effective interaction (Sk35−Skzs*) by employing Tassie and Bohr-Mottelson collective models. The result of the form factors with $(0 + 2)\hbar\omega$ shell is not able to reproduce the data for all momentum transfer regions for both nuclei under study. Introducing the effective charges in both Tassie and Bohr Mottelson to account for the core polarization effect makes a remarkable improvement especially for Bohr Mottelson calculations.

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Conflicts of Interest. The Authors confirm that there are no conflicts of interest.

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ДОСЛІДЖЕННЯ ОБОЛОНКОВОЇ МОДЕЛІ ДЕЯКИХ p-ТА sd-ОБОЛОНКОВИХ ЯДЕР З ГАРМОНІЧНИМ ОСЦИЛЯТОРОМ ТА SKYRME ВЗАЄМОДІЯМИ

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У цьому дослідженні поздовжній заряд $|F_{ch}(q)|^2$ і $|F(C2,q)|^2$формфактори для ядер ^{9}Be і ^{28}Si, що лежать в *p* і *sd* оболонках, вивчаються за допомогою потенціалу гармонічного осцилятора (HO) та ефективна *Skyrme* взаємодії (Sk35−Skzs∗). C0 і C2 з факторів, розрахованих для основного стану 3/2-, 5/2- (2,429 МеВ) і 7/2- (6,380 МеВ) для ^{9}Be, водночас як основний стан 0+ і 2+ (1,779 МеВ) для ядра ^{28}Si. Розрахунки мікроскопічних збурень, які передбачають проміжне збудження однієї частинки, однієї дірки з орбіт ядра та MS на всі верхні орбіти з збудженнями n$\hbar\omega$, використовуються для створення ефективних зарядів, необхідних для врахування «ефекту поляризації ядра». Розрахунки моделі оболонки використовуються в розширеному просторі моделі для включення всіх орбіт 1s, 1p, 2s–1d, 2p 1f з усіканням (0 +2)$\hbar\omega$. Колективна модель Бора-Моттельсона та модель Тассі з правильно оціненими ефективними нейтронними та протонними зарядами враховуються для врахування ефекту внеску ядра. Оцінені форм-фактори порівнювали з наявними виміряними даними, і вони добре збігалися для більшості досліджуваних станів. Можна зробити висновок, що (0 +2)$\hbar\omega$ скорочення є дуже хорошим вибором для вивчення поздовжніх форм-факторів.

- Вибір потенціалу гармонійного осцилятора (HO) та ефективної Skyrme взаємодії (Sk35−Skzs∗) є адекватним для оцінки форми поздовжніх формфакторів.
- Оцінка ефективних зарядів на основі мікроскопічних збурень, які включають проміжні одночастинкові, однодіркові збудження від орбіт ядра та MS до всіх верхніх орбіт зі збудженнями n$\hbar\omega$, є адекватною.
- Скорочення (0 +2)$\hbar\omega$ виявляється дуже успішним для виконання дослідження.

Ключові слова: *модель оболонки; форм-фактор заряду; поздовжні форм-фактори; гармонічний осцилятор; Skyrme взаємодії*