East European Journal of Physics. **1.** 135-145 (2023)
DOI:10.26565/2312-4334-2023-1-17 ISSN 2312-4334

INSINUATION OF ARRHENIUS ENERGY AND SOLAR RADIATION ON ELECTRICAL CONDUCTING WILLIAMSON NANO FLUIDS FLOW WITH SWIMMING MICROORGANISM: COMPLETION OF BUONGIORNO'S MODEL[†]

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Received October 18, 2022; revised December 13, 2022; accepted December 23, 2022

The enriched thermal mechanisms and progressive of nanomaterial has enthused scientists to give devotion to this area in current days. The versatile and synthesizing utilization of such particles embrace energy production, solar systems, heating and cooling monitoring processes, renewable energy systems, cancer treatments, hybrid-powered motors and Nano electronics. Furthermore, in this era of biotechnology and bioengineering, the bio convection of Nano fluids provides for some enthralling applications, such as enzymes, biosensors and biofuels. With such magnetic applications and attentions. A mathematical model is presented for evaluating the electrical conducting Williamson nano fluid with heat and mass transfer over a porous stretched sheet in the existence of bioconvection. The bioconvection of swimming microorganisms, thermal radiation, thermal conductivity and Arrhenius energy are new facets of this investigation. The higher order non-linear governing partial differential equations (PDEs) are solved by applying appropriate similarity variables and resulting couple of ordinary differential equations (ODEs) is produced. The developing set of ODEs is solved numerically by utilizing well known shooting technique with ND solve command in Wolfram MATHEMATICA and compare the result with pvb4c code in MATLAB. The graphs for different physical quantities of interest together with non-dimension velocity, temperature, concentration and density of micro-organisms profiles are discovered for involving parameters like magnetic parameter, Brownian motion, Rayleigh number, Peclet number, Bioconvective Lewis number, parameter of thermophoresis and buoyancy ratio parameter. The influence of numerous parameters on flow and heat transfer characteristics are debated.
Keywords: *Activation energy; Williamson Nano fluids; Chemical reaction; Thermal Radiation; Shooting method; Extending sheet; Thermal conductivity*
PACS: 47.10.-g, 47.10.A-, 47.10.ad

INTRODUCTION

Due to its enormous applications, the mass and heat transfer of boundary layer flow of non-Newtonian fluids through permeable medium due to a extending plate is of significant interest to researchers, engineers and scientists. Some good illustrations of its utilizations are the cooling of nuclear reactors, pipe industry, thinning of copper wire, annealing, solar collection, extraction of metals and extrusion process. The quick and convoluted process in tiny devices and gigantic machinery have generated a large problem of thermal discrepancy. Various additional approaches, like as fans and fins are utilized, but their utility is limited due to the enormous size. Maripala and Kishan [1] considered the effect of chemical diffusion and solar radiation on magneto un-steady flow and heat transport of nanofluid through a porous shrinking plate. Their outcome shows that the energy distribution diminishes with boosted in the suction parameter while nano concentration distribution rise. Over stretched surfaces, the effect of mixed convective mass and heat transport of MHD nanoparticles inserted in permeable sheet. Thermal boundary layer and energy profile diminish when the Prandtl number and dimensionless mass free convection parameter grow, according to their findings investigated by [2-8]. In 1995, Choi [9] proposed that nano-sized particles dispersed in a base fluid, dubbed nanofluid, have a higher heat transfer capacity than fluids lacking nano-sized particles. The present and prospective utilization of nano-sized particles in fluids is discussed by [10]. A single model cannot account for non-Newtonian fluid features. Also, the fundamental Naiver-Stokes equations cannot explain the rheological features of non-Newtonian fluids. A ample models have been established to address this issue. The rheological models that were proposed like Carreau, Ellis, power law, Cross and Williamson fluid model, etc. The Williamson fluid model is an example of a non-Newtonian fluid (liquid/gas) model with shear retreating behaviour, and it was predicted by Williamson [11]. Investigations [12–16] might be mentioned as current inquiries into the flow of magneto Williamson fluid.

The minimal quantity of strength required by chemical reactants to tolerate a significance chemical diffusion is known as activation energy. Bestman [17] investigated the inspiration of heat transfer and energy activation on Natural convection boundary layer through porous plate. Hamid et al. [18] explore the effect of chemical diffusion on time depended flow of electrical conducting Williamson nanofluid in the occurrence of Arrhenius energy. Anuradha and Sasikala [19] examined the effect of energy activation and binary chemical reaction on magneto flow of nanofluid through porous shrinking plate with convective flow. Dhlamini et al. [20] established the mathematical model for mixed convective nanofluid flow in the existence of chemical diffusion and energy with convective boundary conditions. Awad et al. [21] scrutinized the insinuation of Arrhenius energy and binary chemical diffusion on rotating flow of time in depended nanofluid in the occurrence of energy activation. Mustafa [22] and Huang [23] also looked into the effect of

[†] ***Cite as:*** M. Jawad, East Eur. J. Phys. **1**, 135 (2023), https://doi.org/10.26565/2312-4334-2023-1-17

Arrhenius energy on electrical conducting flow of nanoparticle passing through a both permeable horizontal and vertical cylinder. Many researchers like [24–27] describes additional research on the influence of energy activation on MHD flow of non-Newtonian fluids under various geometries. Their figures show that the concentration of nanoparticles rises with rise in activation energy as well.

The sensation of alive microorganisms deeper than water swimming upward in suspensions is known as bioconvection. Bioconvection have significance utilization in biotechnology and biological systems such as enzyme biosensors, purify cultures and living cells [28]. Through a horizontal tube, Raees et al. [29] investigated an time depended flow of magnetized bioconvective nanofluid containing swimming gyrotactic microorganisms. The bioconvective flow of MHD magnetized nano fluid with mass and heat transfer with swimming microorganisms across a rounded vertical cone was quantitatively discovered by Siddiqa et al. [30]. Abbasi et al. [31] described in detail the bioconvection stream of viscoelastic nanoparticles caused by motile microorganisms passing through a turning extending disc with convective boundary condition and zero mass flux as well as prominent parameters' inspirations on Nusselt number, temperature, local density, velocity and Sherwood number. Chu et al. [32] analyzed the impact of stream of bioconvection on electrical conducted fluid over extending plate with chemical reaction, Brownian motion, thermophoresis diffusion, activation energy and significance of motile microorganisms are taken into account. The consequences of a nonlinear thermal radiation and magnetic field on bioconvective magnetized nanofluid flow via the upper surface of a paraboloid of revolution were studied by Makinde et al. [33]. Henda et al. [34] looked considered the magnetic bioconvection flow of time depended fluid past through nonlinear expanding cylinder with a heat source, nonlinear thermal radiation and Arrhenius energy. This investigation illustrates the electrical conducting flow of Williamson nanofluid flow having the insertion of swimming microorganisms through a stretchy sheet. The influence of energy activation, chemical reaction parameter, Buoyant force, and thermal conduction is also a part of this study. The effect of gravitational body forces is also taken under attention in this analysis. The shooting method has been applied to compute the numerical outcomes. Furthermore, a graphical illustration of numerous prominent parameters are also presented in this research.

PROBLEM STATEMENT

Steady two-dimensional inviscid flow of magneto Williamson nanofluid with thermal radiation and Arrhenius energy embedded in a permeable stretchable plate is considered in the occurrence of swimming microorganism and thermal conductivity. The fluids flow owing to a extending sheet with non-zero mass flux. Because of the stretched sheet, it is expected that the flow will behave linearly. Here u and v are the part of velocity along x and y directions. Our mathematical problem characterizes the fundamental equations of mass conservation, equation of flow, temperature equation, nanofluid concentration equation and motile density equation in Cartesian coordinates. The mathematical highly nonlinear governing equations for the stated problems are given below [35-36]:

The equation of mass conservation

$$u_x + v_y = 0. \tag{1}$$

The Equation of momentum

$$uu_x + vu_y = \nu u_{yy} + \frac{1}{\rho_f}\begin{bmatrix} (1-C_\infty)(T-T_\infty)\beta\rho_f - \\ (\rho_p - \rho_f)(C-C_\infty) - \\ \gamma(n-n_\infty)(\rho_m - \rho_r) \end{bmatrix} g - \frac{\sigma_e B_0^2 u}{\rho_f} + \Gamma u_y u_{yy} - \frac{\nu u}{K}. \tag{2}$$

Subjected to

$$v = 0,\ u = ax\ ,\ u \to U = ax$$

The Equation Temperature

$$uT_x + vT_Y = \frac{k}{(\rho c)_f}\left(T_{yy}\right) + \frac{(\rho c)_p}{(\rho c)_f}\left\{\frac{D_T}{T_\infty}\left(T_y\right)^2 + D_B C_y T_y\right\} - \frac{1}{(\rho c)_f} q_{r_y} + \frac{Q_0}{(\rho c)_f}\left(T - T_\infty\right). \tag{3}$$

Subjected to

$$T = T_W \ \ as\ \ y \to 0\ ,$$

$$T \to T_\infty \ \ as\ \ y \to \infty.$$

The Equation Concentration

$$uC_x + vC_y = D_B C_{yy} + \frac{D_T}{T_\infty}T_{yy} - K_0\left(C - C_\infty\right)\left(\frac{T}{T_\infty}\right)^n \exp\left(-\frac{E_a}{kT}\right). \tag{4}$$

Subjected to

$$C = C_W \ \ as\ \ y \to 0\ .$$

$$C \to C_\infty \;\; as \;\; y \to \infty .$$

The Equation of Density of microorganism

$$un_x + vn_y + \frac{bW_c}{C_W - C_\infty}\left[n_y C_{yy} \right] = D_m n_{yy} . \tag{5}$$

Subjected to

$$n = n_W \;\; as \;\; y \to 0 ,$$
$$n \to n_\infty \;\; as \;\; y \to \infty .$$

In these governing equations the component of velocity u and v are assumed in x and y direction respectively, ρ_f is the density of base fluid, ν is the viscosity, σ^* is the electrical intensity, k^* is the absorption constant, β the volume expansion coefficient, g is gravity, ρ_p density of microorganisms particles, $\dot{\gamma}$ represents the volume of the microorganism, n is the concentration of the microorganism in the fluid T is temperature of nanofluid, α is thermal diffusivity, $(\rho c)_f$ heat capacity of liquid, $(\rho c)_p$ effective heat capacity of nanoparticles, q_r is radiative heat flux, W_c is the maximum cell swimming speed, D_B for Brownian diffusivity, D_T for thermophoretic diffusion coefficient, k_c chemical reaction parameter and D_m is the diffusivity of microorganisms.

where $k_c\left(C - C_\infty\right)\left(\frac{T}{T_\infty}\right)^n \exp\left(-\frac{E_a}{kT}\right)$ represents the Arrhenius expression. The temperature dependent thermal conductivity is expressed as

$$k = k_\infty\left(1 + \varepsilon\frac{T - T_\infty}{T_{w-}T_\infty}\right), \tag{6}$$

According to radiative heat flux theory

$$q_r = \frac{-4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{7}$$

where k^* stands for absorption coefficient and σ^* denotes Stefan Boltzman Constant. Using expansion of Talyer's series about T_∞ we get

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 . \tag{8}$$

The utilization of Eqs. (11-12) in (4), we have

$$uT_x + vT_Y = \frac{k}{(\rho c)_f}\left(T_{yy}\right) + \frac{(\rho c)_p}{(\rho c)_f}\left\{\frac{D_T}{T_\infty}\left(T_y\right)^2 + D_B C_y T_y\right\} + \frac{16\sigma^* T_\infty^3}{3k^*(\rho c)_f}\frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{(\rho c)_f}\left(T - T_\infty\right). \tag{9}$$

Let us implement the following similarity approaches, to transform partial differential equations (PDEs) to Ordinary differential equations (ODEs) [30-31]:

$$\psi = (a\nu)^{\frac{1}{2}} x f(\eta),\; \eta = \left(\frac{a}{\nu}\right)^{\frac{1}{2}} y, \text{ and } C = C_\infty + \left(C_w - C_\infty\right)\phi(\eta),\; T = T_\infty + \left(T_w - T_\infty\right)\theta(\eta),\; n = n_\infty + \left(n_w - n_\infty\right)\chi(\eta),$$

where $\psi(x, y)$ is the stream line function defined as $u = \psi_y$ and $v = -\psi_x$, which tropically fulfills the equation of mass conservation and η is the similarity variable. Equations (02) – (05) reduce to

$$f''' + ff'' - f'^2 + Wef'''[\eta]f''[\eta] - (M + K_1)f' + \lambda\left(\theta - N_r\phi - N_c\chi\right) = 0, \tag{10}$$

$$\left(1 + \frac{4Rd}{3}\right)\theta''[\eta] + p_r f[\eta]\theta'[\eta] + p_r Nt\theta'^2[\eta] + p_r Nb\theta'[\eta]\phi'[\eta] + p_r\Delta\theta[\eta] = 0, \tag{11}$$

$$\phi''[\eta] + \Pr Lef[\eta]\phi'[\eta] + \left(\frac{N_t}{N_b}\right)\theta''[\eta] - \Pr Le\gamma\left(1 + \delta\theta[\eta]\right)^n \exp\left(\frac{-E}{1 + \delta\theta}\right)\phi[\eta] = 0, \tag{12}$$

$$\chi''[\eta]-P_e\chi[\eta]\phi'[\eta]-P_e\sigma\phi''[\eta]+P_e\chi'[\eta]\phi''[\eta]+S_c f[\eta]\chi'[\eta]=0 \tag{13}$$

Boundary condition becomes:

$$f(\eta)=S, f'(\eta)=\lambda, \chi(\eta)=1, \theta(\eta)=1, \phi(\eta)=1 \;\; as \;\; \eta\to 0 \tag{14}$$

$$f'(\eta)=1,\; \theta(\eta)=0,\; \phi(\eta)=0,\; \chi(\eta)=0 \;\; as \;\; \eta\to\infty \tag{15}$$

where $\gamma=\frac{K_0}{a}$ is chemical reaction parameter, $Le=\frac{\nu}{D_B}$ Lewis number, $\delta=\frac{T_w-T_\infty}{T_\infty}$ is temperature difference, $\lambda'=\frac{\beta\dot{\gamma}(1-C_\infty)(T_w-T_\infty)x^3}{aU_w}$ mixed convection parameter, $E=\frac{E_a}{kT_\infty}$ is activation energy, $Rd=\frac{4\sigma^*T_\infty^3}{k^*k_\infty}$ is thermal Radiation, $Nr=\frac{(\rho_p-\rho_f)(C_w-C_\infty)}{\beta\rho_f(1-C_\infty)T_\infty\beta}$ is buoyancy ratio parameter, $K_1=\frac{\nu}{aK}$ is porosity parameter, $P_r=\frac{\nu_f}{\alpha}$ is Prandtl number, $Nc=\frac{\dot{\gamma}(\rho_m-\rho_f)(n_w-n_\infty)}{\beta\rho_f(1-C_\infty)T_\infty}$ Rayleigh number, $\sigma=\frac{N_\infty}{N_w-N_\infty}$ is microorganism concentration difference, $Nb=\frac{D_B\tau(C_w-C_\infty)}{\nu}$ is Brownian motion, $\Delta=\frac{Q_0}{a(\rho_C)_f}$ is heat generation/absorption coefficients, $Nt=\frac{D_T\tau(T_w-T_\infty)}{T_\infty\nu}$ is thermophoresis parameters, $Pe=\frac{bW_c}{D_m}$ is Peclet number and $Sc=\frac{\nu}{D_n}$ is Schmidt number.

The physical quantities of interest are defined as

$$C_f=\frac{\tau_w}{\rho U_w^2}, Nu_x=\frac{xq_w}{k(T_w-T_\infty)}, Sh_x=\frac{xq_m}{D_B(C_w-C_\infty)}, Nn_x=\frac{xq_m}{D_n(N_w-N_\infty)} \tag{16}$$

$$C_f\,\mathrm{Re}_x^{1/2}=-f''(0), Nu_x\,\mathrm{Re}_x^{1/2}=-\theta'(0), Sh_x\,\mathrm{Re}_x^{1/2}=-\phi'(0), Nn_x\,\mathrm{Re}_x^{1/2}=-\chi'(0), \tag{17}$$

where $\mathrm{Re}_x^{1/2}=U_w\, x/\upsilon$ is the local Reynolds number.

NUMERICAL TECHNIQUE

In a daily life many mathematical models of equations are highly nonlinear differential equation. We knew that exact solution of extremely nonlinear differential equations is not usually possible. In case of boundary value problem, the shooting method is one of best and well know scheme among all other methods. Another characteristic of current method is to find boundary conditions by utilizing initial approximations. This procedure is straightforward sensitive and free from error or complexity.

CODE OF VALIDATION

Table 1 provides a critical study of the current findings of $-\theta'(0)$ and $-\varphi'(0$ for the different value of Nb (Brownian motion parameter) utilizing the bvp4c package in MATLAB. The critical study of these numerical findings in Table 1 reveals that the scheme is valid, M = 0.5, λ = 0.1, γ = 0, Nc = σ = 0.3, Sc = Pe = Le = 0.2 and Pr = 7. The assessment of outcomes in Table 1 shows the astonishingly considerable compositions of the current inquiry with the bvp4c (MATLAB) results, which motivates the investigator to tackle this problem with changes of nonlinear thermal radiation and chemical diffusion effects using a well-known shooting approach.

Table. 1. Comparison of $-\theta'(0)$ and $-\varphi'(0)$ with different value of Nb.

Parameter	bvp4c	Present [ND solve]	bvp4c	Present [ND solve]
$\boldsymbol{Nb}$	$-\theta'(0)$		$-\varphi'(0)$	
0.1	1.15531	1.15531	0.71015	0.71015
0.2	0.82268	0.82268	0.42569	0.42569
0.3	0.56502	0.56502	0.22570	0.22570
0.4	0.37419	0.37419	0.09421	0.09421
0.5	0.23917	0.23917	0.01730	0.01730

RESULTS AND DISCUSSIONS

Where this section is equipped to explore the act of non-dimensional velocity profile $f'(\eta)$, energy profile $\theta(\eta)$, nanofluid concentration profile $\phi(\eta)$ and density $\chi(\eta)$ under the influence of several prominent parameters like chemical diffusion parameter γ, temperature difference δ, mixed convection parameter λ', energy activation E, thermal Radiation Rd, buoyancy ratio parameter Nr, porosity parameter K_1, Prandtl number P_r, Nc Rayleigh number, microorganism concentration difference σ, Brownian motion Nb, heat generation/absorption coefficients Δ, thermophoresis parameters Nt, bioconvection Lewis number Le, Harman number M and Peclet number Pe. Fig. 1 is demonstrated to assess the influence of Hartman number M on the velocity function $f'(\eta)$. Figure 1 depicts that the supplementing values of M causes retardation in the velocity profile Lorentz forces that are resistive forces are included in Hartman number. As M boost up, the Lorentz force enhanced causing in resistance of the flow of liquid as a result of velocities decline. The effect of Williamson parameter We against velocity distribution is plotted in Fig. 2 the increment in We decreases the fluid speed $f'(\eta)$, it is due to occurrence of buoyancy forces. An inverse relation between stretching λ and velocity field $f'(\eta)$ is obtained by Fig. 3, an increment in values of λ decreases the curve of $f'(\eta)$. Inspiration of Mixed convection Parameter on velocity distribution is plotted in Figure (4). It shows that increase the value of Mixed convection parameter decease velocity curve; it is expected to existence of buoyancy forces. A reverse relation between Nr and $f'(\eta)$ is obtained by Fig. (5), an increment in values of Nr as a result decreases the curve of velocity component $f'(\eta)$ same impact shows for Nc and porosity K are delegated in figures (6-7). Fig. 8 illustrates the change of thermophoresis parameter Nt on temperature distribution. The figure depicts that $\theta(\eta)$ is the increasing functions of thermophoresis parameter for some growing values of Nt. The increasing value of Nt results to raises the thermal conductivity of liquid. The heavy-duty thermal conductivity liquid comprises advanced temperature field $\theta(\eta)$. In thermophoresis, tiny particles of fluid are dragged from hot surface to cold. Thus, caused by departure of many tiny particles from hot surface temperature rises. Fig. 9 indicates the conduct of temperature profile with respect to parametric values of diffusivity ratio *Nb*. When a gradual increment is done in the morals of diffusivity ratio *Nb*, deceleration is obtained in the temperature function. Fig. 9 shows the consequence of shrinking parameter λ on temperature profile. The decrease in the principles of λ in the stretching case resulted in reducing the temperature contour $\theta(\eta)$. Figure (10-11) describes the prominence of porosity parameter K and heat generation/absorption parameter on temperature field $\theta(\eta)$.

The enhancement in K and Δ results in much convective heat transfer and concentration rate. Henceforth rise in the distribution for temperature of fluid. Temperature distribution of nanoparticles is enhanced because of the high temperature. The influence of Prandtl number on temperature field $\theta(\eta)$ illustrated in figure (12). Temperature of nanoparticles drop because of enhancement in Pr. Prandtl number is termed as ratio among thermal conductivity of fluid and thermal diffusivity of a fluid. In consequence, minimum value of Prandtl number consequences in the maximum thermal diffusivity while this causes lesser boundary layer thickness and temperature. Figure (13) demonstrates the impact of thermal radiation Rd on temperature field. These figures depict that $\theta(\eta)$ is the increasing functions of radiation parameter for some growing values of Rd. Fig. 14 displays the effect of stretching or shrinking sheet parameter on concentration profile. On increasing values of λ, a decreasing behavior of concentration is obtained. The Impact of chemical diffusion on concentration profile is shown on Figures (15). The curve of concentration distribution is increased as value of γ increased.

Figure (16) illustrates the influence of thermophoresis parameter Nt on concentration field. These figures depict that $\phi(\eta)$ is the increasing functions of thermophoresis parameter for some growing values of Nt. The increasing value of Nt results to raises the thermal conductivity of liquid. The heavy-duty thermal conductivity liquid comprises advanced concentration profile $\phi(\eta)$. In thermophoresis, tiny particles of fluid are dragged from hot surface to cold. Thus, caused by departure of many tiny particles from hot surface temperature rises and this high temperature points to an increment in the concentration. The concentration profile grows faster and once a minor change, it falloffs. The description for significance of Brownian motion parameter Nb on temperature field of nanoparticles is explored in Figure (17). In Fig. (18), diffusivity ratio parameter *Nb* is depicted to show its behavior on concentration profile. A significant downfall in the curve of concentration profile is obtained when gradual increase is done in diffusivity ratio parameter *Nb.* The Impact of Le on concentration profile is shown on figures (19). The curve of concentration distribution is increased as value of Le increased. Fig (20) denotes the consequences of parameter E called energy activation parameter on concentration profile $\phi(\eta)$. By enhancing the values of E concentration profile boosted up. Fig (21) represents the effects of parameter σ called microorganism concentration difference on density profile $\chi(\eta)$. By enhancing the values of σ density profile retarded. From Figure (22), it is demonstrated that within the increment in bio-convection Peclet number Pe, the density $\chi(\eta)$ is retarded. Here the extreme rapidity of cell-swimming is enriched by raise the value of Pe. This advanced rapidity of cell-swimming is accountable in the lesser performance of $\chi(\eta)$. The graph of Schmidt number Sc on motile density field is displayed in Fig. 23. Density function of motile microorganisms is decreased for gradual increase in Schmidt number.

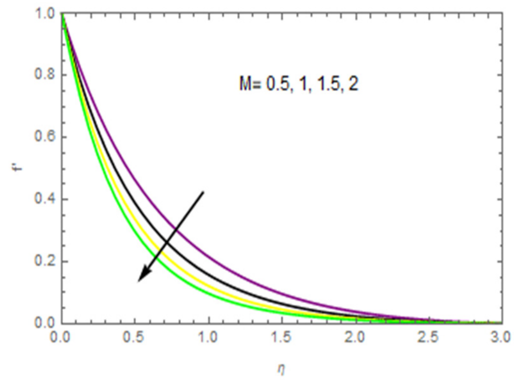

Figure 1. Influence of M on velocity field

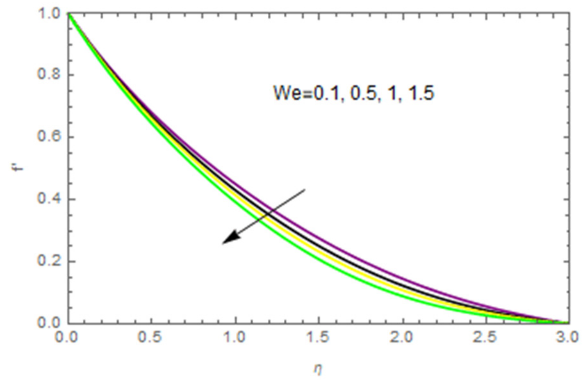

Figure 2. Inspiration of We on velocity variable

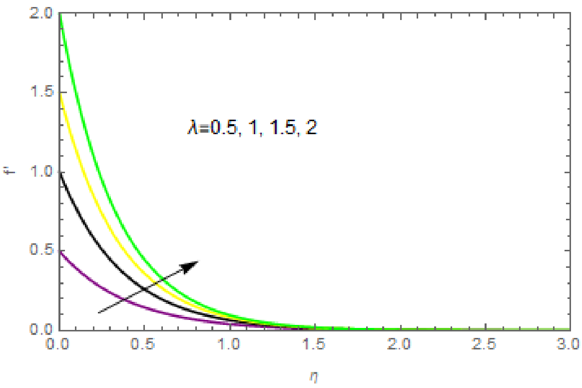

Figure 3. Effect of Stretching parameter on velocity function

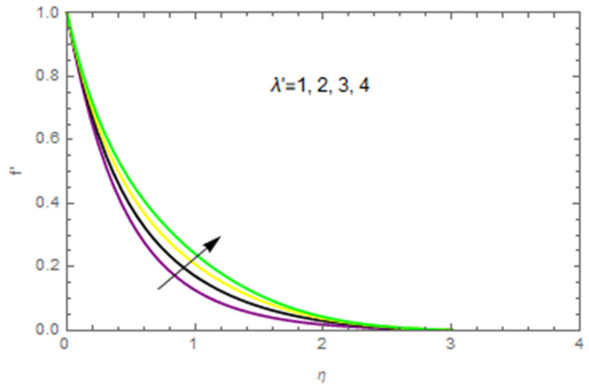

Figure 4. Impact of Mixed convection Parameter on velocity Distribution

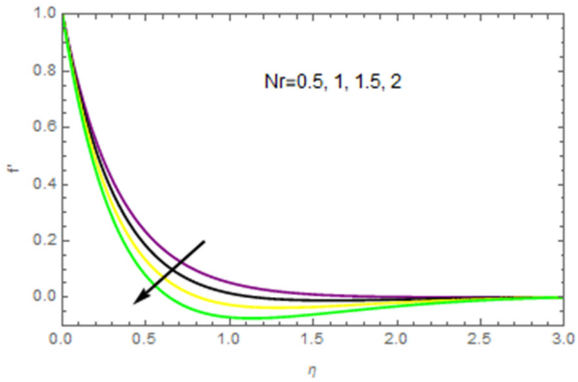

Figure 5. Change of Nr on velocity Distribution

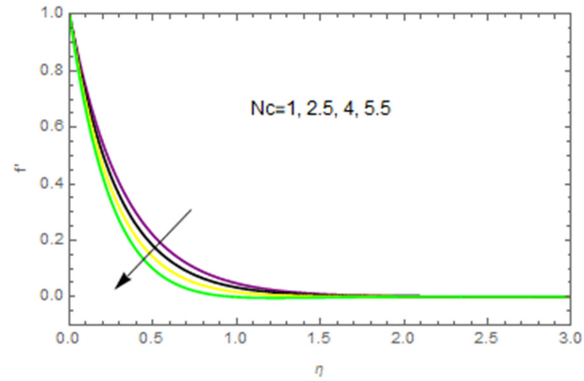

Figure 6. Change of Nc on velocity Distribution

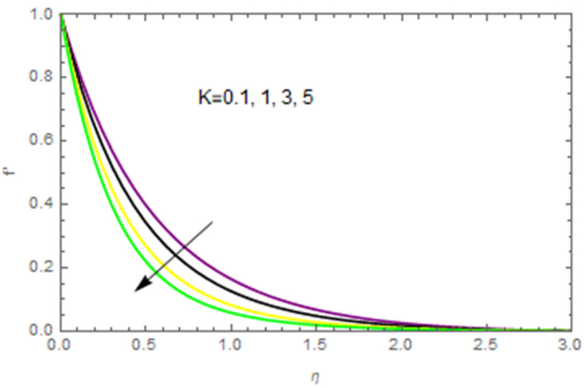

Figure 7. Influence of K on velocity Distribution

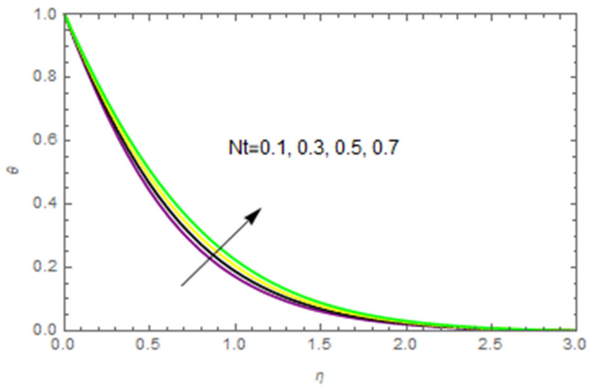

Figure 8. Influence of Nt on Energy Distribution

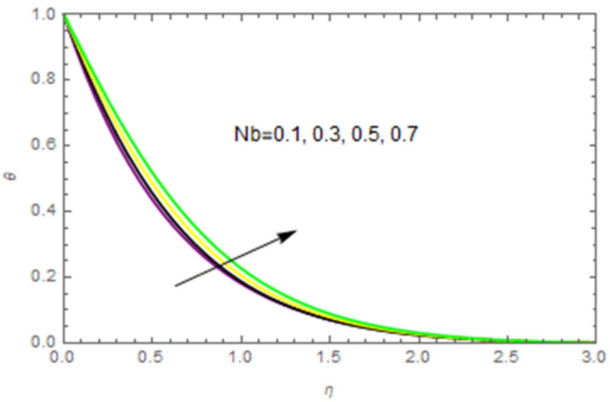

Figure 9. Change of Nb on Energy Distribution

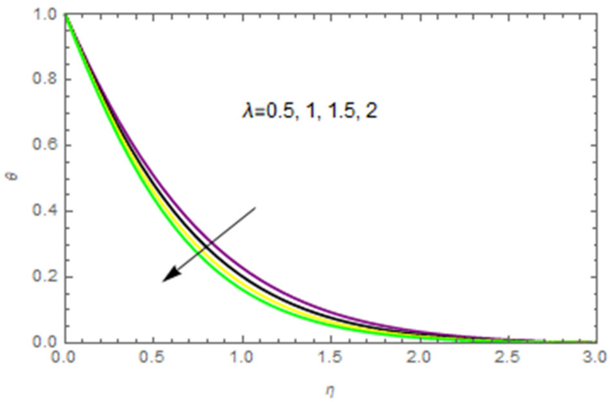

Figure 10. Impact of stretching parameter on Energy field

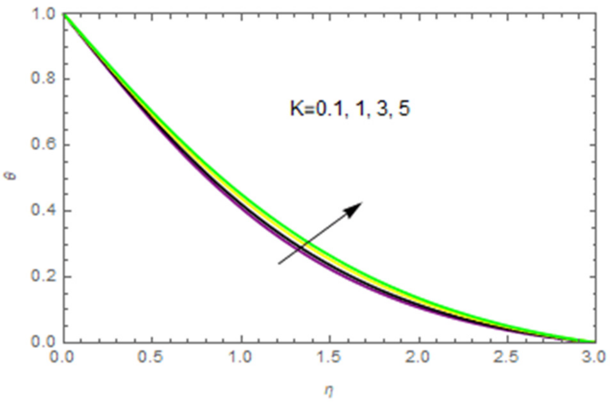

Figure 11. Influence of permeable parameter on Energy field

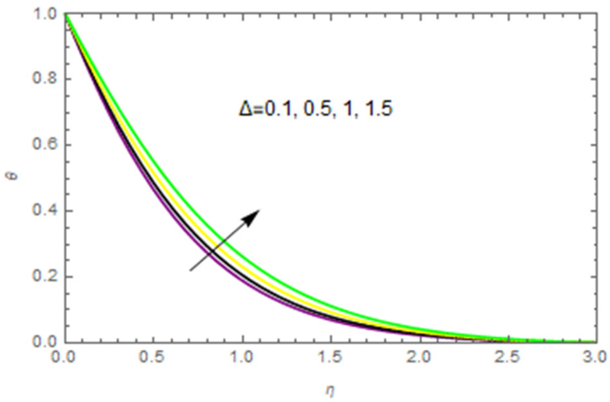

Figure 12. variation of Heat on Energy Distribution

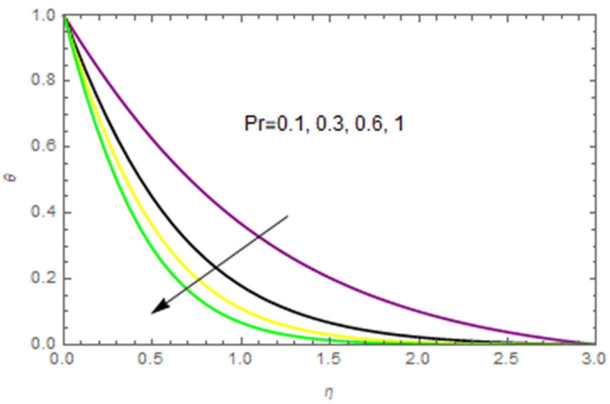

Figure 13. variation of Pr on Energy profile

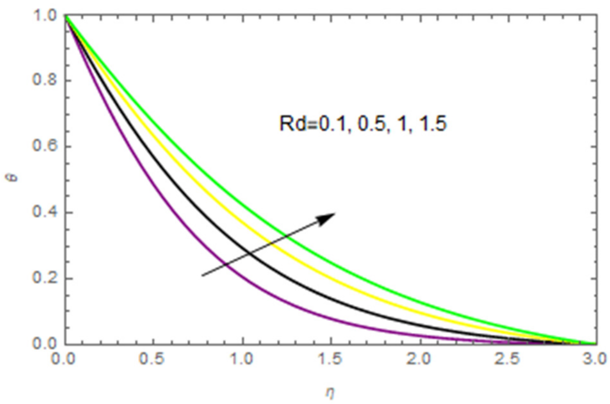

Figure 14. Influence of Rd on Energy function

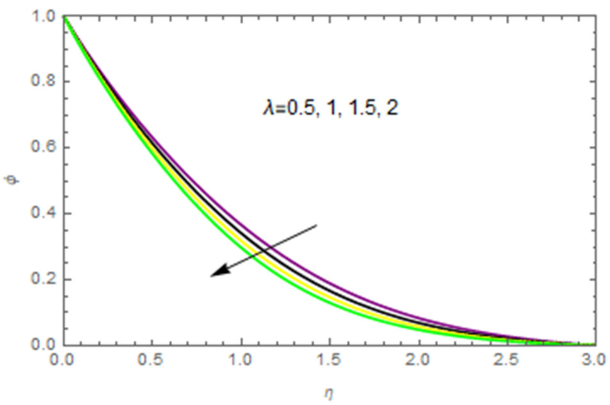

Figure 15. Impact of stretching parameter on Concentration Distribution

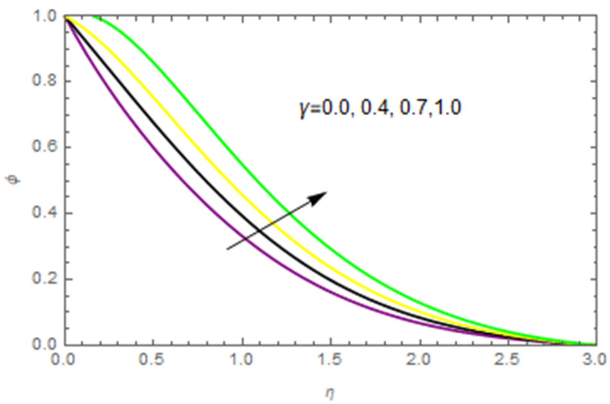

Figure 16. Influence of Chemical Reaction on Concentration Distribution

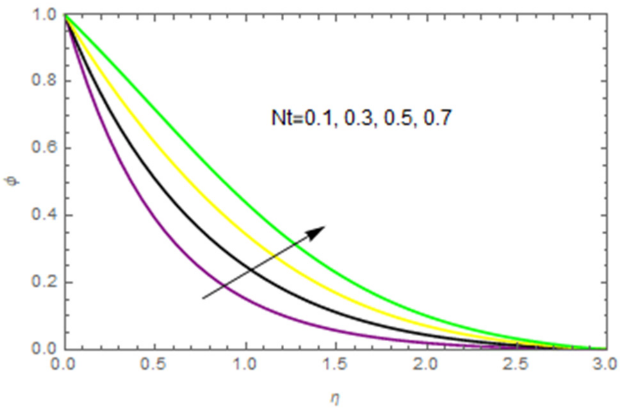

Figure 17. Influence of Nt on Concentration variable

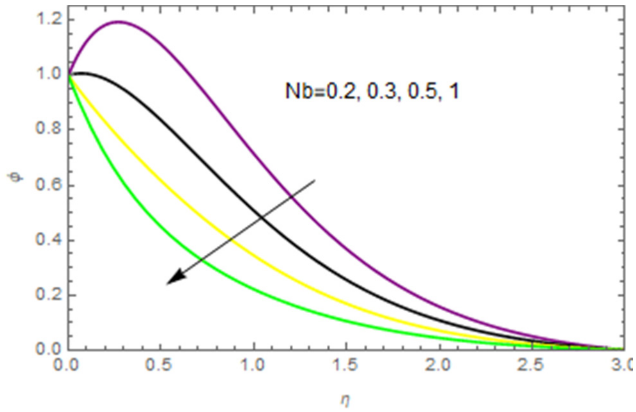

Figure 18. Variation of Nb on Concentration field

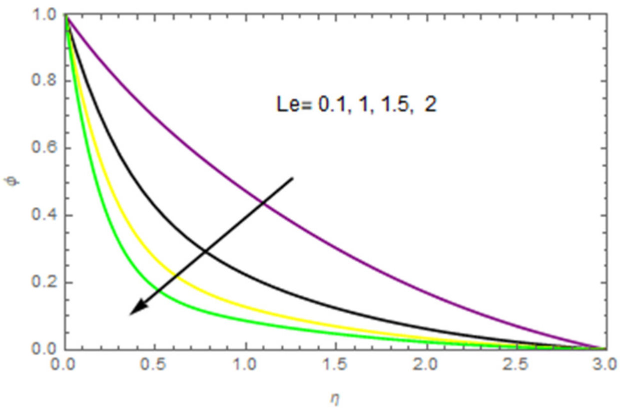

Figure 19. Influence of Le on Concentration Distribution

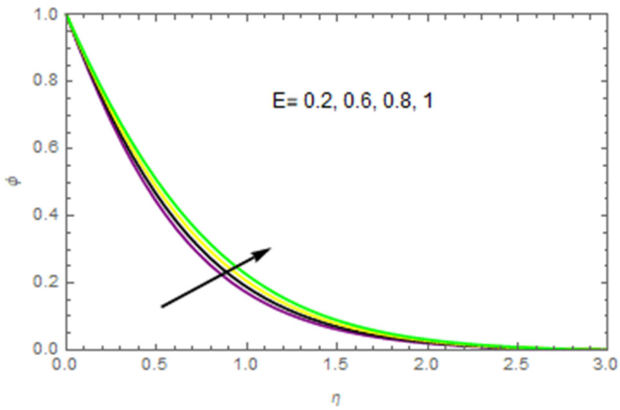

Figure 20. Influence of E on Concentration field

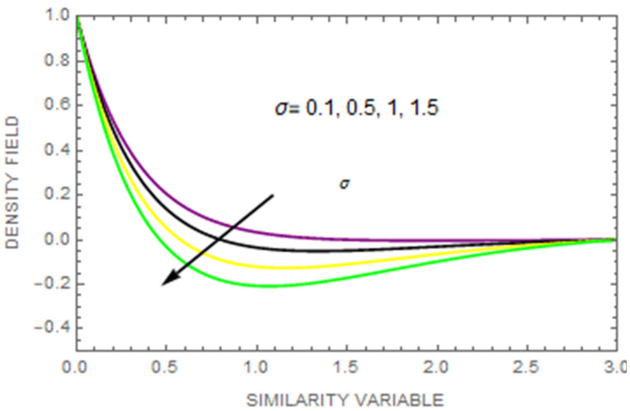

Figure 21. Influence of Microorganism rotation on Density field

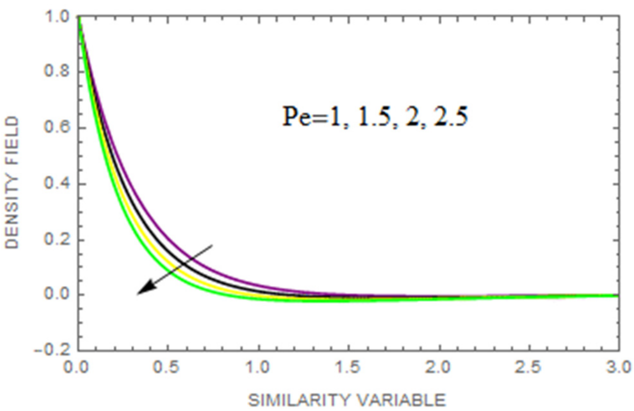

Figure 22. Variation of Pe on Density field

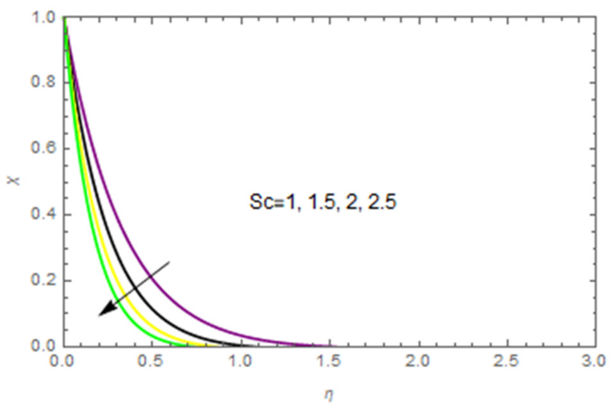

Figure 23. Influence of Sc on Density Distribution

Table 2. Nusselt number, local Sherwood number and density number at the stretching walls using ND solve command in Mathematica.

M	λ	σ	Pe	S	Pr	Nb	Nc	γ	Le	Sc	$N_uRe_x^{\frac{-1}{2}}$	$S_xRe_x^{\frac{-1}{2}}$	$Nn_xRe_x^{\frac{-1}{2}}$
	0.3	0.2	2.0	1	1.0	0.3	0.3	0.1	0.4	0.4	1.16659	0.56552	4.32923
1.0													
2.0											1.15123	0.40950	3.21537
3.0											1.07338	0.16075	1.04172
	0.2										0.17512	0.45780	2.94381
	0.6										0.17228	0.44645	2.85185
	0.8										0.16902	0.41387	2.74916
		0.5									1.13333	0.40568	1.36869
		0.9									1.13333	0.40570	1.41039
		1.3									1.13333	0.40570	1.45210
			1.0								1.13733	0.22030	0.43075
			1.5								1.13733	0.22030	0.47950
			2.0								1.13733	0.22030	0.52876
				1							2.16131	1.43887	4.26398
				2							2.16042	1.43678	4.24719
				3							2.15951	1.43469	4.22999
					1.0						0.45077	0.00966	-0.74797
					2.0						0.68383	-0.08266	-1.23763
					3.0						0.84725	-0.07903	-1.04744
						1					1.15746	0.43021	3.19426
						2					0.82402	0.90844	7.04756
						3					0.56577	1.04348	8.14001
							0.5				1.15742	0.43021	3.19438
							1				0.95049	0.10051	0.67048
							1.5				0.86338	0.02481	0.16697
								0.1			1.15752	0.43031	3.19430
								0.5			1.05198	0.48546	3.63174
								1.0			0.99724	0.51466	3.86424
									0.5		1.08467	0.35129	2.43540
									1.0		0.93459	0.66762	5.01487
									1.5		0.83409	0.97854	7.05759
										1	0.82405	0.90852	7.04755
										2	0.89141	0.75773	5.85983
										3	0.92656	0.68164	5.26339

CONCLUSION

The major findings are given below:

- The flow speed distribution improves with mixed convection parameter λ'. and diminishes with Harman number M .
- Energy distribution increased for thermophoresis Nt, Brownian motion parameters Nb and radiation parameter Rd.
- Concentration enhanced with activation energy E, thermal radiation Rd and buoyancy ratio number Nr.
- The motile microorganism distribution and recced with Picklet number Pe, bio convection Lewis number Le and Schmidt number Sc and increased with bioconvection Rayleigh number.

Conflict of Interest: None

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ВПЛИВ ЕНЕРГІЇ АРРЕНІУСА ТА СОНЯЧНОГО ВИПРОМІНЮВАННЯ НА ЕЛЕКТРОПРОВІДНІСТЬ НАНОРІДИНИ ВІЛЬЯМСОНА З ПЛАВАЮЧИМ МІКРООРГАНІЗМОМ: ЗАВЕРШЕННЯ МОДЕЛІ БУОНДЖОРНО

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Збагачені термічні механізми та прогресивність наноматеріалів спонукали вчених присвятити себе цій галузі в наші дні. Універсальне та синтезуюче використання таких частинок охоплює виробництво енергії, сонячні системи, процеси моніторингу опалення та охолодження, системи відновлюваної енергії, лікування раку, гібридні двигуни та наноелектроніку. Крім того, в цю еру біотехнології та біоінженерії біоконвекція нанорідин забезпечує деякі захоплюючі застосування, такі як ферменти, біосенсори та біопаливо. Представлено математичну модель для оцінки електропровідності нанорідини Вільямсона з тепло- та масообміном через пористий розтягнутий лист за наявності біоконвекції. Біоконвекція плаваючих мікроорганізмів, теплове випромінювання, теплопровідність і енергія Арреніуса є новими аспектами цього дослідження. Нелінійні керуючі диференціальні рівняння з частинними похідними (PDE) вищого порядку розв'язуються шляхом застосування відповідних змінних подібності, і в результаті створюється пара звичайних диференціальних рівнянь (ODE). Розроблений набір ODE розв'язується чисельно за допомогою добре відомої техніки зйомки за допомогою команди ND solve у Wolfram MATHEMATICA та порівнюється з результатом коду pvb4c у MATLAB. Отримані графіки для різних фізичних величин, що представляють інтерес, разом із безрозмірними профілями швидкості, температури, концентрації та щільності мікроорганізмів для включення таких параметрів, як магнітний параметр, броунівський рух, число Релея, число Пекле, біоконвективне число Льюїса, параметр термофорезу і параметра коефіцієнта плавучості. Обговорюється вплив численних параметрів на характеристики потоку та теплообміну.

Ключові слова: *енергія активації; нанорідини Вільямсона; хімічна реакція; теплове випромінювання; метод стрільби; лист, що розширюється; теплопровідність*