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SUPERRADIATION OF MOBILE OSCILLATORS†

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The paper considers the development of the process of superradiance of radiating oscillators interacting with each other by means of an electromagnetic field. The interaction of oscillators occurs both with the nearest neighbors and with all other oscillators in the system. In this case, the possibility of longitudinal motion of oscillators along the system, due to the action of the Lorentz force, is taken into account. It is shown that, regardless of the motion of the oscillators, for example, due to their different masses, the maximum attainable amplitude of the generation field changes little. However, the radiation efficiency depends on how this field is distributed in the longitudinal direction. In the case of a shift of the field maximum towards the ends of the system, the radiation efficiency can noticeably increase. In addition, the direction of the phase velocity of the external initiating field is important, which accelerates the process of phase synchronization of the oscillators. This can also affect the ejection of particles outside the initial region, and here the total number of ejected particles and their speed turn out to be important. It is discussed how the density of oscillators and the size of the region occupied by oscillators will change.
Keywords: Superradiance of moving oscillators, shift of the field maximum in the volume, change in the density of oscillators.
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Interest in the processes of generation of oscillations in superradiance regimes began with the well-known work of R.N. Dicke [1]. Superradiance is usually realized in open systems, when high-frequency energy is removed from the system. To a certain extent, this energy output can be equivalent to dissipative processes of a distributed type.

Previously, dissipative regimes of excitation or generation of an electromagnetic wave in open waveguide resonator systems such as a traveling wave lamp traditionally corresponded to the case of the interaction of the waveguide electromagnetic field with emitter particles, most often with moving beam electrons (see, for example, [2]). Since the system is open, the field leaves the waveguide, which for short systems is equivalent to losses or dissipative processes of a distributed type. The field damping decrement in such a waveguide or resonator without emitters or oscillators for such generation or amplification modes may be greater than the development increment of the generation process in the presence of these active elements [3–5]. In particular, one could consider the case of the interaction of a system of fixed oscillators with the field of an open resonator. In this case, the oscillators did not interact directly with each other, but only through the resonator field common to the system. The superradiance regime (see, for example, [1, 6, 7]), on the contrary, ensured precisely the interaction of oscillators with each other both with their nearest neighbours and with all other oscillators in the system. Moreover, due to sufficiently large distances between the particles, the interaction between them occurs only due to their own electromagnetic fields. The mechanism of phase synchronization of the radiation of such oscillators was discussed in [8–10].

Attempts to discover the similarity between dissipative instability regimes and superradiance in open systems began to be undertaken in [11, 12]. It is remarkable that the systems of equations describing the interaction of electron beams rotating in a magnetic field with the fields of a waveguide at cyclotron resonances, when simplified, were reduced to the descriptions of the interaction of oscillators considered in the above papers [13]. This circumstance indicated the existence of a common phase synchronization mechanism for all these cases. A detailed comparison was made of the dissipative mode of field generation in an open system, a resonator, with the superradiance mode in the same resonator uniformly filled with immobile excited oscillators. The analysis showed that the increments of the processes and the maximum achievable amplitudes of the field of these two regimes practically coincide [14].

If this resonator is filled with an active medium, a dissipative excitation regime is also possible, in which quantum oscillators interact only with the resonator field, and there is no direct interaction between them. The regime of superradiance in the same resonator, when quantum oscillators interact only with each other, is also discussed. It is important to note that at a relatively low density of oscillators, their wave functions do not overlap [15]; therefore, they can affect each other only by their own radiation fields. And here the increments of the processes and the maximum achievable amplitudes of the field of these two regimes practically coincide [16].

In this paper, we consider the behavior of a system of oscillators similar to the case studied earlier [14], but we take into account the possibility of longitudinal motion of oscillators along the resonator due to the action of the Lorentz force. We will be interested in how the maximum achievable amplitude of the generation field will change, how this field will be distributed in the longitudinal direction, how the density of oscillators and the size of the region occupied by oscillators will change.

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SYSTEM OF EQUATIONS TAKING INTO ACCOUNT THE MOTION OF OSCILLATORS

The paper [14] gives equations for slowly changing the amplitude of oscillators and their total field in the case when the oscillators are stationary. Let us consider a system of oscillators [14], but take into account that the electric field $E_x(z,t)$, which arises when the oscillators oscillate, leads to the appearance of a magnetic field $H_y(z,t)$. (All designations correspond to [14]). As a result, the Lorentz force F_z acts on the oscillator, moving the oscillators along the longitudinal axis of the system:

$$\frac{\partial E_x}{\partial z} = -\frac{1}{c}\frac{\partial H_y}{\partial t} = \frac{i\omega}{c}H_y, \quad H_y = \frac{c}{i\omega}\frac{\partial E_x}{\partial z}, \; F_z = -e\frac{v_x}{c}H_y. \tag{1}$$

The electric field of the oscillator (electron) at a point is equal to

$$E_x = 2\pi\cdot\, e\cdot\, A\cdot\, \omega\cdot\, c^{-1}\cdot\, e^{-i\omega t}\left(e^{ik(z-z_0)}\cdot\,\theta(\mathrm{z}-\mathrm{z}_0)+e^{-ik(z-z_0)}\cdot\,\theta(\mathrm{z}_0-\mathrm{z})\right) = 2\pi\cdot\, e\cdot\, A\cdot\, \omega\cdot\, c^{-1}\cdot\, e^{-i\omega t}\cdot\, e^{ik|z-z_0|}.$$

Therefore, the magnetic field of this oscillator is

$$H_y = \frac{c}{i\omega}\frac{\partial E_x}{\partial z} = 2\pi\cdot\, e\cdot\, A\cdot\, \omega\cdot\, c^{-1}\cdot\, e^{-i\omega t}\left(e^{ik(z-z_0)}\cdot\,\theta(\mathrm{z}-\mathrm{z}_0)-e^{-ik(z-z_0)}\cdot\,\theta(\mathrm{z}_0-\mathrm{z})\right),$$

and for the system of oscillators, we obtain the magnetic field of the system

$$H_y(z,t) = \frac{2\pi\cdot\, e\cdot\, \omega\cdot\, M}{c}\cdot\, e^{-i\omega t}\cdot\,\frac{1}{N}\sum_{s=1}^{N}A_s\left(e^{ik(z-z_s)}\cdot\,\theta(\mathrm{z}-\mathrm{z}_s)-e^{-ik(z-z_s)}\cdot\,\theta(\mathrm{z}_s-\mathrm{z})\right),$$

and the force acting on the oscillator at the point

$$F_{zj} = -\pi\cdot\, e^2\cdot\, k^2\cdot\, M\cdot\,\mathrm{Re}\left[A_j^*\cdot\,\frac{1}{N}\sum_{s=1}^{N}A_s\left(e^{ik(z_j-z_s)}\cdot\,\theta(z_j-z_s)-e^{-ik(z_j-z_s)}\cdot\,\theta(z_s-z_j)\right)\right] \tag{2}$$

In contrast to [14], the coordinates of the oscillators change. The equations of motion of oscillators under the action of force (2) have the form

$$\frac{dv_{zj}}{dt} = -\frac{\pi\cdot\, e^2\cdot\, k^2\cdot\, M}{m_1}\mathrm{Re}\left[A_j^*\cdot\,\frac{1}{N}\sum_{s=1}^{N}A_s\left(e^{ik(z_j-z_s)}\cdot\,\theta(z_j-z_s)-e^{-ik(z_j-z_s)}\cdot\,\theta(z_s-z_j)\right)\right], \tag{3}$$

$$\frac{dz_j}{dt} = v_{zj}, \tag{4}$$

Equation (3-4), together with the equation for changing the amplitude of the oscillator ([14], equation (19)) constitutes our system of equations.

In dimensionless form, the system takes the form

$$\frac{d\mathrm{A}_j}{d\tau} = \frac{i\alpha}{2}\cdot\,\left|\mathrm{A}_j\right|^2\mathrm{A}_j - \frac{1}{N}\sum_{s=1}^{N}\mathrm{A}_s\cdot\, e^{i2\pi|Z_j-Z_s|} - \mathrm{E}_0\cdot\, e^{2\pi i Z_j} = \frac{i\alpha}{2}\cdot\,\left|\mathrm{A}_j\right|^2\mathrm{A}_j - \frac{1}{2}\mathrm{E}(Z_j,\tau) - \mathrm{E}_0\cdot\, e^{2\pi i Z_j}, \tag{5}$$

$$\frac{dV_{Zj}}{d\tau} = -\beta\cdot\,\frac{4\alpha}{3}\cdot\,\mathrm{Re}\left(\mathrm{A}_j^*\cdot\,\frac{1}{N}\sum_{s}\mathrm{A}_s\cdot\, e^{i2\pi|Z_j-Z_s|}\cdot\, sign(Z_j-Z_s)\right), \tag{6}$$

$$\frac{dZ_j}{d\tau} = \frac{1}{2\pi}V_{Zj}, \tag{7}$$

here $\gamma = \gamma_0^2/\delta_D = \pi e^2 M/mc$; $\mathrm{E} = eE/m\omega\gamma a_0$, $\mathrm{A} = A/a_0$; $k_0 z = 2\pi Z$; $\tau = \gamma t$; $\gamma_0^2 = \pi e^2 n_0/m = \omega_{pe}^2/4$, $\alpha = 3k_0^2 a_0^2\omega/4\gamma$, $\beta = m/m_1$, $M = b\cdot n_0$, b - is the length of the considered space in the longitudinal direction, n_0 is the density of particles per unit volume, m, m_1 - are the masses of the electron and the oscillator, respectively, E0 is the amplitude of the external field traveling in the positive direction of the Z axis.

The intensity of the electric field of oscillator radiation in dimensionless units is written by the expression

$$\mathrm{E}_x(Z,\tau) = \frac{2}{N}\sum_{s=1}^{N}\mathrm{A}_s\exp\{i2\pi|Z-Z_s|\}. \tag{7}$$

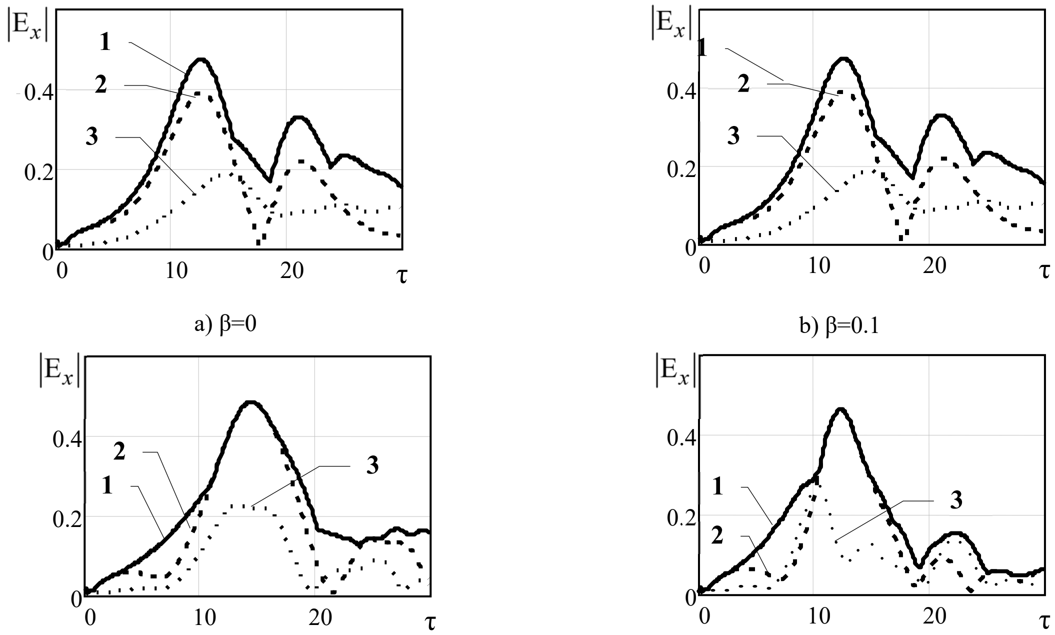
RESULTS OF NUMERICAL CALCULATIONS

In the article [14], the calculations were carried out with the number of particles N=3600 in the case of motionless oscillators. For system (5-8) similar calculations were carried out ($\beta = 0$, fixed oscillators) at N=3600, N=10000, N=20000. The results practically did not differ, therefore, all further calculations were carried out at N=10000.

The following options are selected. The number of particles N =10000, $\alpha = 1$, the amplitude of the additional external field that initiates the superradiance process, E_0=0.02. At the initial moment of time, the oscillators are uniformly distributed along the system, their velocities are equal to zero $V_{zj}(0) = 0$, the amplitude modules of the oscillators are equal to unity $\left|A_j(0)\right| = 1$, their phases ψ_j have random values in the range $(-\pi, \pi)$, additional external field that initiates the process of superradiance E_0=0.02.

The parameter β (the ratio of the mass of the charge to the mass of the oscillator) took the values 0, 0.1, 0.5, 1. Larger values of the parameter β correspond to lighter and more mobile oscillators.

Figure 1(a,b,c,d) shows the time dependence of the modulus of the maximum value of the field in the system and the field at the edges of the system (Z=0 and Z=1) and for different values of the parameter β.



a) β=0 b) β=0.1

c) β=0.5 d) β=1

Figure 1. Dependence of the field modulus in different parts of the system on time τ
$1 - \max\left|E_x\right|$, $2 - \left|E_x(Z=1)\right|$, $3 - \left|E_x(Z=0)\right|$

As can be seen from these figures, taking into account the motion of oscillators does not affect the maximum field strength in the system. At β=0 (the oscillators are immobile) and at β=0.1 (massive and slow-moving oscillators), the field maximum is observed inside the system, which was observed in [14]. But at β=0.5 and β=1, the field maximum is observed at the end at the end of the system (Z=1).

Since the energy output from the system is determined by the value of the field at the ends, in the case when the maximum amplitude is reached at the end, the radiation efficiency is the highest. The change in efficiency can be estimated from the ratio of the squares of the field amplitude at the end to the corresponding value at the maximum. At β=0 and β=0.1 this ratio is equal to 0.66, at β=0.5 and β=1 this ratio is obviously equal to 1.

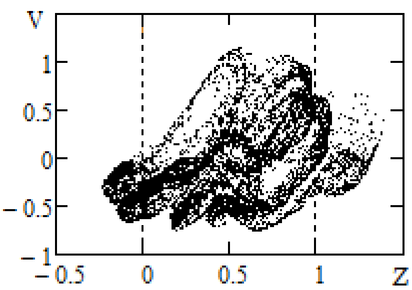


Figure 2. Distribution in space of velocities V of oscillators at β=1 at the moment τ=12.

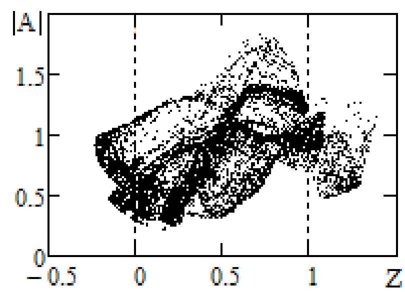


Figure 3. Oscillator amplitude distribution |E| in space at β=1 at the moment τ=12.

Figures 2 and 3 show the spatial distribution of velocities and amplitude modules for all particles at β=1 at the moment of reaching the field maximum τ=12.

As can be seen from the figures, the particles exit from both ends, and particles fly out to a greater distance in the direction of the external field (the wave vector of the external field is oriented in the direction of the Z axis) from the end of the system (Z>1), where the field is maximum, and the particles emitted at the beginning of the system (Z<0), are located more compactly. By the time the field reaches its maximum, τ=12, the coordinate Z<0 for 15% of the particles and Z>1 for 10% of the particles. Those. a larger number of particles flew out through the beginning of the system against the external field. There is an expansion of the area occupied by oscillators and its shift.

Figures 4 and 5 show the time dynamics of particle expansion.

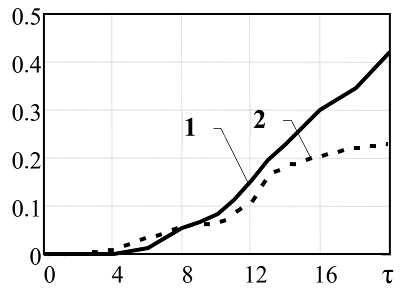


Figure 4. Time dependence of the fraction of particles emitted through the ends of the system, 1- departure through the beginning of the system (Z<0), 2- departure through the end of the system (Z>1)

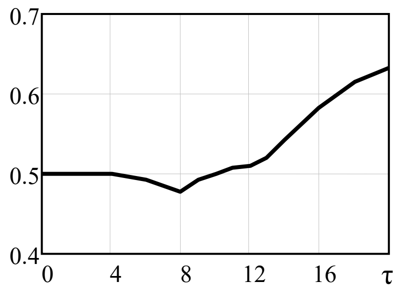


Figure 5. Time dependence of the fraction of particles in the first half of the system (Z<0.5).

It follows from the figures that up to the moment τ=8, the fraction of particles in the first half of the system decreases (Fig. 5) and a greater number of particles leave through the end of the system, the particles move as a whole in the direction of the external field. But from the moment τ=8 the dynamics becomes opposite, the system of particles, continuing to expand, shifts to the beginning against the direction of the external field.

It is important to note that, in the units under consideration, the average spontaneous emission amplitude of 10,000 oscillators is approximately equal to 0.01. On the other hand, the maximum amplitude of the induced emission of these oscillators, whose phases would be completely correlated, would reach unity.

Therefore, the amplitude of the initiating field is only twice the level of spontaneous emission (E0=0.02). The achieved field amplitude in the system is approximately 0.5.

In other words, the energy density of the initiating field is four times greater than the corresponding spontaneous emission level of oscillators with a random phase distribution. The achieved level of radiation, in turn, is four times less than the maximum value of the energy density of all these oscillators, if their phases were completely correlated.

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НАДВИПРОМІНЮВАННЯ РУХОМИХ ОСЦИЛЯТОРІВ

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У роботі розглянуто розвиток процесу надвипромінювання осциляторів, що взаємодіють за допомогою електромагнітного поля між собою. Взаємодія осциляторів відбувається як з найближчими сусідами, так і з усіма іншими осциляторами в системі. При цьому враховано можливість поздовжнього руху осциляторів вздовж системи, зумовленого дією сили Лоренца. Показано, що незалежно від руху осциляторів, наприклад, через їхню різну масу, максимально досяжна амплітуда поля генерації змінюється мало. Однак, ефективність випромінювання залежить від того, як буде це поле розподілено в поздовжньому напрямку. У разі зсуву максимуму поля до торців системи ефективність випромінювання може помітно збільшуватися. Крім того, важливим є напрямок фазової швидкості зовнішнього ініціюючого поля, яке прискорює процес синхронізації фаз осциляторів. Це також здатне впливати на викид частинок за межі початкової області, причому тут виявляється важливим загальна кількість частинок, що виходять, і їх швидкість. Обговорюється як зміниться щільність осциляторів та розмір області, зайнятої осциляторами.

Ключові слова: надвипромінювання рухомих осциляторів, зсув максимуму поля в об'ємі, зміна густини осциляторів