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ON THE MECHANISMS OF FORMATION OF DENSITY CAVITIES UNDER INSTABILITYOF INTENSE LANGMUIR OSCILLATIONS IN A PLASMA[†]

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The paper considers the instability of intense Langmuir oscillations in nonisothermal (Zakharov's model) and cold (Silin's model) 1D plasma. The main attention is paid to the formation of plasma density caverns in the hydrodynamic and hybrid (electrons are described hydrodynamically, ions are described by model particles) representations. In the hydrodynamic representation, with a small number of spectrum modes, large-scale plasma density caverns are observed, which rapidly deepen. This process is supported by the appearance of small-scale perturbations, and phase synchronization of the Langmuir waves of the instability spectrum is observed. This phase synchronization of the spectrum modes is quite capable of fulfilling the role that was previously proposed to be given exclusively to the effect of extrusion of particles from the cavity by the field. In hybrid models, in the region of consideration, ions are described by model particles, the number of which in the one-dimensional case $10^4 \div 5 \cdot 10^4$ (which in the three-dimensional case corresponds to the number of particles $10^{12} \div 10^{14}$). The initial spectrum of perturbations is very wide and rather intense, which leads to an explosive growth of perturbations in the Zakharov model and a rapid development of instability in the Silin model. In this case, in the developed instability regime, the formation of many small-scale plasma density caverns is observed. It is the presence of this small-scale modulation due to the Fermi effect that quickly forms the normal distribution of ions over velocities. In this case, the effect of particle heating due to Landau damping loses its primacy. It is shown that the caverns practically do not change their position; phase changes for the spectral components of the plasma density were not observed. Only individual small-scale caverns demonstrate dynamics similar to the development of caverns in the hydrodynamic representation.
Keywords: intense Langmuir oscillations, description models by Zakharov and Silin, small-scale slow-moving plasma density caverns, phase synchronization.
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1. INTRODUCTION

The traditional idea of the dynamics of the development of instability of intense Langmuir oscillations (the wavelength of which was assumed to be very significant, actually infinite) was associated by many authors of publications on this topic with the idea expressed in [1] about the formation of deepening plasma density caverns filled with the field of the short-wavelength Langmuir spectrum of instability. Indeed, during the development of the instability process, one could see how the spectrum of instability was enriched with short-wavelength Legmuir oscillations. It could also be assumed that the caverns deepened as a result of phase locking of the modes of the short-wavelength spectrum of Langmuir waves. However, this synchronization mechanism itself was practically not discussed. For the reason for the deepening of the plasma density caverns was considered the process of plasma extrusion by the short-wavelength field of the Langmuir spectrum of instability.

In addition, it was assumed that, as a result of the development of instability, particle flows were formed, which took away the energy of the field due to Landau damping. Field and numerical experiments confirmed the appearance of a noticeable inhomogeneity of the plasma due to this instability and demonstrated the transfer of energy from the field to plasma particles, to a greater extent to electrons.The quasi-hydrodynamic description of the process of instability of intense Langmuir oscillations prevailing in many publications under conditions of a relatively narrow spectrum of initial perturbations demonstrated the appearance of individual rapidly deepening plasma density caverns. Moreover, as noted in [1–4], already in the three-dimensional case, the emerging regime with peaking (rapid growth of the short-wavelength Langmuir spectrum with deepening of the caverns) was even called the collapse of Langmuir waves (although the Debye screening limited the minimum size of the bottom of the caverns). Due to the difficulties in numerical simulation of quasi-hydrodynamic models of plasma description [5], they turned to the so-called hybrid description models, where electrons were still represented hydrodynamically as a continuous medium, and ions in a discrete form as particles. According to the opinion expressed by V. E. Zakharov and his colleagues (see [3]), direct modeling of the phenomena of formation of cavities in such problems by the particle method is “the most consistent.” It is clear that the number of modeling particles remained much less than the real number of ions in in two-dimensional and even more so in three-dimensional cases, although in the one-dimensional case, which is of academic interest, the number of modeling particles per unit length was already acceptable.

The use of hybrid models immediately showed the difference between the hydrodynamic description of all plasma particles and the hybrid description, where ions were represented by particles. The discrete description of ions actually

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sharply enhanced the short-wavelength spectrum of initial perturbations and, most importantly, advanced this initial spectrum into the region of perturbations up to the Debye radius of the ions.

This circumstance changed the conditions for the development of instability of intense Langmuir oscillations to such an extent that the picture of the process became different. As is known [2], in the Zakharov model for a nonisothermal plasma, where the electron thermal energy density exceeded the field energy density, the growth rate of perturbations only increased with decreasing scale. In the Silin model of a cold plasma, where the field energy density was higher than the thermal energy density of the plasma, the increment of such instability was significant for perturbations whose scale was comparable to the amplitude of electron oscillations in the field of intense Langmuir oscillations [6, 7]. In these two cases, with a large number of particles describing ions, the perturbation spectrum rapidly increased in the short-wavelength region, forming many small density caverns [8]. Moreover, in the Zakharov model, the growth of the instability spectrum occurred in an explosive manner.

In the hybrid model of Zakharov's nonisothermal plasma, the processes of formation of small-scale cavities immediately dominated, and it was impossible to observe the formation of sharpening regimes - the deepening of only a few large cavities, which was typical for the hydrodynamic description precisely because of the absence of a relatively intense short-wavelength disturbance spectrum in the initial conditions. For it is precisely the discreteness of the description of ions that generated such a wide initial spectrum in the short-wavelength region for the case of a hybrid description.

In Silin's cold plasma model, a decrease in the amplitude of intense Langmuir oscillations shifted the maximum growth rate to the short-wavelength region of the initial spectrum of perturbations with a noticeable amplitude, which arose due to the discreteness of the description of ions. However, this delay in the development of short-wave disturbances allowed the formation of several large-scale caverns. However, their refinement then occurred, and the depth of small-scale caverns turned out to be noticeably greater than in the case of nonisothermal plasma.

These processes of plasma density modulation could no longer be explained only by the displacement of plasma particles from the resulting caverns by the high-frequency Langmuir field, so one should return to consideration of phenomena of phase synchronization of waves of unstable spectra.

The presence of a significant small-scale modulation of the plasma field and density obviously led to the scattering of ions by inhomogeneities and their rapid thermalization. Not only was the transfer of energy from the field to the ions important, but also the formation of a normal energy distribution of the ions, which made it possible to speak about the temperature of the ions. In addition, "tails" appeared - groups of energetic ions, and in cold plasma the energy of such "tails" turned out to be comparable with the energy of ions within the formed normal distribution. It is clear that the role of Landau damping during ion thermalization in hybrid regimes could hardly be dominant; rather, the Fermi scattering mechanism on inhomogeneities manifested itself here [7, 9].

So, the appearance of large-scale deepening density caverns in plasma with instability of intense Lengyur oscillations in the case of describing ions by particles (or, what is the same, in the presence of a sufficiently significant and wide initial short-wavelength spectrum) apparently practically does not occur. Only at the initial stage of instability in a cold plasma can one see such structures that, after grinding, disappear among the mass of small-scale cavities. Therefore, in the presence of a wide intense initial spectrum, it is often impossible to observe peaking regimes in a few large caverns. As it turned out, Landau damping could hardly be considered the cause of ion thermalization.

The aim of this work is a detailed consideration of the process of formation of the spatial modulation of the plasma density. In addition, we will discuss the role of phase synchronization of Langmuir waves of an unstable spectrum in the formation of spatial structures - plasma density caverns. Let us also consider the conditions for the formation of large-scale deepening density caverns.

2. DESCRIPTION MODELS OF V.E. ZAKHAROV AND V.P. SILIN

Let us discuss hydrodynamical and hybrid models of Zakharov and Silin (hybrid models: electrons are represented by quasi-hydrodynamic equations, and ions are represented by particles), descriptions of the instability of intense Leggmuir oscillations in plasma. These models make it possible to see the formation of the Maxwellian velocity distribution of ions and evaluate the efficiency of their heating [7, 9]. However, below we dwell on the consideration of the dynamics of plasma stratification - on the formation of its density caverns.

Let us first consider the case of parametric instability of an external long-wavelength Langmuir field of high intensity for a cold plasma, that is, under conditions when the energy density of the field exceeds the density of the thermal energy of the medium $W / n_0 T_e = |E|^2 / 4\pi n_0 T_e > 1$. The particles are in the field of an external wave, the length of which, for the sake of simplicity of calculations, is set equal to infinity, oscillating with the ion speed is $u_{0\alpha} = -\left(e_\alpha |\mathrm{E}_0| / m_\alpha \omega_0\right)\cos\Phi$. The components of the field strength of the external wave are determined as follows

$$E_0 = -i \cdot \left[|E_0| \exp\left(i\omega_0 t + i\phi\right) - |E_0| \exp\left(-i\omega_0 t - i\phi\right) \right] / 2. \tag{1}$$

For complex slowly varying components $E_n, \overline{E}_n$, respectively, of the HF electric field, LF electric field of the excited short-wavelength instability spectrum, and also for the ion charge density $\nu_{i,n} = e n_{i,n}$, the following system of equations can be written [9].

Silin's hydrodynamic model ($W / n_0 T_e = |E|^2 / 4\pi n_0 T_e > 1$).

$$\frac{\partial E_n}{\partial t} - i\frac{\omega_{pe}^2 - \omega_0^2}{2\omega_0} E_n + \theta \cdot \frac{n^6}{n_M^6} \cdot E_n - \frac{4\pi\omega_{pe}\nu_{in}}{k_0 n} J_1(a_n) \cdot \exp(i\phi) - \\ - i\frac{\omega_0}{2en_0}\sum_m \nu_{in-m} \cdot \left[E_{-m}{*} \cdot J_2(a_{n-m})\exp[2i\phi] + E_m \cdot J_0(a_{n-m}) \right] = 0, \tag{2}$$

$$\frac{\partial^2 \nu_{in}}{\partial t^2} = -\Omega_i^2 \left\{ \begin{array}{l} \nu_{in}\left[1 - J_0^2(a_n) + \frac{2}{3}J_2^2(a_n)\right] + \frac{ik_0 n}{8\pi} J_1(a_n)\left[E_n \cdot e^{-i\phi} - E_{-n}^* \cdot e^{i\phi}\right] + \\ + \frac{n^2 k_0^2}{64\pi^2 en_0}\sum_m J_0(a_n) \cdot E_{n-m} \cdot E_{-m}^* + \frac{nk_0^2}{64\pi^2 en_0} J_2(a_n) \cdot \sum_m (n-m)\left[E_{n-m} \cdot E_m \cdot e^{-2i\phi} + E_{m-n}^* \cdot E_{-m}^* \cdot e^{2i\phi}\right] \end{array} \right\}, \tag{3}$$

$$\frac{\partial E_0}{\partial t} - i\Delta E_0 = -\frac{\omega_0}{2en_0}\sum_m \nu_{i,-m} \cdot \left[E_{-m}^* \cdot J_2(a_m)\exp[2i\phi] + E_m \cdot J_0(a_m) \right]. \tag{4}$$

Hybrid Silin model ($W / n_0 T_e = |E|^2 / 4\pi n_0 T_e > 1$).

$$\frac{\partial E_n}{\partial t} - i\frac{\omega_{pe}^2 - \omega_0^2}{2\omega_0} E_n + \theta \cdot \frac{n^6}{n_M^6} \cdot E_n - \frac{4\pi\omega_{pe}\nu_{in}}{k_0 n} J_1(a_n) \cdot \exp(i\phi) - \\ - i\frac{\omega_0}{2en_0}\sum_m \nu_{in-m} \cdot \left[E_{-m}^* \cdot J_2(a_{n-m})\exp[2i\phi] + E_m \cdot J_0(a_{n-m}) \right] = 0, \tag{5}$$

$$\frac{\partial E_0}{\partial t} - i\Delta E_0 = -\frac{\omega_0}{2en_0}\sum_m \nu_{i,-m} \cdot \left[E_{-m}^* \cdot J_2(a_m)\exp[2i\phi] + E_m \cdot J_0(a_m) \right]. \tag{6}$$

When describing ions by particles, one can use the equations of motion

$$\frac{d^2 x_s}{dt^2} = \frac{e}{M}\sum_n \bar{E}_n \cdot \exp\{ik_0 n x_s\}. \tag{7}$$

where is the intensity of the slowly changing electric field

$$\bar{E}_n = \left(-\frac{4\pi i}{k_0 n}\right)\nu_{in}\left[1 - J_0^2(a_n) + \frac{2}{3}J_2^2(a_n)\right] + \frac{1}{2}J_1(a_n)\left[E_n \cdot e^{-i\phi} - E_{-n}^* \cdot e^{i\phi}\right] - \\ - \frac{ink_0}{16\pi en_0} J_0(a_n)\sum_m E_{n-m} \cdot E_{-m}^* - \\ - \frac{ik_0}{16\pi en_0} J_2(a_n) \cdot \sum_m (n-m)\left[E_{n-m} \cdot E_m \cdot e^{-2i\phi} + E_{m-n}^* \cdot E_{-m}^* \cdot e^{2i\phi}\right]. \tag{8}$$

Here, the arguments of the Bessel functions $a_n = nek_0 E_0 / m_e \omega_0^2, \Phi = \omega_0 t + \phi$, and $\nu_{i,n} = en_{i,n}$, $\nu_{i,n} = en_0 \frac{k_0}{2\pi}\int_{-\pi/k_0}^{\pi/k_0} \exp(-ink_0 x_s(x_0, t))dx_{s0}$ are the components of the ion charge density, the HF field of the spectrum $E = \exp\{-i\omega_0 t\} \cdot \sum_n E_n \cdot \exp\{ink_0 x\}$, and the term $\theta \cdot (n / n_M)^6 E_n$ models the damping of the HF modes of the spectrum by electrons, with $n_M = 20$, $\Delta = (\omega_{pe}^2 - \omega_0^2)/2\omega_0$. In addition, a dispersion term can be added to (2) and (5), which is proportional to $\beta = k_0^2 v_{Te}^2 / 2\omega_0$, $v_{Te}^2 = T_e / m_e$, x_s – the coordinate of the s-th particle simulating the ion. The expressions proportional to $J_0(a_n)$, correspond to slow motions, and the expressions proportional to $J_{\pm 2}(a_n)$, are determined by the contribution to the nonlinearity of the second harmonic, n_0 is the unperturbed plasma density, T_e is the electron temperature, and the ions are assumed to be cold at the initial moment.

Zakharov's hydrodynamic model (supersonic regime) under conditions ([2], see also [7,9]) $W / n_0 T_e = |E|^2 / 4\pi n_0 T_e < 1$.

$$\frac{\partial E_n}{\partial t} - i\frac{\omega_{pe}^2 - \omega_0^2 + k_0^2 n^2 v_{Te}^2}{2\omega_0} E_n + \theta \cdot \frac{n^6}{n_M^6} \cdot E_n - i\frac{\omega_0}{2n_0} \cdot \left[n_{in} E_0 + \sum_{m \neq 0} n_{in-m} E_m \right] = 0, \tag{9}$$

$$\frac{\partial^2 n_{i,n}}{\partial t^2} = -\frac{k_0^2 n^2}{16\pi M}\left[E_n E_0^* + E_0 E_{-n}^* + \sum_{m \neq 0, n} E_{n-m} E_{-m}^* \right]. \tag{10}$$

$$\frac{\partial E_0}{\partial t} - i\frac{\omega_0}{2n_0}\cdot \sum_m n_{i,-m} E_m = 0 . \tag{11}$$

Hybrid model of Zakharov under conditions $W / n_0 T_e = |E|^2 / 4\pi n_0 T_e < 1$.

$$\frac{\partial E_n}{\partial t} - i\frac{\omega_{pe}^2 - \omega_0{}^2 + k^2{}_0 n^2 v_{Te}^2}{2\omega_0} E_n + \theta \cdot \frac{n^6}{n_M^6}\cdot E_n - i\frac{\omega_0}{2n_0}\cdot \left[n_{in} E_0 + \sum_{m\neq 0} n_{in-m} E_m \right] = 0 , \tag{12}$$

$$\frac{\partial E_0}{\partial t} - i\frac{\omega_0}{2n_0}\cdot \sum_m n_{i,-m} E_m = 0 . \tag{13}$$

When describing ions by particles, one can use the equations of motion,

$$\frac{d^2 x_s}{dt^2} = \frac{e}{M}\sum_n \bar{E}_n \cdot \exp\{ik_0 n x_s\} , \tag{14}$$

where is the intensity of the slowly changing electric field

$$\bar{E}_n = -ik_0 n\tilde{\phi}_n = \frac{-ik_0 n n_{i,n} T}{e n_0} + \frac{-ik_0 n e}{4m\omega_p{}^2}\left[E_n E_0^* + E_0 E_{-n}^* + \sum_{m\neq 0,n} E_{n-m} E_{-m}^* \right] . \tag{15}$$

In this case, the ion density perturbations are $n_{in} = n_0 \cdot \frac{k_0}{2\pi}\int_{-\pi/k_0}^{\pi/k_0} \exp\left[-ink_0 \cdot x_s\left(x_0, t\right)\right]\cdot dx_{s0}$, where the term $\theta\cdot E_n \cdot n^6 / n_M^6$ in the equations (9) and (12) models the damping of the HF modes of the spectrum on electrons, and HF field spectrum $E = \exp\{-i\omega_0 t\}\cdot \sum_n E_n \cdot \exp\{ink_0 x\}$.

It is important to note that for the equations of the hydrodynamic model (2)–(4) and the hybrid model (5)–(8) of Silin, taking into account the representation $J_1\left(a_n\right) \approx a_n / 2$, $J_0\left(a_n\right) \approx 1$, $J_2\left(a_n\right) \approx a_n^2 / 8$, coincide with those obtained for the nonisothermal plasma of the hydrodynamic model (7)–(9) and the hybrid model of Zakharov (10)-(13), respectively, up to the detuning value and taking into account the replacements $E_0 \to -iE_0$ and $E_0^* \to iE_0^*$.

From the results of the linear theory [2] (see also [7,9]), it follows that in the Zakharov model, the correction to the frequency normalized to the Langmuir frequency can be written as

$$\left(\delta'\right)^2 = \frac{\left(\Delta'\right)^2}{2} \pm \sqrt{\frac{\left(\Delta'\right)^4}{4} + B\left(\Delta'\right)^2} , \tag{16}$$

Where $B = \frac{1}{2}\frac{m_e}{M}\frac{|E_0|^2}{4\pi n_0 T_e}$. The increment (imaginary part $\delta' = \Omega / \omega_{pe}$) increases with the growth of the wave number of perturbations, reaching its maximum value at large values of the wave number. That is, the growth rate of plasma density perturbations even increases with a decrease in their scale. In Silin's model, at detuning values $\Delta' = \left(m_e 2M\right)^{1/3} J_1^{2/3}\left(a_{n_m}\right)$, the relative increment reaches the values

$$\delta' = \pm\frac{i}{\sqrt[3]{2}}\left(A'\right)^{1/3} = \pm\frac{i}{\sqrt[3]{2}}\left(\frac{m_e}{M}\right)^{1/3} J_1^{2/3}\left(a_n\right). \tag{17}$$

Here $a_n = n\left(ek_0 E_0 / m_e\omega_0^2\right)$ and $k_m = k_0 n_m$. At $a_{n_m} = 1.84$, $\delta'_{\max} = \pm 0.44i\left(\frac{m_e}{M}\right)^{1/3}$. As the instability develops, the amplitude of the Langmuir wave decreases, and the increment maximum shifts to the short-wavelength region [7, 9].

It is useful to note that the values of the perturbation increments in the case of parametric instability in the Zakharov model increase as their scale decreases. Moreover, in the Zakharov model, a decrease in the amplitude of the pump field leads to a decrease in the increments in the entire region of instability. In the Silin model, such a pump depletion process shifts the growth rate maximum to shorter wavelengths without decreasing its value (17). Thus, the process of energy movement to the short-wavelength part of the spectrum in the two models is largely determined by the linear mechanisms of the growth of perturbations.

3. FORMATION OF CAVITIES IN SYSTEMS WITH A SMALL NUMBER OF SPECTRUM MODES AND PARTICLES SIMULATING IONS

In the case of a small number of spectrum modes in Zakharov's hydrodynamic model, several large-scale density caverns are formed, which then deepen. This can be explained by the form of the initial spectrum, which is more clearly represented by relatively long-wavelength perturbations that form the initial scale of the cavity. Since the growth rate of short-wave disturbances is larger, even under conditions of their small initial amplitude they grow rapidly and this leads to their phase synchronization (which will be discussed below) under the action of the field of intense Langmuir oscillations and deepening of the resulting cavity (that is, to the formation in some degree of aggravation). The process of formation of deepening density caverns in Silin's hydrodynamic model develops in a similar way.

As the amplitude of the pump wave decreases, the increment maximum shifts towards larger wavenumbers without changing its value. In addition, the pump wave contributes to phase synchronization of the growing modes of the spectrum, thereby forming the spatial structure of the cavity. The distance between spatial modes (there are only 40 of them here) in the spectrum is inversely proportional to the size of the region under consideration; therefore, only a few caverns form in this region (this can be seen from the position of the spectrum maximum at the beginning of the process). The gradual connection to the formation of cavities of synchronized short-wavelength modes of the spectrum leads to the implementation of the peaking mode and to the deepening of the cavities (see Fig. 1).

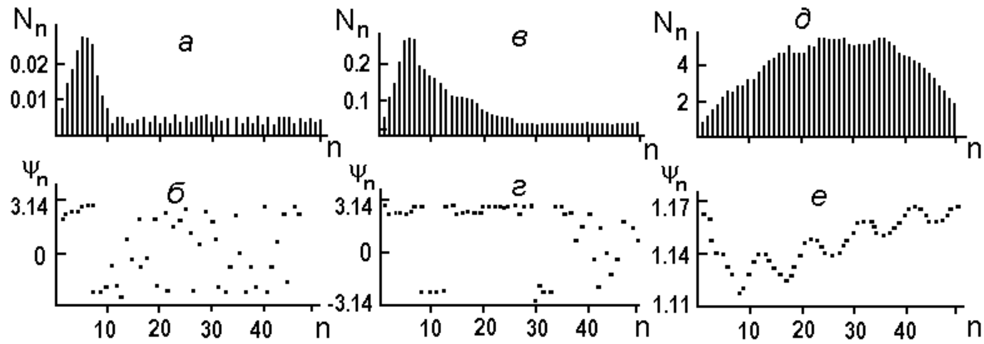

Figure 1. The process of formation of a wave packet of Langmuir waves during instability [10]. One can see phase synchronization (lower figures) and broadening of the spectrum $N_n = N_n \exp(i\Psi_n)$, $E_n = 4\pi ieN_n / 2k_0 n$,where for time points $\tau = 4$; $\tau = 7$; $\tau = 8$

In hybrid models, the presence of a large number of particles simulating ions leads to noticeable amplitudes of the initial wide short-wavelength spectrum. Since the increments in the models of non-isothermal Zakharov plasma only increase with moving to the short-wavelength region, it is not surprising that the process acquires an explosive character (see, for example, [8]). It is clear that refinement of the spatial density structure occurs already at the initial stage of the instability process. The appearance of larger-scale deepening caverns is difficult in this case (see Fig. 2).

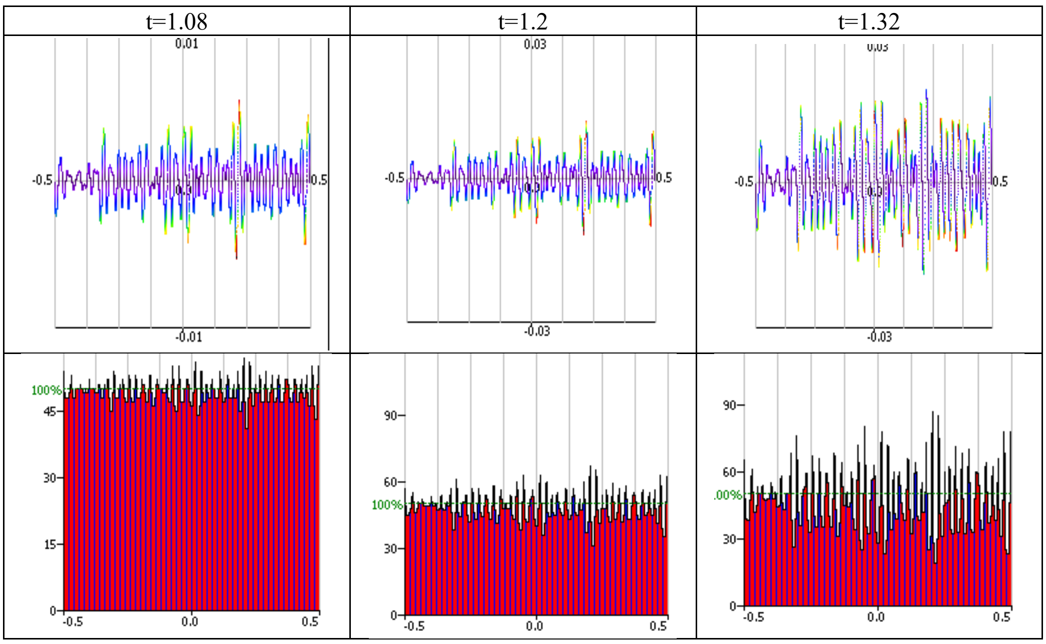

Figure 2. Phase plane (velocity, coordinate) and plasma density at different times for Zakharov's hybrid model. The number of spectrum modes is 201, the number of particles simulating ions is 10,000 [11]

In a cold plasma described by Silin's models, the process of motion of perturbations along the spectrum also develops rapidly at the initial stage of instability. In the Silin hybrid model, with a small number of modes and model particles, one can see the formation of a large-scale deepening cavity and its destruction due to the intersection of particle trajectories [5]. In the nonlinear regime, with a larger number of modes and particles, the caverns are reduced throughout the entire volume of the plasma. Moreover, the scale of even deep caverns remains very small (see Fig. 3).

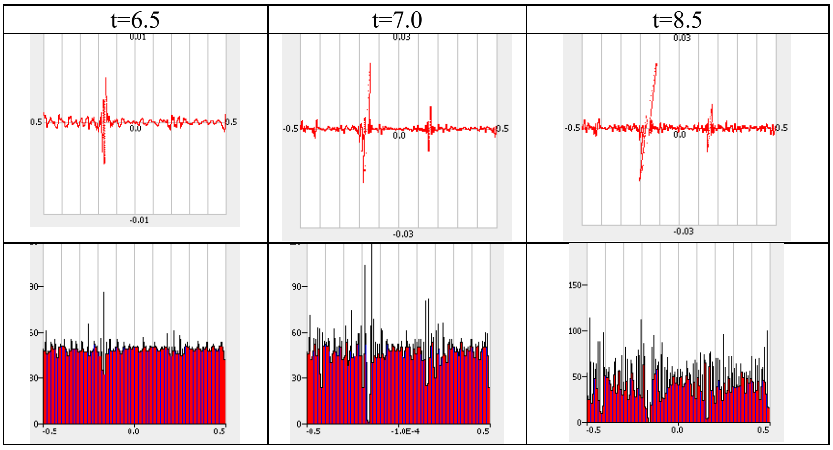

Figure 3. Phase plane (velocity, coordinate) and plasma density at different times for Silin's hybrid model. The number of spectrum modes is 201, the number of particles simulating ions is 10,000 [11]

4. FORMATION OF SPATIAL PLASMA DENSITY DISTRIBUTIONS IN SYSTEMS WITH A LARGE NUMBER OF PARTICLES SIMULATING IONS

Generally speaking, it remains unclear whether the emerging caverns move in space. In other words, do the phases of the amplitudes of charge density and ion density varying slowly with time $\nu_{in} = en_{in}$ and n_{in}. In addition to explaining their dynamics, this circumstance, as shown below, affects the process of synchronization of the Langmuir spectrum of instability. In the calculations, only the obvious condition $n_{i,-n} = (n_{i,n})^*$ was assumed to be satisfied. However, as shown by model calculations for a large number of spectrum modes (the number of which is 1001) and a large number of particles simulating ions, equal to 50,000, phase changes for the spectral components of the plasma density were not noticed. The caverns practically did not change their position, changing only their amplitude (see Fig. 4).

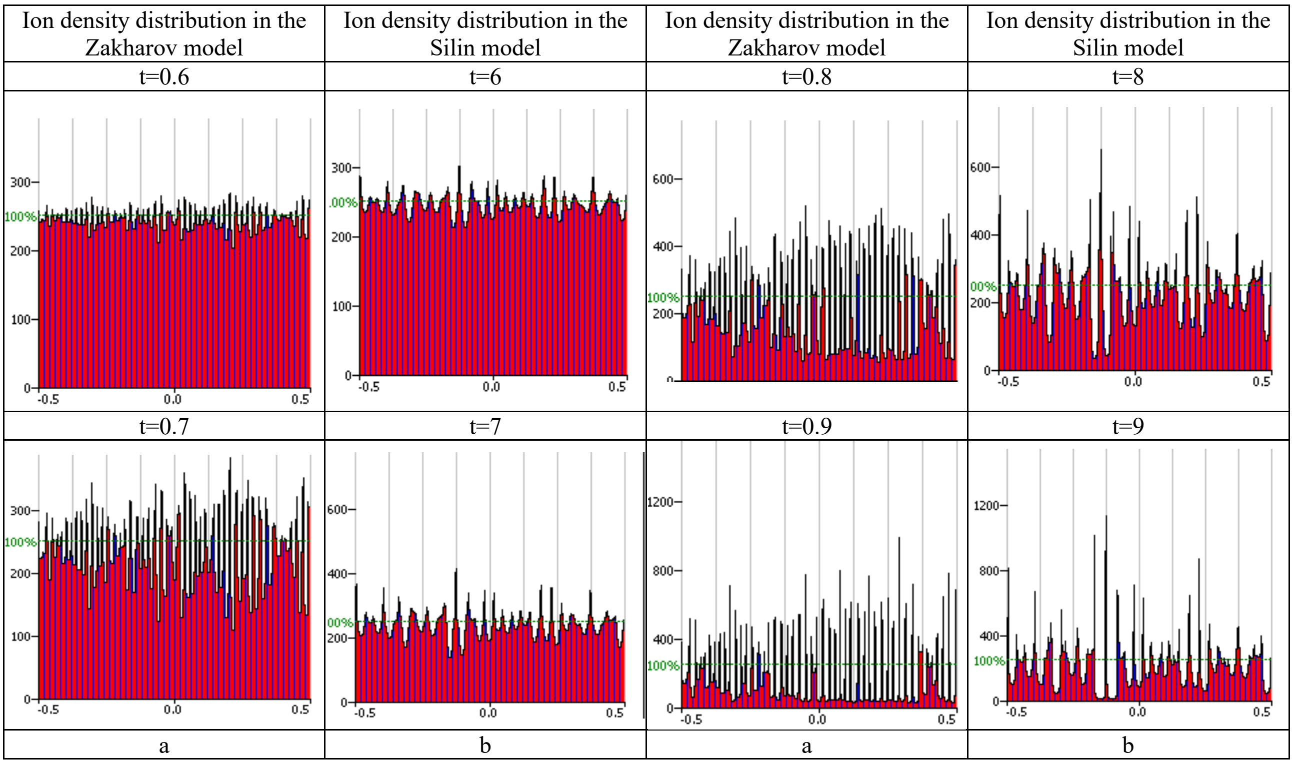

Figure 4. Plasma ion density distribution in the Zakharov (a) and Silin (b) models,
the number of particles simulating ions is 50000, the number of spectra modes is 1001. Absorption in the system is 0.05. [11]

5. PHASE SYNCHRONIZATION MECHANISMS IN THE ZAKHAROV AND SILIN MODELS

The process of phase synchronization of short-wavelength Langmuir waves of the instability spectrum $E_n = |E_n| \cdot \exp\{i\varphi_n\}$ can be illustrated as follows. From equations (7), (10) at the initial stage of instability in the Zakharov model, one can obtain a simple equation for the phase of an individual mode of the RF Langmuir spectrum

$$\frac{\partial \varphi_n}{\partial t} - \Delta_Z = R_Z \cdot \mathrm{Cos}\{\phi - \varphi_n\} \tag{18}$$

where $\Delta_Z = \dfrac{\omega_{pe}^2 - \omega_0^2 + k_0^2 n^2 v_{Te}^2}{2\omega_0}$ and $R_Z = \dfrac{\omega_0 n_{in} E_0}{2 n_0 E_n}$. Obviously, in the absence of changes in the position of the caverns, and at a significant value of $R_Z \propto E_0 / E_n >> 1$, the phases of the RF spectrum modes are able to synchronize.

Accordingly, equations (2), (5) in the initial stage of instability for the phases of the RF modes of the Langmuir spectrum $E_n = |E_n| \cdot \exp\{i\varphi_n\}$ can be written as

$$\frac{\partial \varphi_n}{\partial t} - \Delta_S = R_S \cdot \mathrm{Sin}\{\phi - \varphi_n\}, \tag{19}$$

where $\Delta_S = \dfrac{\omega_{pe}^2 - \omega_0^2}{2\omega_0}$, $R_S = \dfrac{4\pi\omega_{pe} v_{in}}{k_0 n E_n} J_1(a_n) \propto J_1(a_n)/E_n$, and $J_1(a_n)/E_n >> 1$ for rapidly growing modes of the HF spectrum allows the phases of the spectrum to be synchronized.

Despite the development of small-scale modulation of the plasma density in the Zakharov and Silin models, the nature of the formation of cavities, even on small scales, is largely associated with mode locking. This process is illustrated in Fig. 5, which shows the formation of small-scale caverns.

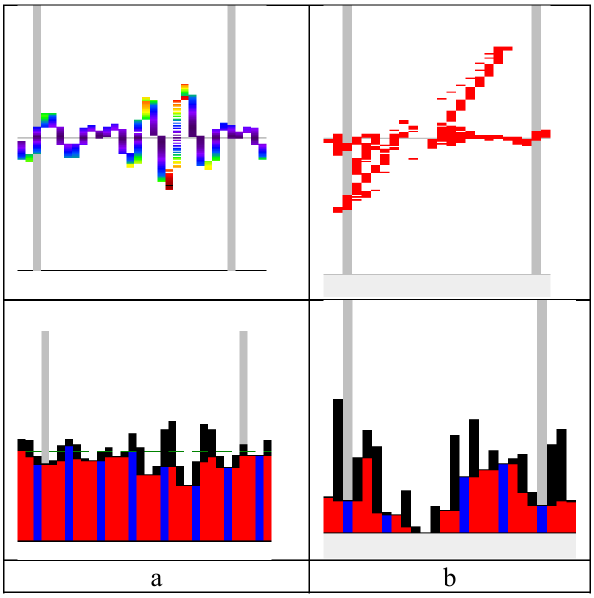

Figure 5. Formation of small-scale plasma ion density caverns in the Zakharov (a) Silin (b) hybrid models. The upper fragments illustrate the phase (velocity and coordinate of particles) space near the caverns, the lower fragments illustrate the particle density distribution [11]

In Zakharov's model for a nonisothermal plasma, expression (13) is valid for the components of the low-frequency field strength, and expression (8) is valid for Silin's cold plasma model. It is important to note that the right-hand sides of these expressions contain terms, the number of which is significant for wide spectra of short-wave disturbances in hybrid models. Therefore, the RF pressure is very high in this case, especially in the Silin model for cold plasma. Perhaps this is why the formation of the small-scale cavity shown in Fig. 5b demonstrates such a dynamic character. In addition, the particle extrusion mode is implemented here, which expands the cavity. It is also possible to switch to the mode of intersection of particle trajectories, which destroys the cavity, as can be seen in [5].

6. CONCLUSION

The paper considers the instability of intense Langmuir oscillations in nonisothermal (Zakharov's model) and cold (Silin's model) 1D plasma. The main attention is paid to the formation of plasma density caverns in the hydrodynamic and hybrid (electrons are described hydrodynamically, ions are described by model particles) representations.

In the hydrodynamic representation, with a small number of spectrum modes, large-scale plasma density caverns are formed, which rapidly deepen. This process is supported by the appearance of small-scale perturbations, and phase synchronization of the Langmuir waves of the instability spectrum is observed (Section 5). This phase synchronization of the spectrum modes is quite capable of fulfilling the role that was previously proposed to be given exclusively to the effect of extrusion of particles from the cavity by the field.

In hybrid models, where ions in the region of consideration are described by model particles, the number of which in the one-dimensional case $10^4 \div 5 \cdot 10^4$ (which in the three-dimensional case corresponds to the number of particles $10^{12} \div 10^{14}$)

the initial spectrum of perturbations is very wide and rather intense, which leads to an explosive growth of perturbations in the Zakharov model [8] and a rapid development of instability in Silin's model. In this case, in the developed instability regime, the formation of many small-scale plasma density caverns is observed. It is important to note that the caverns practically do not change their position; phase changes for the spectral components of the plasma density were not observed. It is the presence of this small-scale modulation due to the Fermi effect that rapidly forms the normal velocity distribution of ions [7]. In this case, the effect of particle heating due to Landau damping loses its primacy [9]. Only individual small-scale caverns demonstrate dynamics (see Fig. 5) similar to the development of caverns in the hydrodynamic representation.

Thus, the notions that plasma density caverns, which form during the instability of intense Langmuir oscillations, first appear on a large scale and only then deepen due to the extrusion of particles by the HF field or due to the development of energy motion along the spectrum are only partly true. In fact, when describing ions by particles due to the wide initial spectrum (due to the discreteness of the ionic component), many small-scale cavities immediately appear and the appearance of large deepening cavities becomes unlikely.

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ПРО МЕХАНІЗМИ ФОРМУВАННЯ КАВЕРН ЩІЛЬНОСТІ ПРИ НЕСТІЙКОСТІ ІНТЕНСИВНИХ ЛЕНГМЮРІВСЬКИХ КОЛИВАНЬ У ПЛАЗМІ

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У роботі розглянуто нестійкість інтенсивних ленгмюрівських коливань у неізотермічній (модель Захарова) та холодній (модель Силіна) 1D плазмі. Основна увага приділена процесу формування каверн щільності плазми у гідродинамічному та у гібридному (електрони описані гідродинамічно, іони – модельними частинками) представленнях. У гідродинамічному поданні при невеликій кількості мод спектру спостерігаються великомасштабні каверни щільності плазми, які швидко поглиблюються. Цей процес підтримується появою дрібномасштабних обурень, причому спостерігається синхронізація фаз ленгмюрівських хвиль спектра нестійкості. Ця синхронізація фаз мод спектру цілком здатна виконати ту роль, яку раніше пропонували віддати виключно ефекту видавлювання полем частинок з каверни. У гібридних моделях у області розгляду іони описані модельними частинками, число яких у одновимірному випадку $10^4 \div 5 \cdot 10^4$ (що у тривимірному випадку відповідає числу частинок $10^{12} \div 10^{14}$). Початковий спектр обурень дуже широкий і досить інтенсивний, що призводить до вибухового зростання збурень у моделі Захарова та швидкого розвитку нестійкості у моделі Силіна. При цьому в розвиненому режимі нестійкості спостерігається формування безлічі дрібномасштабних каверн щільності плазми. Саме наявність цієї дрібномасштабної модуляції за рахунок ефекту Фермі швидко формує нормальний розподіл іонів за швидкостями. В цьому випадку ефект нагрівання частинок за рахунок згасання Ландау втрачає першість. Показано, що каверни мало змінюють свого становища, фазові зміни для спектральних компонентів щільності плазми помічені не були. Тільки окремі дрібномасштабні каверни демонструють динаміку, подібну до розвитку каверн у гідродинамічному представленні.
Ключові слова: інтенсивні ленгмюрівські коливання, моделі опису Захарова та Силіна, дрібномашабні малорухливі каверни щільності плазми, синхронізація фаз.