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SOLUTIONS OF THE SCHRÖDINGER EQUATION WITH HUL NEUTRON POSITION FOR BETA (β^-) DECAY AND NEUTRON EMISSION[†]

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The current study is about the structure of ^{17}B, which has been investigated by the Microscopic Cluster Model. The binding energy and neutron position of two valence neutrons of Beta-decay and neutron emission have been calculated. A cluster configuration of the Halo nucleus inspired me to consider all radioisotopes have cluster configuration before the decay process. The Jacobi coordinates has been used to investigated the ^{17}B nucleus. The Jacobi coordinate is a very well technique to describe such as a three-body system or halo structure. The ^{17}B has Borromean property, so it has been defined in T-configuration in this coordinates. The angle in the figure defines an angle of halo neutron motion around the core. The study has considered a deformation of the core as a high influence on the binding of the valence neutrons.

Keywords: ^{17}B; neutron-halo structure; β^-decay; neutron emission; microscopic cluster model.
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There are many works have been done in investigates of nuclei away from the line of β-stable through the beginning of new facilities and technologies. Experimental studies [l–3] on light atomic nuclei have shown exotic large r.m.s. radius and specifically [4] that has ascribed this to the halo property. Halo structure is defined as the effect of threshold that happens in weakly bound nuclei where the nucleons are connected in very small-range potential. The halo atomic nuclei are confined as more loosely.

The three-body halo nuclei of choice with two neutrons move around the core are drip-line of light nuclei, the (^{17}B= ^{15}B +n +n) nucleus. Actually, the first observation of (Boron-17) is in 1973 [5], interest of the Boron ^{17}B nucleus is exotic properties (neutron emission and β-decayed [6] and an abnormal radius [7,8]). The very large matter radius is a significance of a halo, formed by external nucleons far from a core that increases meaningfully the matter radius [9].

Many studies have been done using the microscopic cluster model and the three-body models [10,11] of ^{17}B depending on different configurations of ^{13}B + di-neutron + di-neutron. The wavefunction was built on the shell of lp with the excitations of the core on the harmonic–oscillator. The angular momentum and parity of ^{17}B have been investigated. The R.M.S. radius and electric quadrupole of (Boron-17) were observed to be around 2.81 fm and around 2.74 e.fm^2, without the ^{15}B+n+n configuration [11]. Ren and Xu [12] investigated the ^{17}B ground state with a three-body model including two neutrons and ^{15}B core. Ren and Xu have calculations regarding a ^{15}B + n interaction that was complicated and explained about the large RMS radius. The shell model spectrum of several boron isotopes has also been used by Warburton and Brown has used several Boron isotopes by the shell model. [13]

Several experimental studies on ^{17}B have been achieved [14, 15]. The separation energy of two neutrons ($S_{2\text{-}n}$) was 1.39 MeV±0.14 MeV for ^{17}B [14] while the radius of ^{17}B was 4.10 fm±0.46 fm [8] and 3.0 fm±0.6 fm [16] by Ozawa in 1993. The abnormal large radius has been taken as a halo structure and loosely bound. The halo phenomenon (especially, the neutron-halo nuclei) is fairly common in nuclei and with high N/Z ratios which lead to β^-*-decay* and neutron emissions. The weakening in the correlation of p-n distribution is a typical of the stable nuclei [17]. In this approach, the skin nucleons can be released from the nuclei to form β^-*-decay* and neutron emissions and move away freely from the other nucleons. The deformation or the core electric quadrupole moment as predicted by the shell model that caused by nucleons outside a close closed shell. [18].

In the current work, the Boron-17 was investigated to determine where is the position of valence neutron to start β^-*-decay* or neutron emissions by using the Microscopic Cluster Model. The hyperspherical harmonic method has been used to solve the model. The core was considered deformed and this deformation has a high role in determining the position of decay. The protons and neutrons far from the magic number in the core shaped the deformations. Thus, the deformation has a clear influence on the energy of valence neutrons. The Jacobi coordinates were space coordinates of ^{17}B nucleus which were used in this calculation, as formerly used for ^{14}Be [21] and ^{11}Li [22].

THEORETICAL ASPECT

The Jacobi coordinates has been used to investigate the ^{17}B nucleus. The Jacobi coordinate is a very well technique to describe such as a three-body system or halo structure as in Fig. 1. The ^{17}B has Borromean property [23], so it has

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been defined in T-configuration in this coordinates. The angle in the figure defines an angle of halo neutron motion around the core.

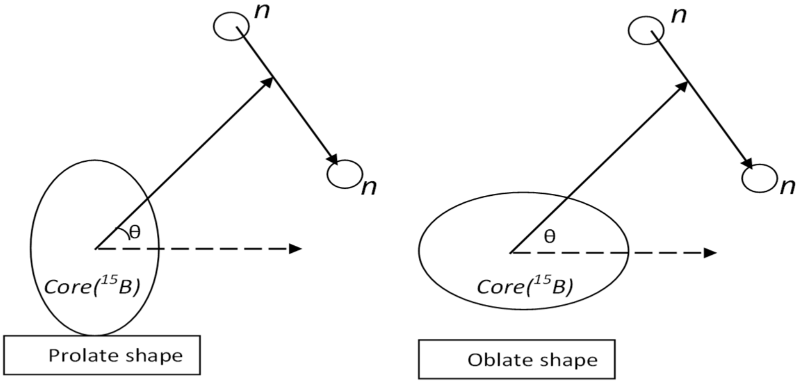

Figure 1. Jacobi coordinates for free-body system with two shapes of the core and angle of two-halo neutrons

The halo structure was defined as the core plus valence neutrons. The total wavefunction of the system is:

$$\Psi^{JM}(x,y,\xi)=\phi_{core}(\xi_{core})\psi(x,y) \tag{1}$$

The Hamiltonian of the core is given as

$$\hat{h}_{core}(\xi_{core})\phi_{core}(\xi_{core})=\varepsilon_{core}\phi_{core}(\xi_{core}) \tag{2}$$

The angular part, the radial part, and spin of the valence neutrons have been included in $\psi(x,y)$.)

The relationship between the coordinates *(x, y)* and the hyperspherical coordinates (hyper-radius ρ and hyper-angle θ) as

$$\rho^2=x^2+y^2 \quad \text{and} \quad \theta=\arctan\left(\frac{x}{y}\right)$$

Hyperspherical expansion is

$$R_n(\rho)=\frac{\rho^{5/2}}{\rho_o{}^3}\sqrt{\frac{n!}{(n+5)!}}L^5_{nlag}(z)\exp\left(\frac{-z}{2}\right) \tag{3}$$

$$z=\rho/\rho_o \quad \text{and} \quad \psi_k^{l_xl_y}(\theta)=N_k^{l_xl_y}(\sin\theta)^{l_x}(\cos\theta)^{l_y}P_n^{l_x+\frac{1}{2},l_y+\frac{1}{2}}(\cos 2\theta) \tag{4}$$

The valence neutrons wave function is

$$\psi_{n,k}^{l_xl_y}(\rho,\theta)=R_n(\rho)\psi_k^{l_xl_y}(\theta) \tag{5}$$
$$\psi(x,y)=\psi_{n,k}^{l_xl_y}(\rho,\theta),$$

where $L^5_{nlag}(z)$ is the associated Laguerre polynomial of the order $nlag=0,1,2,...$ and $P_n^{l_x+\frac{1}{2},l_y+\frac{1}{2}}(\cos 2\theta)$ is the Jacobi polynomial [24, 25].

Total Hamiltonian of the system is

$$\hat{H}=\hat{T}+\hat{h}_{core}(\vec{\xi})+\hat{V}_{core-n1}(r_{core-n1},\vec{\xi})+\hat{V}_{core-n2}(r_{core-n2},\vec{\xi})+\hat{V}_{n-n}(r_{n-n}) \tag{6}$$

$$\begin{aligned}\hat{V}_{core-n}(r_{core-n},\vec{\xi})&=\frac{-V_0}{\left[1+\exp\left(\frac{r_{core-n}-R(\theta,\phi)}{a}\right)\right]}\\&+\frac{-\hbar^2}{m^2c^2}2(l.s)\frac{V_{s.o}}{4r_{core-n}}\frac{d}{dr_{core-n}}\left(\left[1+\exp\left(\frac{r_{core-n}-R_{so}}{a_{so}}\right)^{-1}\right]\right)\end{aligned} \tag{7}$$

$$V_{n-n}(r_{n-n})=-\frac{\hbar^2}{m^2c^2}2(l.s)\frac{V_{s.o}}{4r_{n-n}}\frac{d}{dr_{n-n}}\left(\left[1+\exp\left(\frac{r_{n-n}-R_{so}}{a_{so}}\right)^{-1}\right]\right) \tag{8}$$

with

$$R = R_0\left[1+\beta_2 Y_{20}(\theta,\phi)\right] \tag{9}$$

$$\left\langle r_m^2\right\rangle^{1/2} = \frac{1}{A}\left[\ A_{core}\left\langle r_m^2(core)\right\rangle + \left\langle \rho^2\right\rangle\right] \tag{10}$$

For more information about the formulas [26-28].

RESULTS

The ^{17}B is radioisotope with a half-life of about 5.08(5) ms and its probabilities of decay are

β^-, n (63.0%),
β^- (22.1%),
β^-, 2n (11.0%),
β^-, 3n (3.5%) and
β^-, 4n (0.4%).

The biggest probability is *β^-, n (63.0%)*, which has been focused in the current work. The ^{17}B has two neutrons far from the rest nucleons, which be the reason for that decay. The two valence neutrons surround and move around the deformed core. The Hamiltonian of this system depended on the microscopic of the core and the clusterization of the valence neutrons. This clusterization depended on many factors; one of them is the angle (θ) of valence neutron position. The binding energy of the valence neutrons is calculated depending on Eq.(6).

Eq.(5) defines a wavefunction of the two valence neutrons, whereas ϕ in eq. (2), describes the core wavefunction calculated by the shell model. So the wave function of the valence neutrons has been a function of the angle as appear in eq.(5), where the core is connected to the valence neutrons and deformed. In eq.(9), $Y_{20}(\theta,\phi)$ was taken as

$$Y_{20}(\theta,\phi) = \frac{1}{4}\sqrt{\frac{5}{\pi}}(3\cos^2(\theta)-1)$$

Motion of the valence neutrons around the deformed core make binding energy of these neutrons diverse regarding the angle (θ) and also regarding the shape of deformation (prolate or oblate). In figure (2), the energy diverse from (-0.2MeV to -1.8MeV) with angle from (0° to 90°) that for deformation of $\beta_2=0.7$.

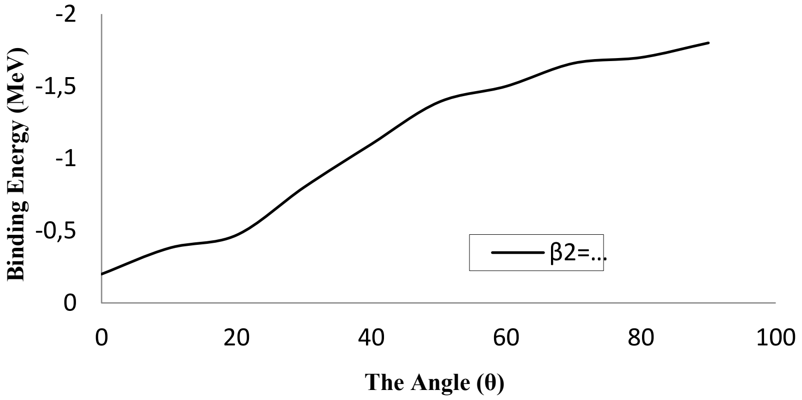

Figure 2. The Binding energy of the valence neutrons as function of the angle with deformation parameter (β_2=0.7)

The Figures (2,3,4 and 5) have considered the core is prolate and its deformation parameter β_2 (0.7, 0.5, 0.3 and 0.1) respectively. Regarding to the structure of the shell model, the core of ^{17}**B** (it is ^{15}**B**) has three protons outside the first closed shell and two neutrons outside the second closed shell. That reason to make a high influence for an electric quadrupole moment. The electric quadrupole moment of this nucleus is $Q=Q_j+Q_c$, it collected from Q_j that caused from the two neutrons which has been neglected in this study and Q_c of the core. In general $Q_c>>Q_j$ [29, 30]

$$Q_c = Q'\left[\ (3\Omega^2/2J^2)-\frac{1}{2}\right] \tag{11}$$

Eq. (11) can drive as

$$Q_c = Q'\ \frac{J}{2J+3}\left[\frac{3\Omega^2}{J(J+1)}-1\right] \tag{12}$$

The J is angular momentum, Q' can be equal to

$$Q' = \frac{4}{5}\delta Z R^2 \quad (13)$$

The Ω is the projection of *j*
and δ has relation with the parameter β_2 as

$(\beta_2 = 2/3(4\pi/5)^{1/2}\delta)$ [28, 29].

From the eq. (12) and eq. (13), the proton has a high effect on deformation, so we expect the deformation parameter about 0.2 to 0.4 if it is prolate and about -0.2 to -0.4 if it is oblate. However, the experimental data is β_2=0.437 [31]. Depending on the experimental data and from the fig. (2), the lowest value of energy is 0.3MeV in the angle (θ=0).

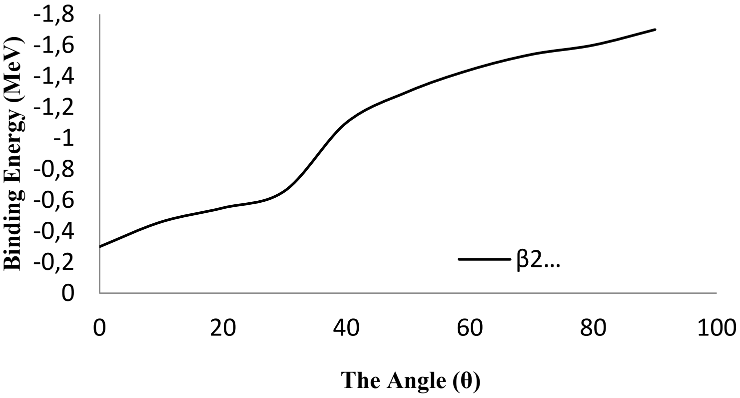

Figure 3. The Binding energy of the valence neutrons as function of the angle with deformation parameter (β_2=0.5)

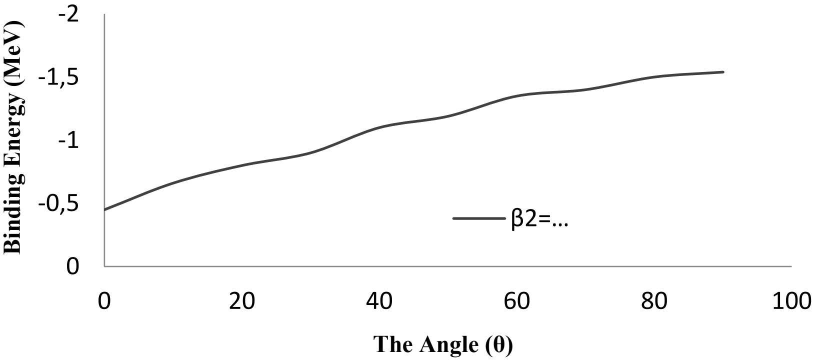

Figure 4. The Binding energy of the valence neutrons as function of the angle with deformation parameter (β_2=0.3)

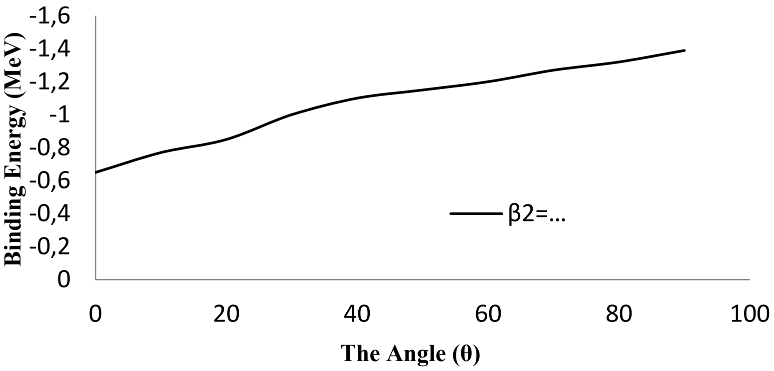

Figure 5. The Binding energy of the valence neutrons as function of the angle with deformation parameter (β_2=0.1)

The oblate shape has been taken into consideration. The figures (6,7,8 and 9) have considered the core is oblate and its deformation parameter β_2 (-0.7, -0.5, -0.3, and -0.1) respectively.

The procedure applied within this work has relied on the use of influence the total angular moment, atomic number, and angle characterizes both the clusters and the parent nucleus. All possible deformation parameters and angles in clusterization channels have been studied. The point of starting is always the deformation parameter of the core. The deformation theoretical values that have been studied instead of experimental values is modified by the space of our study: if only experimental values have been used, several probable clusterization canals cannot be done because of the lack of values of deformation for both or one of the clusters.

The many energies of the neutrons regarding relative motion was calculated as seen in figures using the virtual description. The predilection of diverse cluster configurations is categorized by the reciprocal forbiddingness. Using deformation of the ground state for the parent makes all cluster configurations possible and turn out to be allowed theoretically. A clear configuration towards radioactivity (or neutron emission) may be deduced from these figures. Always keep in mind that "forbidden" in the theoretical structural analysis should mean restrained. We should expect that looks to be of certain advantage: the cluster configuration of low-lying of prolate nucleus isn't meaning the pole-to-pole; instead of both clusters are tended with regard to the axis.

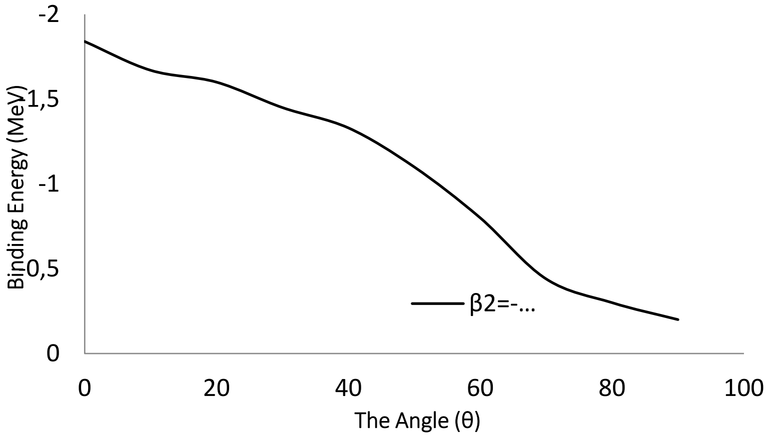

Figure 6. The Binding energy of the valence neutrons as function of the angle with deformation parameter (β_2=-0.7**)**

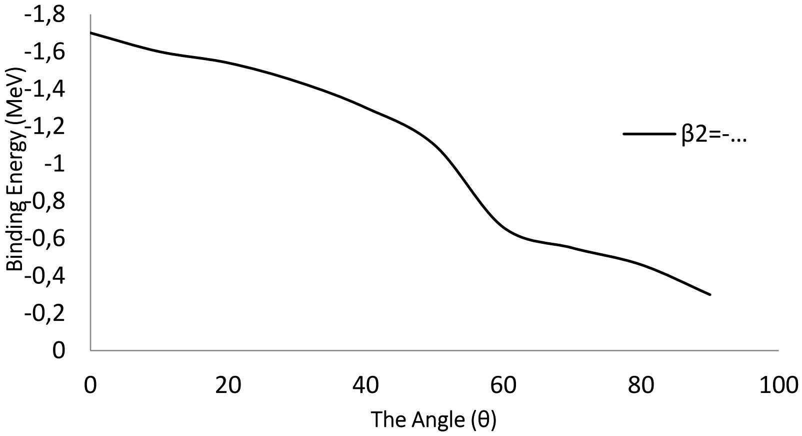

Figure 7. The Binding energy of the valence neutrons as function of the angle with deformation parameter (β_2=-0.5)

Figure 8. The Binding energy of the valence neutrons as function of the angle with deformation parameter (β_2=-0.3)

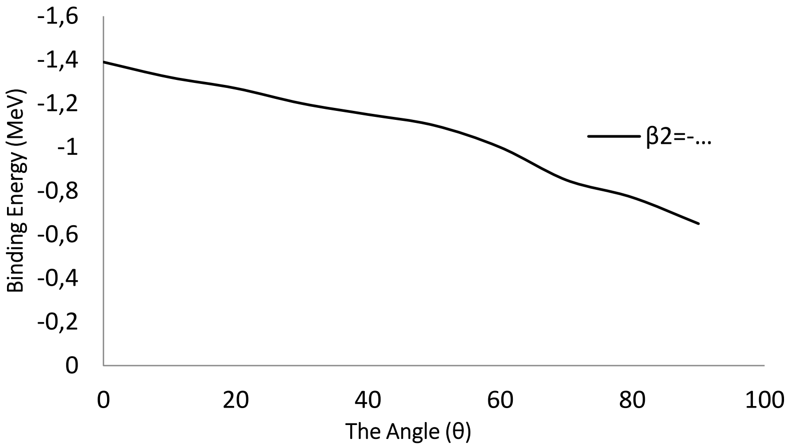

Figure 9. The Binding energy of the valence neutrons as function of the angle with deformation parameter (β_2=-0.1)

This clusterization configuration is of particular interest for starting point of decay or neutron emission as seen in a binary neutron emission channel. The pole-pole shapes, favored by calculation of penetrability, are highly Pauli principle forbidden as seen in results.

So they only can be a small compounds in the wave function of the ground state of the system. Also, if there are allowed clusterizations have been addressed in the case when one change the core deformation to superdeformation or hyperdeformation which connected to the valence neutrons.

The results are presented the clusters of neutrons and the core are considered to have deformation of the ground state. It is exciting to get that in these situations have allowed clusterizations also. In a superdeformed case; the allowed clusterizations connected to mostly to two particular areas wherein: (a) both neutron clusters have big prolate electric quadrupole deformation (around the region with *Zlight*,36), (b) first cluster with prolate deformation (electric quadrupole), and the second with oblate shape (around with *Zlight*,22). For the case of the hyperdeformed more channels are exposed, and from Figs. 2,3,4 and 5 and figs.6,7,8 and 9 a clear inferred to symmetric clusterization.

One of the interesting questions is if clusterization of the Halo nuclei exists in other radioactivity nuclei before the decay process. The clusterization is predicted for radioisotopes at excitation states close the relative multiple Beta decays. Actually, the main point is to determine when and where the decay starting. The position of the valence neutrons to decay or emission is our goal. However, the results determined the starting point of decay regarding energies.

The sizes of nuclei and the nucleon distributions give us the important parameters which connected extremely to the strong or weak interaction and decay. Also, the main idea of this study is to drive the clusterization of nucleons in Halo nuclei to other nuclei, which have radioactivity and large radii. The idea that Halo nuclei and radioisotopes having radii greater than those of the stable nuclei. These nuclei can be presented in a clusterization system.

From the results and regarding the decay energy of ^{17}B, the figures 4, 5, 8, and 9 have been excluded. The figures 2 and 3 represent the prolate shape and the figures 6 and 7 represent the oblate shape. Regardless of the experimental deformation parameter and from equation (13), the theoretical value of deformation is around ($\beta_2=0.5$). from the Figure 3, the energy is about (-0.3MeV) with angle ($\theta=0$).

By normalization, the experimental data of deformation parameter has been used to get the energy, which is about (-0.387MeV).

CONCLUSIONS

In this study, the halo structure and decay process have been investigated by clusterization configuration. The Microscopic Cluster Model (MCM) was the main tool and logical form to describe such as this system. This model gave us a wide range to take into consideration all components even freedom degrees of the core. The results obviously refer to the two considerations, first based on the angle of clusters of the microscopic cluster structure and the second on freedom degrees of the core.

The movement of the valence neutrons around the core represented various energies based on the angles. From that variance, we can determine the location of the valence neutron to emission or to decay. The deformation selection rule has a similar propensity for affecting on the energy. The core deformation parameter has high influence on the decay process. Our results clearly indicate to that the asymmetric binary microscopic cluster forms are preferred for the ground state. I can strongly say, we can expand this approach to all radio nuclei to appoint the energies and the neutron position of decay.

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РІШЕННЯ РІВНЯННЯ ШРЕДІНГЕРА З ПОЛОЖЕННЯМ НЕЙТРОНІВ ХУЛА ДЛЯ БЕТА (В-) РОЗПАДУ ТА ЕМІСІЇ НЕЙТРОНІВ

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Нинішнє дослідження стосується структури ^{17}B, яка була досліджена за допомогою моделі мікроскопічного кластера. Розраховано енергію зв'язку та положення нейтронів двох валентних нейтронів бета-розпаду та нейтронного випромінювання. Кластерна конфігурація ядра Halo надихнула мене розглянути, що всі радіоізотопи мають кластерну конфігурацію до процесу розпаду. Координати Якобі були використані для дослідження ядра ^{17}B. Координата Якобі є дуже вдалою технікою для опису, наприклад, системи трьох тіл або структури гало. ^{17}B має властивість Борромея, тому він був визначений у Т-конфігурації в цих координатах. Кут на малюнку визначає кут руху нейтронів гало навколо ядра. У дослідженні деформація ядра розглядається як так що має великий вплив на зв'язування валентних нейтронів.

Ключові слова: ^{17}B, нейтронно-галоструктура, β--розпад, нейтронна емісія, мікроскопічна кластерна модель.