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GENERAL ANALYSIS OF THE REACTION $e^{+}+e^{-}\rightarrow N+\bar{N}+\pi^{0}$ †

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The general analysis of the reaction $e^{+}+e^{-}\rightarrow N+\bar{N}+\pi^{0}$, in the case of longitudinally polarized electron beam, has been performed in the one-photon-annihilation approximation, accounting for the polarization states of the final nucleon. This analysis is useful for the description of the continuum (non-resonant) and resonant (with different possible vector mesons or excited baryons in the intermediate virtual states of the Feynman diagrams) contributions. The conservation of the hadron electromagnetic currents and P-invariance of the hadron electromagnetic interaction were used to express the matrix element in terms of the six complex independent invariant amplitudes. The general structure of the hadronic tensor for the case of unpolarized final hadrons and polarized nucleon has been derived. The spin-independent part of the hadronic tensor is determined by five structure functions and the spin-dependent one by 13 structure functions. The transversal, longitudinal and normal components of the nucleon polarization four-vector are expressed by means of the four-vectors of the particle momenta. The five independent invariant variables which describe the reaction have been introduced. The limits of the changing of these variables have been considered. The kinematical double invariant variables regions are given in the figure. The kinematics, suitable to study the invariant mass distributions, is investigated.
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About 20 years ago, the BES started a baryon resonance program [1] at Beijing Electron-Positron Collider (BEPC) [2,3]. The major experimental results on N^{*} resonances from $e^{+}e^{-}$ annihilations for last 20 years and some of their interesting phenomenological implications are reviewed in [1].

The measurements of time-like region electromagnetic form factors of hadrons can be done in the electron-positron annihilation process, which provides a key to understanding quantum chromodynamics effects in bound states. For example, there is great progress in the study of baryon electromagnetic form factors in the time like regions, both on the experimental [4–6] and theoretical sides [7–9]. The measurement of the $e^{+}e^{-}$ reactions permits to study also the excited hyperon states, such as Λ^{*},Σ^{*} and Ξ^{*} [10,12]. Up to now, the N^{*} production from $e^{+}e^{-}$ annihilations has been studied only around charmonium region.

The reaction $e^{+}+e^{-}\rightarrow p+\bar{p}+\pi^{0}$, where N^{*} resonances can manifest themselves as intermediate states of corresponding Feynman diagrams, was measured with BESIII detector at the BEPCII collider. In the experiment [13], this reaction has been studied in the vicinity of the $\psi(3770)$ resonance. The Born cross section of $e^{+}+e^{-}\rightarrow\psi(3770)\rightarrow p+\bar{p}+\pi^{0}$ has been extracted allowing the continuum production amplitude to interfere with the resonance production amplitude. Later, the measurement of this reaction was performed at higher energies [14], namely at 13 center of mass energies, $\sqrt{s}$, from 4.008 to 4.600 GeV (in the vicinity of the $Y(4260)$ resonance).

Some interesting results on the N^{*}'s production have been obtained. The $N^{*}(1440)$ peak was observed, for the first time, directly from πN invariant mass spectrum (due to the absence of the strong Δ peak). Besides several well known N^{*} resonances around $1520MeV$ and $1670MeV$, three new N^{*} resonances above $2GeV$ were found using partial wave analyses. The measurement of the $\psi(2S)\rightarrow\bar{p}p\pi^{0}$ channel (by CLEO Collaboration) found a similar strong $N^{*}(1440)$ peak [15]. There is no obvious $N^{*}(1440)$ peak for $e^{+}e^{-}\rightarrow\bar{p}p\pi^{0}$ in the vicinity of the $\psi(3770)$ [13].

The time-like region became accessible with the advent of high-precision, high-intensity $e^{+}e^{-}$ colliders at intermediate energies. New data from BESIII, collected in a high-precision energy scan in 2015, will offer improved precision over a large q^{2} range. The coming upgrade of the BEPCII collider up to c.m.s. energies of $4.9\,GeV$ will allow to study more details of the N^{*} production. The topics which planned to study at BESIII in the near future can be found in [16].

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This paper opens the series of works devoted to the general analysis,in the one-photon-annihilation approximation, of the differential cross section and polarization observables in the process $e^+ + e^- \to N + \bar{N} + \pi^0$, where $N(\bar{N})$ is proton (antiproton) or neutron (antineutron). We intend to account for the continuum (non-resonant) and resonance (with different possible vector mesons or excited baryons in intermediate virtual states of Feynman diagrams) contributions. Here we concentrate on the detailed analysis of the hadronic tensor and kinematics suitable to calculate invariant variables distributions.

FORMALISM

We study the reaction

$$e^-(k_1) + e^+(k_2) \to N(p_1) + \bar{N}(p_2) + \pi^0(k), \tag{1}$$

where the notation of the particle four-momenta is indicated in the parenthesis. In one-photon annihilation approximation $q = k_1 + k_2 = p_1 + p_2 + k$ is the four-momentum of the virtual photon.

The matrix element, in this approximation, can be written down as an contraction of the leptonic $\left(el_\mu\right)$ and hadronic $\left(eJ_\mu\right)$ currents

$$M = \frac{e^2}{q^2} l^\mu J_\mu,\ l^\mu = \bar{v}(k_2)\gamma^\mu u(k_1), \tag{2}$$

where $k^2 = m^2$, $p_1^2 = p_2^2 = M^2$, $m(M)$ is the pion (nucleon) mass. Further, we neglect the electron mass where it is possible.

Then, the square of the matrix element can be written as

$$|M|^2 = \frac{16\pi^2\alpha^2}{q^4} L^{\mu\nu} H_{\mu\nu},\ L^{\mu\nu} = l^\mu l^{\nu *},\ H_{\mu\nu} = J_\mu J_\nu^*. \tag{3}$$

The structure of the hadronic tensor

The hadronic tensor $H_{\mu\nu}$ has the following general form for the case when the polarizations of the final particles are not measured

$$H_{\mu\nu}(0) = H_1\tilde{g}_{\mu\nu} + H_2\tilde{k}_\mu\tilde{k}_\nu + H_3\tilde{p}_\mu\tilde{p}_\nu + H_4(\tilde{p}_\mu\tilde{k}_\nu + \tilde{p}_\nu\tilde{k}_\mu) + iH_5(\tilde{p}_\mu\tilde{k}_\nu - \tilde{p}_\nu\tilde{k}_\mu), \tag{4}$$

where $\tilde{g}_{\mu\nu} = g_{\mu\nu} - q_\mu q_\nu / q^2$, $\tilde{k}_\mu = k_\mu - (k\cdot q/q^2)q_\mu$ and $\tilde{p}_\mu = p_\mu - (p.q/q^2)q_\mu$, $p = p_1 - p_2$. $H_i\ (i = 1-5)$ are the so-called structure functions depending on three invariant variables s_1, s_2 and $s \equiv q^2$ (see below).

The leptonic tensor $L_{\mu\nu}$ has the following form in the case when electron beam is polarized

$$L_{\mu\nu} = -q^2 g_{\mu\nu} + 2(k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu}) + 2im_e(\mu\nu\eta q), \tag{5}$$

where $(\mu\nu ab) = {}_{\mu\nu\ \sigma}a\ b^\sigma$ and η_μ is the spin four-vector of the electron (we chose ${}^{0123} = -{}_{0123} = +1$), m_e is the electron mass.

At chosen normalization, the differential cross section of the process (1), in terms of the leptonic and hadronic tensors, has the following form (we neglect the electron mass for the initial particles leptonic current)

$$d\sigma = \frac{\alpha^2}{8\pi^3 q^6} L^{\mu\nu} H_{\mu\nu}\, dR_3,\ dR_3 = \frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2}\frac{d^3k}{2E}\delta(k_1 + k_2 - p_1 - p_2 - k), \tag{6}$$

where $E_1\,(E_2)$ is the nucleon (antinucleon) energy and E is the pion one.

In the case when the nucleon polarization is measured, we can use the following form of the hadronic tensor

$$H_{\mu\nu} = \frac{1}{2}H_{\mu\nu}(0) + T_{\mu\nu},$$

where the tensor $T_{\mu\nu}$ depends on the nucleon polarization 4-vector S^μ and it can be written as the sum of the symmetrical $T_{\mu\nu}^{(s)}$ and antisymmetrical $T_{\mu\nu}^{(a)}$ parts.

The symmetrical part can be written as follows

$$T^{(s)}_{\mu\nu}(S) = Im\{T_{kk}\,\tilde{k}^{\mu\nu}_k + T_{kp}\,\tilde{k}^{\mu\nu}_p + T_{pk}\,\tilde{p}^{\mu\nu}_k + T_{pp}\,\tilde{p}^{\mu\nu}_p + T_G\,\tilde{G}^{\mu\nu} + T_K\,\tilde{K}^{\mu\nu} + T_P\,\tilde{P}^{\mu\nu} + T_{KP}\,KP^{\mu\nu}\}, \tag{7}$$

where

$$\tilde{k}^{\mu\nu}_k = \tilde{k}(\nu kqS) + \tilde{k}(\mu kqS),\ \tilde{k}^{\mu\nu}_p = \tilde{k}(\nu pqS) + \tilde{k}(\mu pqS),$$

$$\tilde{p}^{\mu\nu}_k = \tilde{p}^{\mu}(\nu kqS) + \tilde{p}^{\nu}(\mu kqS),\ \tilde{p}^{\mu\nu}_p = \tilde{p}^{\mu}(\nu pqS) + \tilde{p}^{\nu}(\mu pqS),$$

$$\tilde{G}^{\mu\nu} = \tilde{g}^{\mu\nu}(kpqS),\ \tilde{K}^{\mu\nu} = \tilde{k}^{\mu}\tilde{k}^{\nu}(kpqS),\ \tilde{P}^{\mu\nu} = \tilde{p}^{\mu}\tilde{p}^{\nu}(kpqS),$$

$$KP^{\mu\nu} = [\tilde{k}^{\mu}\tilde{p}^{\nu} + \tilde{p}^{\mu}\tilde{k}^{\nu}](kpqS),\ (\mu abc) = {}_{\mu\nu\ \sigma}a^{\nu}b\ c^{\sigma},\ (abcd) = {}_{\mu\nu\ \sigma}a^{\mu}b^{\nu}c\ d^{\sigma}.$$

The antisymmetrical part is

$$T^{(a)}_{\mu\nu}(S) = i\,Re\{T_s(\mu\nu qS) + T_{pps}(pS)(\mu\nu pq) + T_{pqs}(qS)(\mu\nu pq) + T_{kps}(pS)(\mu\nu kq) + T_{kqs}(qS)(\mu\nu kq)\}. \tag{8}$$

The invariant amplitudes

The general form of the matrix element (2) can be chosen by analogy with the process of the pion electroproduction on the nucleons [17]. If the gauge invariance and the space parity conservation take place, we have

$$M = \frac{e^2}{q^2}\varphi^+_{\pi}\,\Sigma^6_{i=1}\,\bar{u}(p_1)\gamma_5 M_i\, v(p_2)\,A_i,\ \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3, \tag{9}$$

where φ_π is the pion wave function and the M_i structures have the following form

$$M_1 = \frac{1}{2}\gamma^{\mu}\gamma^{\nu}F_{\mu\nu},\ M_2 = p^{\mu}k^{\nu}F_{\mu\nu},\ M_3 = \gamma^{\mu}k^{\nu}F_{\mu\nu},\ M_4 = (\gamma^{\mu}p^{\nu} - 2M\gamma^{\mu}\gamma^{\nu})F_{\mu\nu}, \tag{10}$$

$$M_5 = q^{\mu}k^{\nu}F_{\mu\nu},\ M_6 = q^{\mu}\gamma^{\nu}F_{\mu\nu},\ F_{\mu\nu} = l_{\mu}q_{\nu} - l_{\nu}q_{\mu}.$$

The invariant amplitudes $A_i(i = 1-6)$ are the complex functions of three independent variables: for example, $q^2 -$ the square of the total invariant mass of the final hadrons, and $s_{1,2} = (p_{1,2} + k)^2$ -the square of the invariant masses of the $N\pi^0$ and $\bar{N}\pi^0$ systems.

Equations (9) and (10) mean that, in general case, J_μ can be written as follows

$$J_\mu = \varphi^+_{\pi}\,\bar{u}(p_1)\gamma_5\hat{O}_\mu\, v(p_2), \tag{11}$$

where the matrix $\hat{O}_\mu$ has the form

$$\hat{O}_\mu = (k\cdot q\,p_\mu - p\cdot q\,k_\mu)A_2 - q^2\tilde{k}_\mu A_5 + (k\cdot q\,A_3 + p\cdot q\,A_4 - q^2\,A_6)\gamma_\mu + \tag{12}$$

$$+(A_6\,q_\mu - A_4\,p_\mu - A_3\,k_\mu)\hat{q} + (A_1 - 4M\,A_4)(\gamma_\mu\,\hat{q} - q_\mu).$$

The hadronic structure functions in Eq. (4), which are independent on the nucleon polarization states, can be written, in general case, in terms of the invariant amplitudes as follows

$$H_1 = 2\{[m^2q^2 + (p\cdot q)^2 - (k\cdot q)^2]\,|A_{14}|^2 + p^2\,|k\cdot q\,A_3 + p\cdot q\,A_4 - q^2\,A_6|^2 +$$

$$+4Mp\cdot q\,Re[k\cdot q\,A_3 + p\cdot q\,A_4 - q^2\,A_6]A^*_{14}\},\ A_{14} = A_1 - 4MA_4, \tag{13}$$

$$H_2 = 2\{(p\cdot q)^2\,|A_4|^2 + [q^2(p^2 + q^2) - (p\cdot q)^2]\,|A_3|^2 + (q-k)^2[(p\cdot q)^2\,|A_2|^2 + q^4\,|A_5|^2]$$

$$+q^2(q^2\,|A_6|^2 - |A_{14}|^2)\} + 4\,Re\{-p\cdot q(p\cdot q\,A_2 + q^2\,A_5)A^*_{14} + p\cdot q(q^2\,A_3 - 2Mp\cdot q\,A_2)A^*_4 +$$

$$+q^2[2M(p\cdot q\,A_2+q^2A_5)-q^2A_3-p\cdot q\,A_4]A_6^*-2Mq^2p\cdot q\,A_2A_3^*+q^2[p\cdot q(q-k)^2A_2-2M(q^2A_3+p\cdot q\,A_4)]A_5^*\}, \quad (14)$$

$$H_3=2\{(k\cdot q)^2[(q-k)^2|A_2|^2-|A_3|^2]+q^2|A_{14}|^2-q^4|A_6|^2+[(k\cdot q-q^2)^2+q^2p^2]|A_4|^2\}+$$

$$+4Re\{k\cdot q(q^2-k\cdot q)A_2(A_{14}^*+2MA_4^*)+q^2(2MA_4A_{14}^*+k\cdot q\,A_3A_6^*)\}, \quad (15)$$

$$H_4=2\{k\cdot q\,p\cdot q(|A_3|^2-(q-k)^2|A_2|^2)+p\cdot q(q^2-k\cdot q)|A_4|^2\}+2Re\{2Mq^2k\cdot qA_2A_3^*+$$

$$+[p\cdot q(2k\cdot q-q^2)A_2+2Mq^2A_3-q^2(q^2-k\cdot q)A_5]A_{14}^*+q^2[(k\cdot q-q^2)A_4-2Mk\cdot qA_2-p\cdot qA_3]A_6^*+$$

$$+q^2[-k\cdot q(q-k)^2A_2+2M(k\cdot q-q^2)A_4]A_5^*+[2Mp\cdot q(2k\cdot q-q^2)A_2+q^2(q^2+p^2-k\cdot q)A_3]A_4^*\}, \quad (16)$$

$$H_5=-2q^2Im\{[2M\,A_3-q\cdot p\,A_2-(q^2-k\cdot q)A_5]A_{14}^*+[-2Mk\cdot qA_2+q\cdot p\,A_6-$$

$$-(p^2+q^2-k\cdot q)A_4]A_3^*+[k\cdot q(q-k)^2A_2-2M(k\cdot q-q^2)A_4]A_5^*+[(q^2-k\cdot q)A_4+2Mk\cdot qA_2]A_6^*-2Mq\cdot p\,A_2A_4^*\}. \quad (17)$$

The relations between the invariant amplitudes and the hadronic structure functions in Eqs. (7) and (8), which depend on the nucleon polarization states, are more complicated and read

$$T_{kk}=[p\cdot q(A_4-2MA_2)-q^2(2MA_5+A_6)+\frac{1}{2q\cdot p_1}[q^2(p^2+q^2-k\cdot q)+p\cdot q(k\cdot q-p\cdot q)]A_3]A_{14}^*$$

$$+\frac{p^2}{2q\cdot p_1}(k\cdot q\,A_3+p\cdot q\,A_4-q^2A_6)(p\cdot q\,A_2^*+q^2A_5^*), \quad (18)$$

$$T_{kp}=[q^2A_6-p\cdot q\,A_4+\frac{(p\cdot q)^2+m^2q^2-k\cdot q(p\cdot q+q^2)}{2q\cdot p_1}A_3]A_{14}^*+$$

$$2M(p\cdot q\,A_3A_4^*-q^2A_3A_6^*)+\frac{(p_1+p_2)^2}{2q\cdot p_1}(k\cdot q\,A_3+p\cdot q\,A_4-q^2A_6)(p\cdot q\,A_2^*+q^2A_5^*), \quad (19)$$

$$T_{pk}=[k\cdot q(2MA_2+A_3)-q^2A_6-(k\cdot q-q^2-\frac{p^2q^2}{2q\cdot p_1})A_4]A_{14}^*-\frac{k\cdot q\,p^2}{2q\cdot p_1}(k\cdot q\,A_3+p\cdot q\,A_4-q^2A_6)A_2^*, \quad (20)$$

$$T_{pp}=[q^2A_6-k\cdot q\,A_3-[p\cdot q-\frac{(p\cdot q)^2+q^2m^2-(k\cdot q)^2}{2q\cdot p_1}]A_4]A_{14}^*+$$

$$2M(q^2A_6-k\cdot q\,A_3)A_4^*-\frac{k\cdot q(p_1+p_2)^2}{2q\cdot p_1}(k\cdot q\,A_3+p\cdot q\,A_4-q^2A_6)A_2^*, \quad (21)$$

$$T_K=\frac{1}{q\cdot p_1}\{[q^2A_6-p\cdot q\,A_4-(p\cdot q+q^2)A_3](p\cdot q\,A_2^*+q^2A_5^*)-q^2A_3A_{14}^*\}, \quad (22)$$

$$T_P=\frac{1}{q\cdot p_1}\{k\cdot q[q^2A_6-(k\cdot q-q^2)A_4-k\cdot q\,A_3]A_2^*+q^2A_4A_{14}^*\}, \quad (23)$$

$$T_{KP}=-\frac{1}{2q\cdot p_1}\{[q^2(p\cdot q+k\cdot q)A_6-k\cdot q(2p\cdot q+q^2)A_3-p\cdot q(2k\cdot q-q^2)A_4]A_2^*+$$

$$q^2(A_4-A_3)A_{14}^*+q^2[q^2A_6-k\cdot q\,A_3+(q^2-k\cdot q)A_4]A_5^*\}, \quad (24)$$

$$T_G=2(k\cdot q\,A_3+p\cdot q\,A_4-q^2A_6)A_{14}^*, \quad (25)$$

$$T_s=2Mp\cdot q(k\cdot q|A_3|^2+p^2|A_4|^2+q^2|A_6|^2+|A_{14}|^2)+$$

$$[2Mp\cdot q(k\cdot q-m^2)A_2+[k\cdot q(p^2-k\cdot q)+(p\cdot q)^2+m^2q^2]A_3+p\cdot q(4M^2+p^2)A_4+$$

$$2M[(k\cdot q)^2-m^2q^2]A_5+[(k\cdot q-q^2)^2-(p\cdot q)^2-4M^2q^2]A_6]A_{14}^*+$$

$$[[(4M^2-p^2)(p\cdot q)^2+p^2k\cdot q\,(k\cdot q-q^2)]A_2-2M[p\cdot q\,(k\cdot q+q^2)A_3+((p\cdot q)^2+p^2q^2)A_4]+$$

$$(4M^2-p^2)p\cdot qq^2A_5]A_6^*+[-(4M^2-p^2)p\cdot q\,k\cdot q\,A_3+p^2[(k\cdot q)^2-m^2q^2]A_4]A_5^*+$$

$$[p^2\,p\cdot q(k\cdot q-m^2)A_2+2M[p^2\,k\cdot q+(p\cdot q)^2]A_3]A_4^*+p^2\,k\cdot q\,(k\cdot q-m^2)A_2A_3^*, \quad (26)$$

$$T_{pps}=2M(k\cdot q|A_3|^2+p\cdot q|A_4|^2)+p\cdot q\,(A_3+A_4)A_{14}^*+$$

$$[(k\cdot q-p\cdot q)(k\cdot q-q^2)A_2-2Mq^2(A_3+A_4)]A_6^*+[(k\cdot q)^2-m^2q^2]A_4A_5^*+$$

$$[(k\cdot q-m^2)\,p\cdot q\,A_2+2M(k\cdot q+p\cdot q)A_3]A_4^*+(k\cdot q-m^2)\,k\cdot q\,A_2A_3^*+(k\cdot q-q^2)(k\cdot qA_3-q^2A_6)A_5^*, \quad (27)$$

$$T_{pqs}=2M(k\cdot q|A_3|^2+q^2|A_6|^2+p\cdot q\,A_3A_4^*)+2(q\cdot p_2\,A_6-k\cdot p_2\,A_3)A_{14}^*+$$

$$[(4M^2-p^2)(p\cdot q\,A_2+q^2A_5)-2M[(k\cdot q+q^2)A_3+p\cdot q\,A_4]]A_6^*-(4M^2-p^2)\,k\cdot q\,A_3A_5^*, \quad (28)$$

$$T_{kps}=[2M[(k\cdot q-p\cdot q)\,A_2-q^2A_5]+(q^2-k\cdot q)(A_3+A_4)]A_{14}^*+$$

$$p\cdot q[(p\cdot q-k\cdot q)\,A_2+q^2\,A_5]A_6^*+(p\cdot q-p^2)(k\cdot q\,A_2A_3^*+p\cdot q\,A_2A_4^*)+$$

$$[-k\cdot q\,p\cdot q\,A_3+[-k\cdot q\,p\cdot q+q^2(p\cdot q-p^2)]A_4]A_5^*, \quad (29)$$

$$T_{kqs}=-2M|A_{14}|^2+[2M[(k\cdot q-q^2)A_5-p\cdot q\,A_2]+2q\cdot p_2\,(A_3-A_6)-p^2A_4]A_{14}^*+$$

$$p^2[k\cdot q\left(A_2A_6^*-A_2A_3^*\right)-p\cdot q\,A_2A_4^*+\left(k\cdot q-q^2\right)A_4A_5^*]. \quad (30)$$

The nucleon polarization 4-vector

In the rest frame of the nucleon $(\mathbf{p}_1=0)$ its polarization four-vector has the form $S_r^\mu=(0,\mathbf{n})$, $\mathbf{n}^2=1$, and, in general case, 3-vector $\mathbf{n}$ has three independent components: two in the plane $(\mathbf{q},\mathbf{k})$ and one along 3-vector $[\mathbf{k}\times\mathbf{q}]$. It means that in arbitrary Lorentz system 4-vector S^μ can be expressed by means of 4-vectors of the particle momenta and expanded by three independent 4-vectors: longitudinal S_L^μ, transversal S_T^μ and normal S_N^μ.

Let us choose the longitudinal polarization such that in the rest frame $\mathbf{n}=-\mathbf{q}/|\mathbf{q}|$. It can be expressed in terms of 4-vectors p_1^μ and q^μ, and has the following form

$$S_L^\mu=\frac{q\cdot p_1\,p_1^\mu-M^2\,q^\mu}{M\,K},\ K=\sqrt{(q\cdot p_1)^2-q^2M^2},\ S_L\cdot p_1=0,\ S_L^2=-1. \quad (31)$$

Note also that in c. m. s. of the process (1), where $\mathbf{q}=0$,

$$S_L^\mu=\left(\frac{|\mathbf{p}_1|}{M},\frac{E_1\,\mathbf{p}_1}{M|\mathbf{p}_1|}\right).$$

The transversal polarization was chosen to be orthogonal to the longitudinal one, that is

$$S_T^\mu\cdot S_L^\mu=0,\to S_T\cdot p_1=0,\ S_T\cdot q=0,\ S_T^2=-1.$$

The relation $S_T \cdot q = 0$ indicates that the polarization 4-vector S_T^μ is expressed in terms of the 4-vectors $\tilde{p}_1$, $\tilde{p}_2$ and $\tilde{k}$. Only two 4-vectors are independent since we have the following relation $\tilde{p}_1 + \tilde{p}_2 + \tilde{k} = 0$. Choosing $\tilde{p}_1$ and $\tilde{k}$ one can obtain

$$S_T^\mu = \frac{(q^2\, k\cdot p_1 - q\cdot p_1\, k\cdot q)\,\tilde{p}_1^\mu + [(q\cdot p_1)^2 - q^2 M^2]\tilde{k}^\mu}{K\,N}, \tag{32}$$

where

$$N = \sqrt{-(\mu k p_1 q)(\mu k p_1 q)}\,,\ N^2 = 2k\cdot q\, k\cdot p_1\, q\cdot p_1 - q^2\,(k\cdot p_1)^2) - M^2 (k\cdot q)^2 - m^2 (q\cdot p_1)^2 + q^2 M^2 m^2\,.$$

In both coordinate system (the rest system and c. m. s.) the 4-vector S_T^μ has not the time component and its space component is

$$\frac{[\mathbf{q}\times[\mathbf{k}\times\mathbf{q}]]}{|[\mathbf{q}\times[\mathbf{k}\times\mathbf{q}]]|},\ \frac{[\mathbf{p}_1\times[\mathbf{k}\times\mathbf{p}_1]]}{|[\mathbf{p}_1\times[\mathbf{k}\times\mathbf{p}_1]]|}$$

in the rest frame and the c. m. s., correspondingly.

It is clear that the normal polarization is

$$S_N^\mu = \frac{(\mu k p_1 q)}{N} = (0, \frac{[\mathbf{k}\times\mathbf{p}_1]}{|[\mathbf{k}\times\mathbf{p}_1]|})\,(in\, the\, c.m.s.) = (0, \frac{[\mathbf{q}\times\mathbf{k}]}{|[\mathbf{q}\times\mathbf{k}]|})\,(in\, the\, rest). \tag{33}$$

KINEMATICS

We define five independent invariant variables as follows

$$s = (k_1 + k_2)^2 = (p_1 + p_2 + k)^2,\ s_1 = (p_1 + k)^2 = (k_1 + k_2 - p_2)^2, \tag{34}$$

$$s_2 = (p_2 + k)^2 = (k_1 + k_2 - p_1)^2,\ t_1 = (k_1 - p_1)^2 = (p_2 + k - k_2)^2,\ t_2 = (k_2 - p_2)^2 = (p_1 + k - k_1)^2.$$

The scalar products of the 4-momenta in the process can be written in terms of these invariants as

$$2k_1\cdot p_2 = s - s_1 + t_2 - m_e^2,\ 2k_2\cdot p_1 = s - s_2 + t_1 - m_e^2,\ 2k_1\cdot k = s_1 + t_1 - t_2 - M^2, \tag{35}$$

$$2k_2\cdot k = s_2 + t_2 - t_1 - M^2,\ 2p_1\cdot p_2 = s - s_1 - s_2 + m^2,\ 2k_1\cdot k_2 = s - 2m_e^2,$$

$$2k_2\cdot p_2 = M^2 + m_e^2 - t_2,\ 2k_1\cdot p_1 = M^2 + m_e^2 - t_1,\ 2k\cdot p_1 = s_1 - M^2 - m^2,\ 2k\cdot p_2 = s_2 - M^2 - m^2.$$

The limits of the changing of the invariant variables can be obtained from the condition of positivity of the quantity $(-\Delta) = (k_1 k_2 p_1 p_2)^2$, where Δ is the Gramian determinant. It has a form

$$\Delta = \frac{1}{16}\begin{vmatrix} 2m_e^2 & s - 2m_e^2 & M^2 + m_e^2 - t_1 & s - s_1 + t_2 - m_e^2 \\ s - 2m_e^2 & 2m_e^2 & s - s_2 + t_1 - m_e^2 & M^2 + m_e^2 - t_2 \\ M^2 + m_e^2 - t_1 & s - s_2 + t_1 - m_e^2 & 2M^2 & s - s_1 - s_2 + m^2 \\ s - s_1 + t_2 - m_e^2 & M^2 + m_e^2 - t_2 & s - s_1 - s_2 + m^2 & 2M^2 \end{vmatrix}.$$

Taking into account the azimuthal symmetry relative to the line of the colliding electron-positron beams, the phase space of the final particles can be written as [18]

$$d\,R_3 = \frac{\pi}{16(s - 2m_e^2)}\frac{dt_1\, dt_2\, ds_1\, ds_2}{\sqrt{-\Delta}}, \tag{36}$$

Note, that we can neglect, with the very high accuracy, the electron mass in our calculations.

All the scalar products in hadronic part depend on the variables s, s_1 and s_2

$$q^2 \equiv s,\; p^2 = 2M^2 - m^2 + s_1 + s_2 - s,\; q\cdot p = k\cdot p = \frac{s_1 - s_2}{2},\; k\cdot q = \frac{s_1 + s_2}{2} - M^2,\; d_1 = s_2 - M^2,\; d_2 = s_1 - M^2.$$

In further works we are going to concentrate on the double differential distributions. To study the (s_1,t_2) or (s_2,t_1) -distributions, it is enough to measure the 4-momentum p_1 or p_2, respectively. To investigate the (s_1,s_2), (s_1,t_1), (s_2,t_2), (t_1,t_2) -ones, we have to measure both p_1 and p_2.

Let us consider the ranges of the invariant variables to study the (s_1,s_2) distribution. In this case, it is necessary to integrate over t_1 and t_2. From the positivity of the quantity $(-\Delta)$, we conclude

$$t_{1-} \le t_1 \le t_{1+},\; t_{1\pm} = \frac{A(s,s_1,s_2,t_2) \pm 2\sqrt{B(s,s_1,s_2)\,C(s,s_1,t_2)}}{(s+s_1-M^2)^2 - 4ss_1}, \tag{37}$$

$$A(s,s_1,s_2,t_2) = m_e^2[2M^4 - M^2(3s_1+s_2) + ss_2 - 2m^2 s - s_1(s - s_1 - s_2)]$$

$$-M^2[m^2 s + s_1(s_2 - 2s - t_2) + t_2(2s - s_2)] - t_2[s(s_1 + s_2 - s - 2m^2) + s_1 s_2] + m^2 s(s - s_1) + M^6 - M^4(s + s_1 + t_2) - s_1 s_2(s - s_1),$$

$$B(s,s_1,s_2) = s_1 s_2(s_1 + s_2 - s) + 2M^6 - M^4(s + s_1 + s_2 + m^2)$$

$$+M^2[ss_2 + s_1(s - 2s_2) + m^2(s_1 + s_2 - 2s)] + m^4 s + m^2[s(s - s_1 - s_2) - s_1 s_2],$$

$$C(s,s_1,t_2) = s[t_2(s - s_1 + t_2 - M^2) + M^2 s_1] + m_e^2[M^4 - M^2(s + 2s_1) - s(s_1 + 2t_2) + s_1^2 + m_e^4 s].$$

The expression under the square root in Eq. (37) factorizes, and the limits on the variable $t_2(s_2)$ can be found from the condition $C(s,s_1,t_2) \ge 0$, $(B(s,s_1,s_2) \ge 0)$. For the variable t_2 they read

$$t_{2-} \le t_2 \le t_{2+}, t_{2\pm} = \frac{1}{2}\left[M^2 + 2m_e^2 - s + s_1 \pm \sqrt{\left(1 - \frac{4m_e^2}{s}\right)[(s + s_1 - M^2)^2 - 4ss_1]}\right] \tag{38}$$

As concerns the s_2 limits, we have

$$s_{2-} \le s_2 \le s_{2+},\; s_{2\pm} = \frac{1}{2s_1}\left(D(s,s_1) \pm \sqrt{F(s,s_1)G(s,s_1)}\right), \tag{39}$$

$$D(s,s_1) = M^4 - M^2(s - 2s_1 + m^2) + m^2(s + s_1) + s_1(s - s_1),$$

$$F(s,s_1) = (s + s_1 - M^2)^2 - 4ss_1,\; G(s,s_1) = (s_1 + m^2 - M^2)^2 - 4m^2 s_1.$$

Both expressions $F(s,s_1)$ and $G(s,s_1)$ have not to be negative, and we obtain

$$(m+M)^2 \le s_1 \le (\sqrt{s} - M)^2. \tag{40}$$

It is clear that the inequalities (38), (39) and (40) define the regions (s_1,s_2) and (s_1,t_2) which are plotted in Figs. 1 a) and 1 b), correspondingly. Because of the symmetry of the Gramian determinant at $(s_1 \quad s_2, t_1 \quad t_2)-$ permutation one can use the above inequalities to obtain also the region (s_2,t_1).

It is interesting to investigate the distribution over the nucleon-antinucleon invariant mass squared $s_{12} = (p_1 + p_2)^2 = 2M^2 + m^2 + s - s_1 - s_2$. For this goal, we define firstly the region (s_1,s_{12}) and use the inequality (see Eq.(37))

$$B(s,s_1,s_2 = 2M^2 + m^2 + s - s_1 - s_{12}) \ge 0$$

to obtain the limits on the variable s_1 at fixed values of the variable s_{12}

$$s_{1-} \le s_1 \le s_{1+}, s_{1\pm} = \frac{1}{2}[2M^2 + m^2 + s - s_{12} \pm \sqrt{(1-\frac{4M^2}{s_{12}})[(s+m^2-s_{12})^2 - 4m^2 s]}]. \tag{41}$$

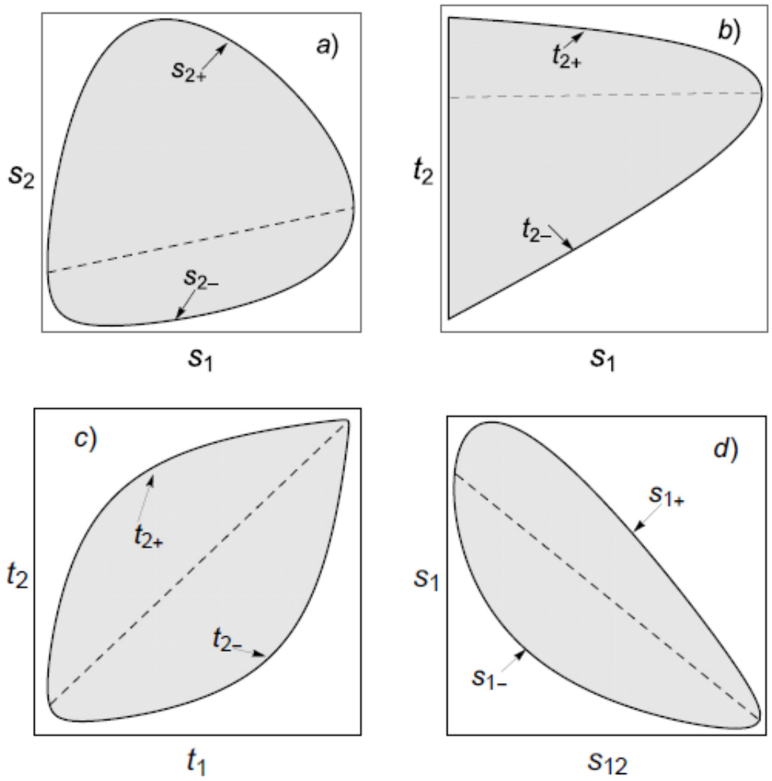

Figure 1. The kinematical double invariant variables regions calculated at $s = 10\,GeV^2$
(a) $-(s_1,s_2)$, (b) $-(s_1,t_2)$, (c) $-(t_1,t_2)$ and (d) $-(s_{12},s_1)$.

Taking into account that the expression under square root in Eq.(41) has not to be negative, one finds the limits on the variable s_{12}

$$4M^2 \le s_{12} \le (\sqrt{s} - m)^2.$$

As concerns the region (t_1,t_2), the corresponding boundaries are more complicated and the analytical expressions for them require additional short notation. We introduce

$$G(x,y,z,u,v,w) = -\frac{1}{2}\begin{vmatrix} 2u & u-v+x & u+w-y \\ u-v+x & 2x & w+x-z \\ u+w-y & w+x-z & 2w \end{vmatrix},$$

$$s_{1-} < s_1 < s_{1+},$$

$$s_{1\pm} = \frac{(a \pm b)}{(m_e^2 - s_2^+)^2 - 4t_1 s_2}, b = 2\sqrt{G\left(s,t_1,s_2,m_e^2,m_e^2,M^2\right)G\left(t_2,s_2,t_1,M^2,m_e^2,m^2\right)},$$

$$a = s_2^-\left[\ s\left(t_1 - m^2\right) + M^2\left(t_+ - M^2\right) - s_2 t_2\right] + s\left(t_2 s_2^+ - 2M^2 t_1\right) + m_e^2\left[\ m_e^2\left(s - 2M^2\right) + \right.$$

$$\left. + m^2\left(s - 2M^2 + 2s_2\right) + M^2\left(M^2 + t_+ + 2s_2^+\right) + st_- - s_2\left(t_+ + s_2\right)\right],$$

$$\frac{-\lambda_{12}\lambda_2 + a_1}{2t_2} < s_2 < \frac{\lambda_s\lambda_1 + b_1}{2m_e^2},$$

where

$$a_1 = t_-(t_2 - m_e^2) + m^2(-M^2 + m_e^2 + t_2) + M^2 t_+, b_1 = s(t_1 - M^2) + m_e^2(s + 2M^2),$$

$$s_2^{\pm} = s_2 \pm t_1, t_{\pm} = t_1 \pm t_2, \lambda(x,y,z) = x^2 - 2xy - 2xz + y^2 - 2yz + z^2,$$

$$\lambda_{1,2} = \sqrt{\lambda\left(t_{1,2}, m_e^2, M^2\right)}, \lambda_{12} = \sqrt{\lambda\left(t_1, t_2, m^2\right)}, \lambda_s = \sqrt{\lambda\left(s, m_e^2, m_e^2\right)}.$$

The boundaries of the region (t_1, t_2) are determined by the equation

$$\frac{-\lambda_{12}\lambda_2 + a_1}{2t_2} = \frac{\lambda_s \lambda_1 + b_1}{2m_e^2}. \tag{42}$$

It is clear that, at such high energies, the electron mass cannot influence the kinematics, and to simplify following calculations we go to the limit $m_e -> 0.$ The Eq.(42), in this limiting case, reads

$$\frac{t_1\left(M^2 - s - t_1\right)}{M^2 - t_1} = \frac{(t_2 - M^2)(\lambda_{12} + m^2) + t_2 t_- + M^2 t_+}{2t_2},$$

and gives

$$t_2^- < t_2 < t_2^+$$

$$t_2^{\pm} = \frac{a_2 \pm b_2}{2\left(M^2 - t_1\right)\left(M^2 - s - t_1\right)}, a_2 = M^2[2t_1(s + t_1) - m^2 s + 2M^4 - M^2(s + 4t_1)] +$$

$$+ s t_1(m^2 - s - t_1), b_2 = s\{M^4[M^4 + 4t_1(t_1 - M^2) + 2t_1(s + t_1)] +$$

$$+ t_1^2(s + t_1)(s + t_1 - 4M^2) + m^2(t_1 - M^2)[m^2(t_1 - M^2) + 2M^4 - 2t_1(s + t_1)]\}^{\frac{1}{2}},$$

$$\frac{a_3 - b_3}{2} \le t_1 \le \frac{a_3 + b_3}{2},\ a_3 = 2M(M + m) + m^2 - s,\ b_3 = \sqrt{s - m^2}\sqrt{s - (m + 2M)^2}.$$

The regions (t_2, t_1) and (s_1, s_{12}) are plotted in the lower row in Fig. 1.

In addition, the dependence on the invariant mass of the $N\bar{N}$ -system is also of the great interest. It is evident, that to study this dependence, it is enough to measure the pion 4-momentum k only. This ensures, at least, investigations of the double distributions over invariants $\bar{t}_1 = (k_1 - k)^2$, $s_{12} = (p_1 + p_2)^2$ or $\bar{t}_2 = (k_2 - k)^2, s_{12}$. To perform the corresponding calculations, it is necessary to investigate the Gramian determinant using $\bar{t}_1$ (or $\bar{t}_2$) and s_{12} variable of five independent invariant variables. In present paper such kind of distributions are not considered but we hope to study them in the next publications.

The matrix element squared of the process (1), as well as the differential cross section, are defined by the convolution of the leptonic and hadronic tensors (see Eq. (6)), that can be expressed in terms of the invariant amplitudes and chosen invariant variables. To calculate the contribution of any dynamical mechanism to the cross section, it is enough to know only the corresponding invariant amplitudes and this simplification is an essential advantage of the developed here formalism.

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ЗАГАЛЬНИЙ АНАЛІЗ РЕАКЦІЇ $e^+ + e^- \to N + \bar{N} + \pi^0$

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У наближенні однофотонної анігіляції виконано загальний аналіз реакції $e^+ + e^- \to N + \bar{N} + \pi^0$ у випадку поздовжньо поляризованого пучка електронів з врахуванням поляризаційних станів кінцевого нуклона. Цей аналіз є корисним для опису внесків континума (не резонасний) та резонансного (з різними можливими векторними мезонами або збудженими баріонами у проміжних віртуальних станах діаграм Фейнмана). Для виразу матричного елемента у термінах шести комплексних незалежних інваріантних амплітуд було використано збереження електромагнітних струмів адронів та P-інваріантність електромагнітної взаємодії адронів. Була визначена загальна структура адронного тензора у випадку неполяризованих кінцевих адронів і поляризованого нуклона. Спіннезалежна частина адронного тензора визначається п’ятьма структурними функціями, а спінзалежна – 13 структурними функціями. Поперечна, повздовжня та нормальна компоненти нуклонного чотиривектора поляризації виражені у термінах чотиривекторів імпульсів частинок. Застосовані п’ять незалежних інваріантних змінних що описують реакцію. Досліджені межі існування цих змінних. Кінематичні області подвійних інваріантних змінних приведені на рисунку. Досліджена кінематика яка зручна для дослідження розподілів по інваріантній масі.

Ключові слова: поляризаційні явища, електрон, іваріантні амплітуди, адронний тензор, кінематика.