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LEVEL STRUCTURE OF ^{58}Cu WITHIN MODIFIED SURFACE DELTA-INTERACTION†

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The nuclear shell model with modified surface delta interaction MSDI was used to calculate the values of energy levels of the copper nucleus ^{58}Cu. The neutron and proton in the model space $1p_{3/2}$ $0f_{5/2}$ $1p_{1/2}$ of the copper nucleus occur outside the closed core ^{56}Ni. This research investigates the excitation energy and angular momentum. As a consequence, theoretical approaches are used to uncover a collection between excitation energies and classical coupling angles θa,b at various orbitals. Finally, we demonstrate that our results are supported by experimental evidence: Excitation energies have two major functions, both of which are influenced by classical coupling angles but are unaffected by angular momentum I.
Keywords: energy levels, modified surface delta interaction, ^{58}Cu, classical coupling angle
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Many research have focused on nuclear isotopes, and the element copper will be discussed in this topic [1,2]. According to Ref [3], Martin and Breckon measured ^{58}Cu in 1952. In the last 50 years, many approaches for the effective interaction of nuclei in the "s-d and f-p shells" have been presented. Using the Hamada-Johnston potential, Kuo and Brown estimated the effective two-body interaction in the *s-d* and *f-p* shells [4]. Richter et al. "New effective interactions for the *0f1p* shell used experimental approaches to derive 195 two-body matrix elements and four single particle energies in the fp-shell, using Wildenthal's nonlinear fit general practice. [3] Talmi employed the surface delta interaction to determine the parameters of nuclear states on a magic core with few' nucleons'. Talmi's theory is based on the following assumptions: First, there is an inert core model of tight shell that works with central forces on valence nucleons; and second, there is 'residual interaction' between the valence nucleons that is induced by two-body forces. Schiffer [5] considered nuclei in which two (hole or particle) are present in addition to a closed shell. Essentially, Schiffer emphasizes that the general behavior of the effective interaction in terms of the angle between the interacting nucleons' angular momenta is a trait that was subsequently linked to the effective interaction's short-range nature [6,7]. In Ref [8] stated the angle between the proton and neutron angular momentum vectors ja and jb. In the mass range A =50 to 102, J. Kostensalo and J. Suhonen [9] calculated the characteristics pairing interaction for even-even) reference nuclei. We used modified surface delta interaction and surface delta interaction to investigate the excitation energies for states two-hole and two-particle [10,11,12], one particle – one hole [13,14]. In conclusion, investigations accept the new study's purpose, primarily by the use of MSDI, which predicts low-lying levels structure of ^{58}Cu nuclei.

THEORY

The Schrödinger equation has been essential steps to a particular appropriate Hamiltonian, so that a typical shell-model of effective Hamiltonian may be stated as [11,12].

$$\mathrm{H} = \sum_{k=l} H_0 + \sum_{k\le l} V_{kl} \tag{1}$$

where $\sum_{k\langle l} V_{kl}$ is the residual 2-body interaction, which exists in addition to the average shell-model potential, and we can express this as:

$$\sum_{k<l} V_{kl} = \sum_{IM} \sum_{j_a \ge j_b} \sum_{j_c \ge j_d} \langle j_a j_b | V | j_c j_d \rangle_I \, a^+_{IM}(j_a j_b) a^+_{IM}(j_c j_d) \tag{2}$$

If ρ_j is single particle energy $\langle j_a j_b |V| j_c j_d \rangle = V^{I,T}_{ab,cd}$ is the matrix element [11,12,14]

If the 2 particles occupy the same level, the energy relative to the closed shell is:

$$\langle H \rangle = 2\rho_j + \langle j_a j_b | V | j_c j_d \rangle \tag{3}$$

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Several single-particle levels should be considered the basis for describing low-lying states; if there are two states, they should be indicated by $\left| j_a j_b IM \right\rangle$ and $\left| j_c j_d IM \right\rangle$ then their energies with respect to the core are given by [11,13,15]

$$\left\langle H \right\rangle_{11} = \rho_{j_a} + \rho_{j_b} + V^{IT}_{abab} \tag{4}$$

$$\left\langle H \right\rangle_{22} = \rho_{j_c} + \rho_{j_d} + V^{IT}_{cdcd} \tag{5}$$

$$\left\langle H \right\rangle_{12} = \left\langle H \right\rangle_{21} = V^{IT}_{abcd} \tag{6}$$

To estimate the matrix element for the residual nucleon-nucleon interaction using the MSDI potential [15,16]

$$V_{a,b} = -4\pi A_T \delta\, \Omega_{a,b}\, \delta\left(\hat{r}(a)\right) - R_0 \delta\left(\hat{r}(b) - R_0\right) + B\, \tau_a.\tau_b \tag{7}$$

where $\hat{r}(a)$, $\hat{r}(b)$ are the position vectors of interacting particles, R_0 is the nuclear radius[14] the strength of interaction A_T .The correction term $B\,\tau_a.\tau_b$ is introduced to account for the splitting between the groups of levels with different isospin. Such a form of interaction is called MSDI. The antisymmetrized matrix element of V^{IT}_{acdb} is given by[10,15,16]

$$\begin{aligned} V^{I,T}_{ab,cd} = & -\frac{A_T}{2(2I+1)} \times \sqrt{\frac{(2j_a+1)(2j_b+1)(2j_c+1)(2j_d+1)}{(1+\delta_{ab})(1+\delta_{cd})}} \times \\ & [(-1)^{l_a+l_b+j_c+j_d} h_I(j_a j_b) h_I(j_c j_d) \left[1-(-1)^{l_c+l_d+I+T}\right] - \\ & [k_I(j_a j_b) k_I(j_c j_d)] \left[1+(-1)^T\right] + \{\left[2T(T+1)-3\right]\ B + C\ \}\delta_{a,c}\delta_{b,d} \end{aligned} \tag{8}$$

where it is $h_I(j_a j_b) = \left\langle j_b \tfrac{1}{2} j_a \tfrac{-1}{2} \middle| I0 \right\rangle$ $k_I(j_a j_b) = \left\langle j_a \tfrac{1}{2} j_b \tfrac{1}{2} \middle| I1 \right\rangle$; where $\langle\ |\ \rangle$ is the Clebsh -Gordon coefficients

The comportment of the diagonal 2 - body matrix element as a function of the spin I ofparticle - particle) state is very distinctive when their value are plotted in a property way .Consider (particle – particle) in orbits j_a and j_b with $I = j_a + j_a$ one can write then [11,10,14]

$$I^2 = (j_a + j_a)^2 = j_a^2 + j_b^2 + 2\times(j_a j_b)\cos\ \theta_{a,b} \tag{9}$$

where $\theta_{a,b}$ is the angle between the vectors j_a and j_b. Since the length of vector j is given by $\sqrt{j(j+1)}$ one obtains from eq (10) in a classical picture[6,8,13,16]

$$\cos\ \theta_{a,b} = \frac{I(I+1) - j_a(j_a+1) - j_b(j_b+1)}{2\sqrt{j_a(j_a+1) j_b(j_b+1)}} \tag{10}$$

The I-dependence of the matrix element V^{IT}_{abcd} can thus be plotted as a function of the angle $\theta_{a,b}$ .The radial overlaps of the particle orbits for light nuclei differ from those for heavy nuclei. The proton -neutron configurations correspond to nucleon pair having mixed isospin and one find [13,16]

$$E_{I(p,n)} = 0.5\left\{\left(V^{IT}_{abcd}\right)_{\substack{T=1\\I}} + \left(V^{IT}_{abcd}\right)_{\substack{T=0\\I}}\right\} \tag{11}$$

Plotting the excitation energy of these states as a function of the corresponding angle $\theta_{a,b}$ determined as specified by Eq.(10). For neutron and proton in various orbits the absolute value of average two body energy is given by[10,15]:

$$\overline{E} = \left| \sum_{I} (2I+1) E_I \left\{ \sum_{I} (2I+1) \right\}^{0.5} \right| \tag{12}$$

With E_I defined by Eq. (11) .

RESULTS AND DISCUSSION

The main properties of nuclear structure for ground bands of ^{58}Cu nuclei were calculated using MSDI in this paper. The valence nucleons of these nuclei are dispersed in model space $1p_{3/2}$, $0f_{5/2}$, $1p_{1/2}$. MSDI was used to determine the energy levels and classical coupling angles $\theta_{a,b}$ for(neutron-proton) in these calculations. The nuclear shell model for nucleus^{58}Cu has one (neutron and proton) outside the inert core ^{56}Ni. The nucleon's that have taken up residence in the model space $1p_{3/2}$, $0f_{5/2}$, $1p_{1/2}$.

The original MSDI Hamiltonian is also modified to account for the ground state energy. Energy levels were obtained using the equations (4,5,6 and 8). As a result, it's discovered that the experimental value's acceptability is quite high. Configurations mixing between the orbits are used to include the neutron and proton contributions.

To find energy levels, use the single particle energy .For proton particle , $\rho 1p_{3/2}$ = -0.6901MeV, $\rho 0f_{5/2}$ =0.3381MeV, and $\rho 1p_{1/2}$ =0.4161MeV may be used; however, for state neutron particle, $\rho 1p_{3/2}$ = -10.2543MeV, $\rho 0f_{5/2}$ =-9.4861MeV, and $\rho 1p_{1/2}$ =-9.1422MeV can be used. The use of particle energies inside the space of the aforementioned model, [17,18]. The angular momentum possibilities for this nucleus range from 0 to 5.

Table 1. A comparison between a theoretical result and experimental result excitation energies, MeV, for ^{58}Cu nucleus by using MSDI

I^{π}	Energy Levels	Energy Levels	I^{π}	I^{π}	Energy Levels	Energy Levels	I^{π}
Theor. Res.		Exp. Res. [19]		Theor. Res.		Exp. Res. [19]	
1^+_1	0.0000	0.0000	1+	2^+_4	3.111	----	----
		0.2029 0.4436 1.051	0+ 3+ 1+	1^+_5	3.26	3.230	----
2^+_1	1.43	1.427	2+	2^+_5	3.28	3.2802	(0+ to 4+)
1^+_2	1.663	1.549	(4+)	1^+_6	3.29	3.310 3.4210 3.4601	---- (7+) (1)+
0^+_1	1.79	----	----	3^+_4	3.5111	3.5126	----
3^+_1	1.8214	1.647	(3+)	5^+_1	3.5743	3.570	----
2^+_2	1.8723	1.652	2+	0^+_2	3.6322	3.6779	(1)+
3^+_2	1.9211	----	----	3^+_5	3.6356		
4^+_1	2.0801	2.0650 2.070 2.170 2.249 2.270	(5+)	2^+_6	3.7645	3.717 3.820	(1)+
1^+_3	2.556	----	----	4^+_3	3.9545	3.890	----
		2.690	4+	3^+_6	4.1403	4.010	----
2^+_3	2.7404	2.7502 2.780 2.8152 2.840 2.9206 2.949	(4+) (5+) (1)+	2^+_7	4.1823	4.0656	(7+)
3^+_3	3.0821	2.9309	(0+ to 4+)	2^+_8	4.5755	4.210 4.4414 4.720 5.065 12.034	 (8+) (1)+ (1)+
1^+_4	3.0834	----	----	1^+_7	12.4011	12.45	----
4^+_2	3.0911	----	----	0^+_3	12.543	12.520	(15+)

Theoretically, the energy level (1.8214, 3.0821and 3.28) MeV was uncertain at the state (3^+_1, $3^+_3$and 2^+_5) .The energy of the states ($2^+_1$and 2^+_2) near the experimental data. The levels 2^+_3, 1^+_6, 3^+_4,5^+_1,2^+_6,4^+_3 and $1^+_7$with practical energies (2.780, 3.310, 3.5126, 3.570, 3.820, 3.890 and 12.45) MeV ,respectively correspond to state for which the angular momentum and/or parity of the corresponding state are not established experimentally .

The new energy levels which are expected for this nucleus in the states 0^+_1, 3^+_2, 1^+_3, 1^+_4, 4^+_2 and 2^+_4were not well established experimentally.

The particle orbits have a low overlap in the style of the curve in Fig.1 for $\theta_{a,b}$ =90, resulting in a weak interaction. The orbits of (neutron- proton)interacting in opposing directions have a high overlap for $\theta_{a,b}$ =180. Because the nuclear force has a limited range, the contact will be strong. This interaction explains why the curves in Fig.1 (A, B and C) have an appositive slope for differences ranging from 180 to 90 degrees. For smaller angles, the Pauli principle expresses itself. The two isospin coupling metrics must be recognized for $\theta_{a,b}$ = 0.0 and.

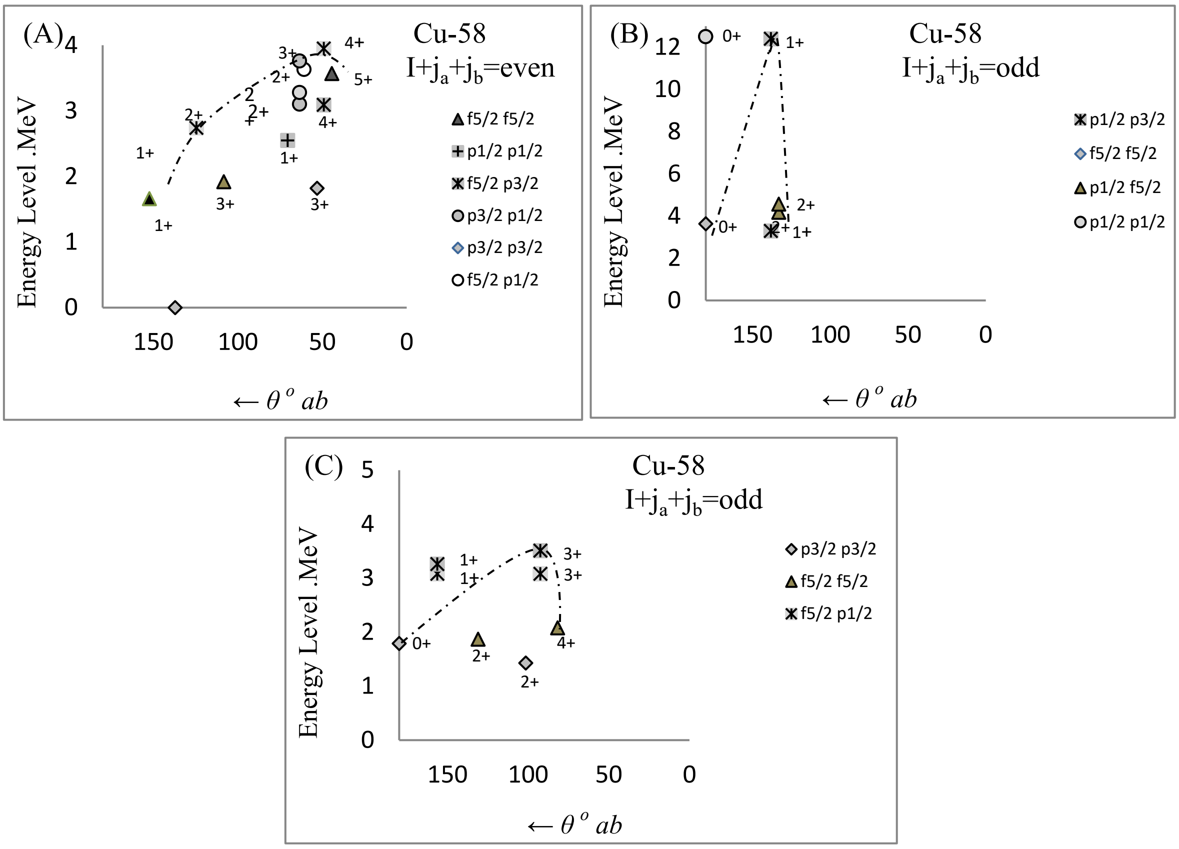

Figure 1. The relationship between classical coupling angles of an even and an odd cases, with the energy levels of all possible states

The particle occupies a spatially symmetric particle state in the $\theta_{a,b}$ =180 scenario, which results in a significant negative matrix element due to the high short-range attraction. When T=1, the (neutron- proton) constitute a spatially antisymmetric instance, and their relative space increases when the angle is reduced to 0.0.

The curves displayed in Fig. 1 A. may be created by plotting the excitation energy of these states as a function of the identical angle supplied by Eq. (10) for even states and Fig. 1 (B and C) for odd states of effective interaction determined from the data. Curvature is a measure of short-range attractive force.

Table 2 shows the lowest angle corresponds to the greatest J reading .The angular momentum of the even cases 5_1^+ which represents the highest angular momentum inside model space $0f_{5/2}0f_{5/2}$ is 44.4153 degrees as seen in table 3.3. The angle value is 152.3395 with angular momentum 1^+_2 in model space $0f_{5/2}0f_{5/2}$ reflects the lowest angular momentum. The angle of 81.7867 degrees is the angular momentum of the odd states 4_1^+,which represents the maximum angular momentum. The angle value 180.0000 corresponds to the lowest angular momentum 0_1^+in the same model space.

Table 2. According to cases of angular momentum I , all possible states of the semi-classical coupling angle value

I^π	Configuration ^{58}Cu	state	$\theta^O_{a,b}$	I^π	Configuration ^{58}Cu	state	$\theta^O_{a,b}$
1^+_1	$(\frac{3}{2}\frac{3}{2};\frac{3}{2}\frac{3}{2})$	Even	137.1665	1^+_5	$(\frac{3}{2}\frac{5}{2};\frac{3}{2}\frac{5}{2})$	Odd	156.4218
2^+_1	$(\frac{3}{2}\frac{3}{2};\frac{3}{2}\frac{3}{2})$	Odd	101.5369	2^+_5	$(\frac{3}{2}\frac{1}{2};\frac{3}{2}\frac{1}{2})$	Even	63.4349

I^{π}	Configuration ^{58}Cu	state	$\theta^{o}_{a,b}$	I^{π}	Configuration ^{58}Cu	state	$\theta^{o}_{a,b}$
1^+_2	$(\frac{5}{2}\frac{5}{2};\frac{5}{2}\frac{5}{2})$	Even	152.3395	1^+_6	$(\frac{3}{2}\frac{1}{2};\frac{3}{2}\frac{1}{2})$	Odd	138.1896
0^+_1	$(\frac{3}{2}\frac{3}{2};\frac{3}{2}\frac{3}{2})$	Odd	180.0000	3^+_4	$(\frac{3}{2}\frac{5}{2};\frac{3}{2}\frac{5}{2})$	Odd	92.5013
3^+_1	$(\frac{3}{2}\frac{3}{2};\frac{3}{2}\frac{3}{2})$	Even	53.13010	5^+_1	$(\frac{5}{2}\frac{5}{2};\frac{5}{2}\frac{5}{2})$	Even	44.4153
2^+_2	$(\frac{5}{2}\frac{5}{2};\frac{5}{2}\frac{5}{2})$	Odd	131.0823	0^+_2	$(\frac{5}{2}\frac{5}{2};\frac{5}{2}\frac{5}{2})$	Odd	180.0000
3^+_2	$(\frac{5}{2}\frac{5}{2};\frac{5}{2}\frac{5}{2})$	Even	108.3176	3^+_5	$(\frac{5}{2}\frac{1}{2};\frac{5}{2}\frac{1}{2})$	Even	60.7940
4^+_1	$(\frac{5}{2}\frac{5}{2};\frac{5}{2}\frac{5}{2})$	Odd	81.7867	2^+_6	$(\frac{3}{2}\frac{1}{2};\frac{3}{2}\frac{1}{2})$	Even	63.4349
1^+_3	$(\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2})$	Even	70.5287	4^+_3	$(\frac{3}{2}\frac{5}{2};\frac{3}{2}\frac{5}{2})$	Even	49.1066
2^+_3	$(\frac{3}{2}\frac{5}{2};\frac{3}{2}\frac{5}{2})$	Even	124.5667	3^+_6	$(\frac{5}{2}\frac{1}{2};\frac{5}{2}\frac{1}{2})$	Even	60.7940
3^+_3	$(\frac{3}{2}\frac{5}{2};\frac{3}{2}\frac{5}{2})$	Odd	92.5013	2^+_7	$(\frac{5}{2}\frac{1}{2};\frac{5}{2}\frac{1}{2})$	Odd	133.0881
1^+_4	$(\frac{3}{2}\frac{5}{2};\frac{3}{2}\frac{5}{2})$	Odd	156.4218	2^+_8	$(\frac{5}{2}\frac{1}{2};\frac{5}{2}\frac{1}{2})$	Odd	133.0887
4^+_2	$(\frac{3}{2}\frac{5}{2};\frac{3}{2}\frac{5}{2})$	Even	49.1066	1^+_7	$(\frac{3}{2}\frac{1}{2};\frac{3}{2}\frac{1}{2})$	Odd	138.1896
2^+_4	$(\frac{3}{2}\frac{1}{2};\frac{3}{2}\frac{1}{2})$	Even	63.4349	0^+_3	$(\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2})$	Odd	180.0000

CONCLUSION

For energy levels, theoretical and experimental levels are desirable. As a result, there are much too many experimental excitation energies that are confirmed by calculations and new energy levels is found. As a result, MSDI theoretical calculations are generally consistent with experimental findings. Angler momentum's minimum values agree with the greatest angle. Finally, this highlights an essential fact: The MSDI is enough to show the nuclear structure of ^{58}Cu nuclei.

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REFERENCES

[1] D.V. Kutnii, D.D. Burdeynyi, S.A. Vanzha, and N.V. Rud, East Eur. J. Phys. **2**, 104 (2020). https://doi.org/10.26565/2312-4334-2020-2-09
[2] T.A. Younis, NeuroQuantology, **19**(1), 30 (2021). http://doi.org/10.14704/nq.2021.19.1.NQ21005
[3] T. Hamada, and I.D. Johnston, Nucl. Phys. **34**(2), 382 (1962). https://doi.org/10.1016/0029-5582(62)90228-6
[4] W. M. Martin, and S. W. Breckon, Canadian Journal of Physics, **30**(6), 643 (1952). https://doi.org/10.1139/p52-060
[5] A. Molinari, M.B.Johnson, H.A. Bethe, and W.M. Alberico, Nucl. Phys. A, **239**(1), 45 (1975). https://doi.org/10.1016/0375-9474(75)91132-X
[6] P. Van Isacker, EPJ Web Confer. **178**, 05002 (2018). https://doi.org/10.1051/epjconf/201817805002
[7] E. Caurier, G. Martínez-Pinedo, F. Nowacki, A. Poves, and A. P. Zuker, Rev. Mod. Phys. **77**, 427 (2005), https://doi.org/10.1103/RevModPhys.77.427
[8] J.P. Schiffer, Ann. Phys. **66**, 798 (1971). https://doi.org/10.1016/0003-4916(71)90082-0
[9] J. Kostensalo, and J. Suhonen, Phys. Rev. C, **95,** 014322 (2017). https://doi.org/10.1103/PhysRevC.95.014322
[10] D.N. Hameed, and A.K. Hasan, Indian Journal Physics, **95**, 1833 (2021). https://doi.org/10.1007/s12648-020-01873-6
[11] D.N. Hameed, and A.K. Hasan, J. Phys.: Conference Series. **1963**, 012062 (2021). https://doi.org/10.1088/1742-6596/1963/1/012062
[12] A.K. Hasan, and D.N. Hameed, NeuroQuantology, **19**(5), 61 (2021). http://doi.org/10.14704/nq.2021.19.5.NQ21049
[13] D.N. Hameed, and A.K. Hasan, Energy levels of nuclei 40Sc and 40K as a function of semi-classical coupling angle θ1,2 within the modified surface delta-interaction" Nucl. Phy. At. Energy, **20**(2), 146 (2019)
[14] D.N. Hameed, and A.K. Hasan" Energy Levels Of Isobaric Nuclei (^{16}N, ^{16}F) Within The Modified Surface Delta-Interaction Model" Ukrainian Journal of Physics, **63**, 579 (2018). http://dx.doi.org/10.15407/ujpe63.7.579
[15] R.D. Lawson Theory of the Nuclear Shell Model (Clarendon Press, Oxford, 1980).
[16] P.J. Brussaard, and P.W.M. Glauddemans, Shell-Model Applications in Nuclear Spectroscopy (North-Holland, Amsterdam, 1977).
[17] G. Audi, A.H. Wapstr, and C. Thibault, Nuclear Physics A, **729**, 337 (2003). https://doi.org/10.1016/j.nuclphysa.2003.11.003
[18] M.R. Bhat, Nuclear Data Sheets, **85**(3), 415 (1998). https://inis.iaea.org/search/searchsinglerecord.aspx?recordsFor=SingleRecord&RN=30021608
[19] Nesaraja, Caroline D., Scott D. Geraedts, and Balraj Singh. "Nuclear data sheets for A= 58." Nuclear Data Sheets, **111**(4), 897 (2010). https://doi.org/10.1016/j.nds.2010.03.003

СТРУКТУРА РІВНІВ ^{58}Cu З МОДИФІКОВАНОЮ ПОВЕРХНЕЮ ДЕЛЬТА-ВЗАЄМОДІЇ

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Для розрахунку значень енергетичних рівнів ядра міді ^{58}Cu використано модель ядерної оболонки з модифікованою поверхневою дельта взаємодією MSDI. Нейтрон і протон в модельному просторі $1p_{3/2}$ $0f_{5/2}$ $1p_{1/2}$ ядра міді зустрічаються поза замкнутим ядром ^{56}Ni. У цій роботі досліджується енергія збудження та кутовий момент. Як наслідок, для виявлення сукупності між енергіями збудження та класичними кутами зв'язку θa,b на різних орбіталях використовуються теоретичні підходи. Нарешті, ми демонструємо, що наші результати підтверджуються експериментальними доказами: енергії збудження мають дві основні функції, на обидві з яких впливають класичні кути зв'язку, але на них не впливає кутовий момент I.

Ключові слова: енергетичні рівні, модифікована поверхнева дельта взаємодія, ^{58}Cu, класичний кут зв'язку