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FAST ELECTROMAGNETIC WAVES ON METAMATERIAL'S BOUNDARY: MODELING OF GAIN[†]

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The paper presents the results of the study of properties of fast surface electromagnetic waves that propagate along the flat interface between the active metamaterial and air (or vacuum). The case of homogeneous and isotropic metamaterial is considered. The dispersion properties, the wave spatial attenuation, the phase and group velocities, as well as the spatial distribution of the electromagnetic field of the eigen TE and TM modes of such a waveguide structure are studied in the frequency range where the metamaterial has a simultaneously negative permittivity and permeability. It is shown that fast surface electromagnetic waves can exist in this waveguide structure and their properties are studied. It is shown that the phase speed of TM mode is several times higher than the speed of light in vacuum, while the phase speed of TE mode is slightly higher than the speed of light in vacuum. The TM mode is a direct wave in which the phase and group velocities have the same direction. It is obtained that the group velocity of the TM mode varies from zero to the about half of speed of light in vacuum, and reaches a minimum at a certain value of wave frequency, which depends on the characteristics of the metamaterial. It is shown that the penetration depth of the TM mode into the metamaterial is much smaller than into the vacuum. The TE mode is a backward wave with opposite directed phase and group velocities. The absolute value of the group velocity of the TE mode is about six times less than the speed of light in vacuum. In contrast to the TM mode the penetration depth of the TE mode into the metamaterial is much greater than in vacuum. The obtained properties of the fast surface electromagnetic waves can be used for modeling and design of modern generation and amplification devices containing metamaterials.
Keywords: metamaterial, fast surface electromagnetic wave, dispersion, attenuation, spatial wave structure
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The metamaterials are referred to the materials consisting of a small-size cells that play the role of atoms for electromagnetic waves. It's possible for such artificial materials to get various combinations of electromagnetic characteristics including simultaneously negative value of permittivity and permeability, so-called left-handed materials [1].

The generating and amplifying devices use the backward electromagnetic waves (phase and group velocities are opposite) and the fast electromagnetic waves (phase velocity exceeds the speed of light in a vacuum) [2-4].

In our previous papers [5,6] we has been studied the properties of slow surface electromagnetic waves on the left-handed materials. In the presented work we will study the fast surface electromagnetic waves on the left-handed materials.

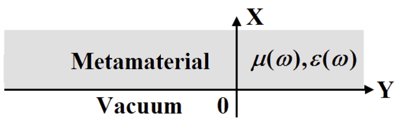

Figure 1. The geometry of problem

TASK SETTINGS

Let us investigate the electromagnetic waves of a surface type that propagate along a plane waveguide structure in the Z - axis direction (Fig. 1). The semi-infinite air (or vacuum) region with permittivity $\varepsilon_v = 1$ and permeability $\mu_v = 1$ takes place at $x \le 0$. The semi-infinite region $x \ge 0$ is filled by double negative metamaterial (DNM) that characterized by the commonly used the effective permittivity $\varepsilon(\omega)$ and permeability $\mu(\omega)$ [5]:

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega - i\nu_G)}, \tag{1}$$

$$\mu(\omega) = 1 - \frac{F\omega^2}{\omega(\omega - i\gamma_G) - \omega_0^2}, \tag{2}$$

where ω - is angular wave frequency, ν_G, γ_G - are effective electric and magnetic gain, respectively.

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The modeling was carried out for the metamaterial with $\omega_p / 2\pi = 10$ *GHz*, $\omega_0 / 2\pi = 4$ *GHz*, and $F = 0.56$ [7]. Under these conditions it is possible to obtain $\varepsilon(\omega) < 0$ and $\mu(\omega) < 0$ simultaneously in the frequency range $1 < \Omega = \omega/\omega_0 < 1.5$.

Let us consider the fast electromagnetic waves that propagate along interface between the metamaterial and air (or vacuum) and possess the exponentially decreasing field far away from the interface. The spatial-temporal dependence of the wave components has such a form:

$$E, H \propto E(x), H(x)\exp[i(k_3 z - \omega t)], \tag{3}$$

where the amplitudes of the wave fields decrease exponentially from the boundary and k_3 is complex wavenumber. Real part of k_3 characterizes the wavelength and imagine part – the decrement in the propagation direction.

Under the considered assumptions the set of Maxwell equations split into two independent sub-systems. One of them describes the TE-waves that contain (H_x, E_y, H_z) -components and another – TM-waves that contain (E_x, H_y, E_z) -components.

The boundary conditions (the continuity of the tangential wave field components at metameterial-vacuum interface $x = 0$) gives the dispersion equation as for the TM-mode:

$$\kappa_{\mathrm{v}}\varepsilon(\omega) + \varepsilon_{\mathrm{v}}\kappa_{\mathrm{m}} = 0, \tag{4}$$

as for the TE-mode:

$$\kappa_{\mathrm{v}}\mu(\omega) + \mu_{\mathrm{v}}\kappa_{\mathrm{m}} = 0, \tag{5}$$

where, $\kappa_v = \sqrt{k_3^2 - k^2}$, $\kappa_m = \sqrt{k_3^2 - \varepsilon(\omega)\mu(\omega)k^2}$ - are the transverse wave vectors in vacuum and metamaterial regions, respectively, $k = \omega / c$ and c is the light speed in vacuum.

The TM-wave possesses the components that have the following form in air (or vacuum) region $x < 0$:

$$\begin{aligned} \mathrm{H_y(x)} &= \mathrm{e}^{\kappa_{\mathrm{v}}\mathrm{x}}\mathrm{H_y}(0) \\ \mathrm{E_x(x)} &= \frac{\mathrm{e}^{\kappa_{\mathrm{v}}\mathrm{x}}\mathrm{k_3 H_y}(0)}{\mathrm{k}}, \\ \mathrm{E_z(x)} &= \frac{\mathrm{i}\mathrm{e}^{\kappa_{\mathrm{v}}\mathrm{x}}\kappa_{\mathrm{v}}\mathrm{H_y}(0)}{\mathrm{k}}. \end{aligned} \tag{6}$$

In the metamaterial region $x > 0$ the components of TM-wave has such form:

$$\begin{aligned} H_y(x) &= \mathrm{e}^{-\kappa_m x}H_y(0), \\ E_x(x) &= \frac{\mathrm{e}^{-\kappa_m x}k_3 H_y(0)}{\varepsilon(\omega)k}, \\ E_z(x) &= \frac{-\mathrm{i}\mathrm{e}^{-\kappa_m x}\kappa_m H_y(0)}{\varepsilon(\omega)k}. \end{aligned} \tag{7}$$

The TE-wave components of the following form in vacuum region $x < 0$:

$$\begin{aligned} \mathrm{E_y(x)} &= \mathrm{e}^{\kappa_{\mathrm{v}}\mathrm{x}}\mathrm{E_y}(0), \\ \mathrm{H_x(x)} &= -\frac{\mathrm{e}^{\kappa_{\mathrm{v}}\mathrm{x}}\mathrm{k_3 E_y}(0)}{\mathrm{k}}, \\ \mathrm{H_z(x)} &= -\frac{\mathrm{i}\mathrm{e}^{\kappa_{\mathrm{v}}\mathrm{x}}\kappa_{\mathrm{v}}\mathrm{E_y}(0)}{\mathrm{k}}. \end{aligned} \tag{8}$$

and in the metamaterial region $x > 0$:

$$\begin{aligned} E_y(x) &= \mathrm{e}^{-\kappa_m x}E_y(0), \\ H_x(x) &= -\frac{\mathrm{e}^{-\kappa_m x}k_3 E_y(0)}{\mu(\omega)k}, \\ H_z(x) &= \frac{\mathrm{i}\mathrm{e}^{-\kappa_m x}\kappa_m E_y(0)}{\mu(\omega)k}. \end{aligned} \tag{9}$$

MAIN RESULTS

To analyze obtained equations let’s introduce such dimensionless variables: the frequency $\Omega = \omega / \omega_0$, the wavenumber $\beta = \mathrm{Re}(k_3)c/\omega_0$, the normalized decrement $\alpha = \mathrm{Im}(k_3)c/\omega_0$, the normalized electric $\nu = \nu_G / \omega_0$ and magnetic $\gamma = \gamma_G / \omega_0$ gains, and distance $\zeta = x\,\omega_0 / c$.

At first, two obtained dispersion equations for TM- and TE-modes (4,5) were solved numerically for metamaterial with $\nu = 0$, $\gamma = 0$ (Fig. 2).

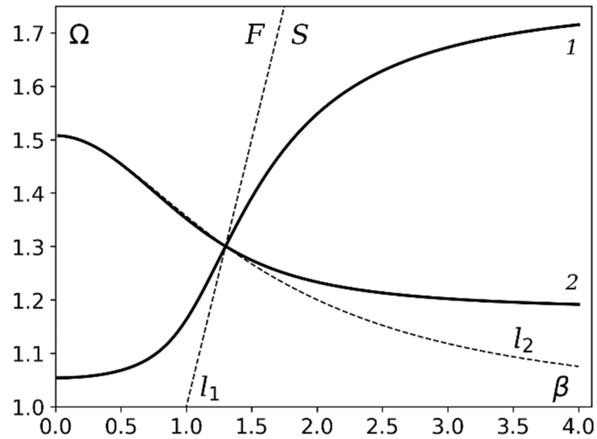

Figure 2. Normalized frequency as a function of the normalized wavenumber for TM and TE modes when $\nu = 0$, $\gamma = 0$ in equations (4,5).

The wave marked with number 1 corresponds to the wave of TM-polarizations, and by the number 2 – TE-wave. The dashed lines correspond to the conditions $\kappa_\nu = 0$ and $\kappa_m = 0$. These lines separate the regions where waves are fast or slow and bulk or surface.

It was previously shown [8] that the slow surface electromagnetic waves can exist in the right and upper region with respect to $\kappa_\nu = 0$ (line marked l_1 in Fig. 2) and $\kappa_m = 0$ (line marked l_2 in Fig. 2) lines. This region is marked by the letter *S* (see Fig. 2). The results obtained are in a good accordance with previously obtained results [8]. The fast electromagnetic waves can exist in the left region with respect to line l_1 that is marked by the letter *F* (Fig. 2). To study the properties of such fast waves it is necessary to take into account the non-zero values of ν and γ parameters in metamaterial permittivity $\varepsilon(\omega)$ and permeability $\mu(\omega)$ expressions (1, 2), because in the case of “ideal” metamaterial one obtain non-physical solutions of the dispersion equation. The results of the numerical solution of dispersion equations (4, 5) for this case for fast waves are presented in Figs. 3-6.

The numerical solutions of the dispersion equation (4) for fast TM-waves for the case when $\nu, \gamma \neq 0$ are presented in the Figs 3,4. These results have shown that the increase of the gain ν and γ leads to the strong decrease of wavelength under the fixed wave frequency value in the region of the rather low wave frequencies $\Omega < 1.15$ (see, Fig. 3). In the region when $1.15 < \Omega < 1.3$ the influence of ν and γ parameters variation is much smaller (see, Fig. 3). The dependence of the normalized decrement α versus ν and γ values are presented in the Fig. 4. This decrement has negative sign and the increase of gain ν and γ values leads to the α absolute value increase (see, Fig. 4).

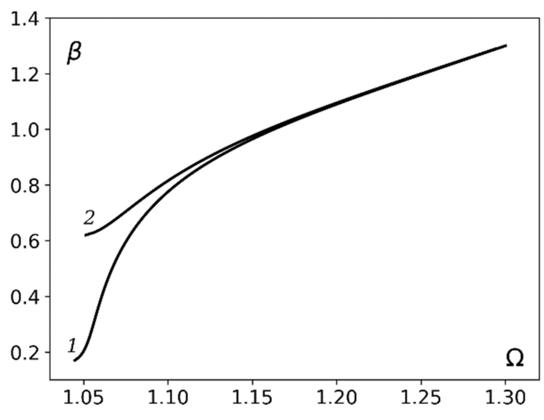

Figure. 3. The normalized wavenumber β versus wave frequency Ω for the fast TM- wave. The line 1 corresponds to $\nu = 0.01$, $\gamma = 0.01$ and the line 2 corresponds to $\nu = 0.05$, $\gamma = 0.05$

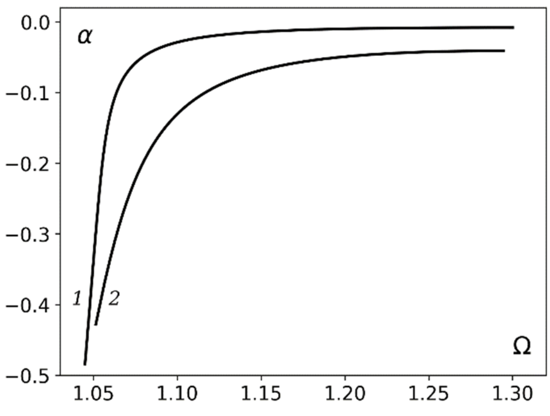

Figure 4. The normalized decrement α versus wave frequency Ω for the fast TM-waves. The parameters and curve numbering are the same as for the Fig. 3

The results of the numerical analysis of the dispersion equation (5) for fast TE-waves for the case $\nu, \gamma \neq 0$ are presented in the Figs 5,6. It is necessary to mention that simultaneously variation of ν, γ parameters make an extremely small influence on the TE-wave dispersion (see Fig. 5). But the influence of mentioned parameters of the TE-wave normalized decrement is much larger.

The increase of the parameters ν and γ values twice from 0.005 up to 0.01 leads to the increase on the normalized decrement α almost twice in the whole frequency range (see Fig. 6). It is also necessary to mention that for the fast TE-waves the normalized decrement $\alpha > 0$ and is much smaller by absolute value than α for the TM-wave.

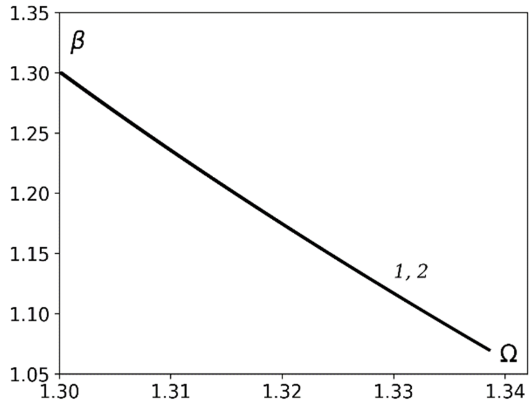

Figure 5. The normalized wavenumber β versus wave frequency Ω for the fast TE- wave. The coincident lines 1,2 corresponds to $\nu = 0.005$, $\gamma = 0.005$ and $\nu = 0.01$, $\gamma = 0.01$

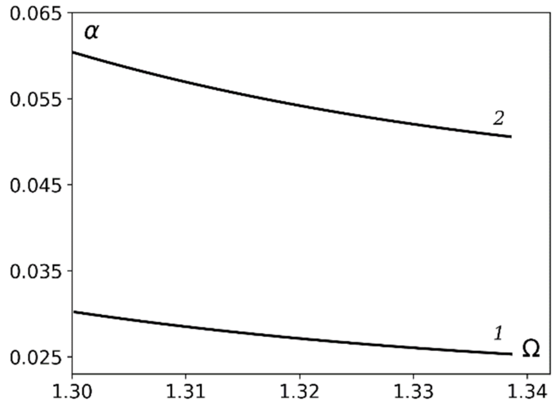

Figure 6. The normalized decrement α versus wave frequency Ω for the fast TE-modes. The parameters and curve numbering are the same as for the Fig. 5.

It was also studied the normalized phase ($V_{ph} = \Omega / \beta$) and group ($V_{gr} = d\,\Omega / d\beta$) velocities of the fast waves considered. The dependence of the V_{ph} for the fast TM-wave versus normalized frequency Ω for the same parameters as for the Fig. 3 are presented in the Fig. 7. It is shown that the wave is really fast wave ($V_{ph} > 1$) in appropriate frequency range. The simultaneously increase of the parameters ν and γ leads to the essentially TM-wave slowing in the region of rather low wave frequencies Ω ($\Omega < 1.1$ in Fig. 7).

The dependence of the V_{gr} for the fast TM-wave versus normalized frequency Ω for the same parameters as for the Fig. 3 are presented for the Fig. 8.

While the phase velocity is essenctially greater than the speed of ligth (Fig. 7) the group velocity is much smaller than c (Fig. 8). TM-mode is forward, as can be seen from the figure above, i.e. the velocities V_{ph} and V_{gr} have the same sign. The normalized group velocity increases with the simultaneously increasing of the parameters ν and γ. It is necessary to mention that V_{gr} possesses the some minimum value in the region on rather small Ω values (see lines $1,2$ in the Fig. 8 in the region $1.05 < \Omega < 1.1$). This minimum values shifts to the upper Ω values with the increase of the ν and γ.

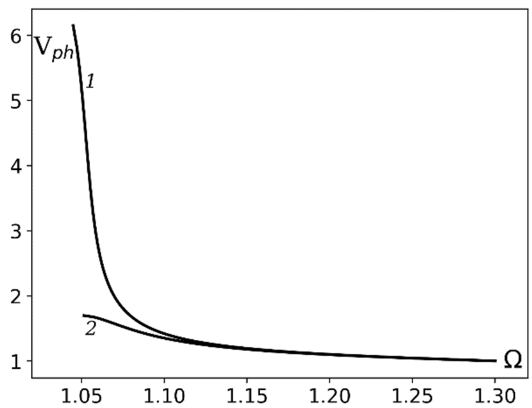

Figure 7. The normalized phase velocity of the fast TM- wave versus normalized frequency Ω. The parameters and curve numbering are the same as for the Fig. 3

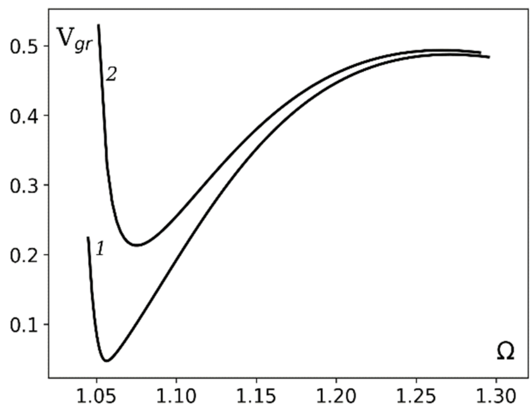

Figure 8. The normalized group velocity of the fast TM-wave versus normalized frequency Ω. The parameters and curve numbering are the same as for the Fig. 3

The dependence of the V_{ph} for the fast TE-wave versus normalized frequency Ω for the same parameters as for the Fig.5 are presented for the Fig. 9. It is shown that the wave is fast wave but more slowly than TM-wave. The characteristic feature of the TE-wave is almost linearly dependence of the V_{ph} versus Ω (see Fig. 9) in the whole frequency range of fast wave existence. The phase velocity of the TE-wave is practically independent upon changing ν and γ.

The dependence of the V_{gr} for the fast TE-wave versus normalized frequency Ω for the same parameters as for the Fig. 5 are presented for the Fig. 10. The group velocity is negative, so the fast TE-wave is backward (V_{ph} and V_{gr} have the different signs). The absolute value of V_{gr} of TE-wave is much smaller than V_{gr} of TM-wave. This group velocity has weak dependence upon changing ν and γ and has almost linearly dependence versus Ω.

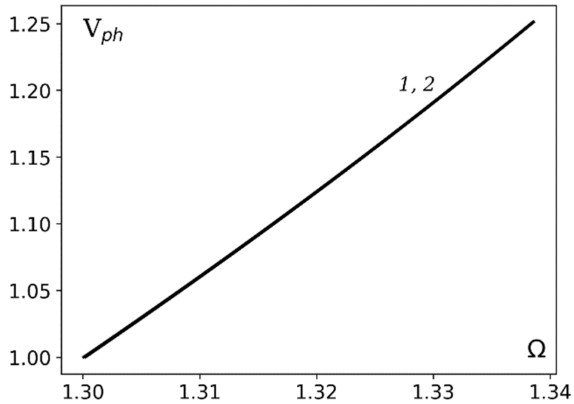

Figure 9. The normalized phase velocity of the fast TE-wave versus normalized frequency Ω. The parameters and curve numbering are the same as for the Fig. 5

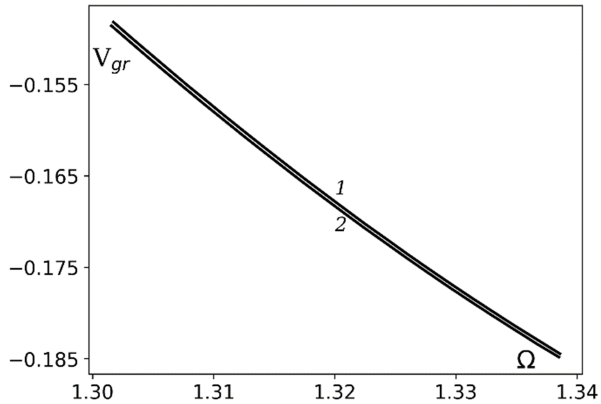

Figure 10. The normalized group velocity of the fast TE-wave versus normalized frequency Ω. The parameters and curve numbering are the same as for the Fig. 5

The absolute values of the normalized wave field components (upon the $H_y(0)$) for fast TM-wave for $\beta = 0.21$ (where wave phase velocity is much greater than c) are presented in the Fig. 11.

The absolute values of the normalized wave field components (upon the $E_y(0)$) for fast TE-wave for $\beta = 1.1$ (where wave phase velocity is much greater than c) are presented in the Fig. 12. It is necessary to mention that the normalized skin depth of the fast TM-wave in metamaterial region is much smaller that skin depth of the fast TE-wave.The obtained results can be important for modeling and designing of modern devices based on metamaterials.

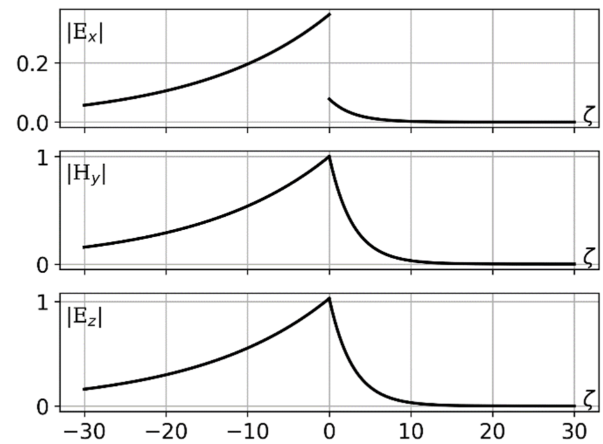

Figure 11. The absolute values of the normalized wave field components (upon the $H_y(0)$) for fast TM-wave for $\beta = 0.21$

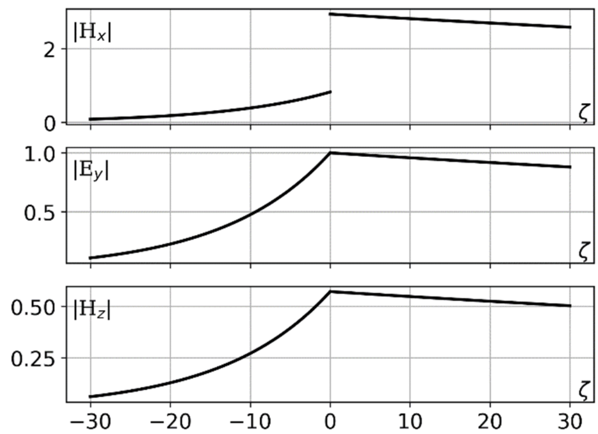

Figure 12. The absolute values of the normalized wave field components (upon the $E_y(0)$) for fast TE-wave for $\beta = 1.1$

CONCLUSIONS

It was shown that fast eigen TE- and TM-waves can exist in the considered waveguide structure. It was studied the dependence of the dispersion properties, wave attenuation, phase and group velocities, and spatial wave field structure of waves considered upon the metamaterial parameters. It is demonstrated that the eigen fast TM-wave is the direct wave and fast eigen TE-wave is backward wave.

The obtained results can be important for modeling and designing of modern generating and amplifying devices based on metamaterials.

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ШВИДКІ ЕЛЕКТРОМАГНІТНІ ХВИЛИ НА МЕЖІ МЕТАМАТЕРІАЛУ: МОДЕЛЮВАННЯ ПІДСИЛЕННЯ

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В статті представлено результати дослідження властивостей швидких поверхневих електромагнітних хвиль, що поширюються уздовж пласкої поверхні розділу між активним метаматеріалом та повітрям (або вакуумом). Розглянуто випадок однорідного та ізотропного метаматеріалу. Досліджено дисперсійні властивості, просторове загасання хвилі, фазову та групову швидкості, а також просторовий розподіл електромагнітного поля власних TE та TM мод такої хвилеводної структури. Показано, що в цій хвилеводній структурі можуть існувати швидкі поверхневі електромагнітні хвилі та досліджено їх властивості. Показано, що фазова швидкість TM-моди у кілька разів перевищує швидкість світла у вакуумі, тоді як фазова швидкість TE-моди незначно перевищує швидкість світла у вакуумі. TM-мода є прямою хвилею, в якій фазова та групова швидкості однаково спрямовані. Отримано, що групова швидкість TM-моди змінюється від нуля до половини швидкості світла у вакуумі, та досягає мінімуму за деякого значення частоти хвилі, що залежить від характеристик метаматеріалу. Показано, що глибина проникнення TM-моди в метаматеріал значно менша ніж у вакуум. TE-мода є зворотною хвилею, в якій фазова та групова швидкості протилежно направлені. Абсолютне значення групової швидкості TE-моди приблизно в шість разів менше за швидкість світла у вакуумі. На відміну від TM-моди, глибина проникнення TE-моди в метаматеріал значно більша, ніж у вакуум. Визначені властивості швидких поверхневих електромагнітних хвиль можуть бути використані для моделювання та проектування сучасних приладів генерації та підсилення, що містять метаматеріали.

Ключові слова: метаматеріал, швидка поверхнева електромагнітна хвиля, дисперсія, загасання, просторова структура хвилі