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FORMATION OF OPTICAL IMAGES WITH SYNCHROTRON RADIATION FLUX OF RELATIVISTIC ELECTRONS IN THE X-RAY GENERATOR "NESTOR"[†]

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When setting up physical experiments involving the use of the polarization properties of synchrotron radiation (SR) or a monoenergetic photon beam, detailed calculation of the spectral angular distribution of SR and its polarization components is of interest. Consideration of the electron beam size shows that in real conditions the radiation propagating in the plane of the equilibrium orbit will not be completely polarized, and the shape and dimensions of the angular distribution of radiation will be distorted. The motion of electrons in the uniform magnetic field and SR of the beam of relativistic particles in the storage ring of "NESTOR" are considered. The effect of the size of the electron beam with the energy of E=225 MeV in the 6-dimensional space on the formation of images of the flux of quanta of SR is analyzed. It is shown that the main contribution to the formation of images is made by the two-dimensional distribution of particles along the vertical direction axis and vertical oscillations. A software simulation code has been developed, the use of which made it possible to simulate the process of optical image formations by the flux of SR quanta (Этого предложения нет в русской аннотации). The formation of images of the radiation of electrons with an energy of E=225 MeV with change in the longitudinal distance L to the registration plane is considered. It is determined that at small longitudinal distances the main contribution to the image is made by the vertical distribution of particles in the beam. With an increase in the basic distance L, the contribution of the distribution of particles over vertical oscillations increases, which becomes decisive for large L value. Numerical simulation of image formation has been carried out. For the base distance of 300 cm and beam parameters with the vertical root mean square size σ_y of 0.2 mm and a vertical root mean square size $\sigma_{y'}$ of 0.15 mrad, the family of angular distributions is presented in the form of two-dimensional histograms for wavelengths $\lambda = 0.5\,\lambda_c$, $\lambda = \lambda_c$, $\lambda = 2\,\lambda_c$, where λ_c, is the critical wavelength of SR. The dimensions of the optical window are obtained, the size of which makes it possible to reliably register the entire flux of SR quanta for the indicated registration characteristics.

Keywords: electron storage ring, electron beam, synchrotron radiation, angular distribution, polarization, σ component, π component, formation of optical image.
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The movement of relativistic electrons in a magnetic field is accompanied by the emission of synchrotron radiation (SR) quanta. This radiation has many remarkable properties. These include, first of all, its unconditional reproducibility and metrological calculability [1, 2]. The problem of analytically describing the properties of SR in the ideal case has received a complete solution [3]. Practical application of SR implies the possibility of calculating the parameters of the flux of SR quanta in real conditions.

MAIN RESEARCH SUBJECT

Ideally, the emitting particle moves in a magnetic field along circular reference orbit. In practice, an intense flux of SR quanta is emitted by a distributed electron beam, conducted trough an extraction channel, and recorded at a selected base distance in the image plane.

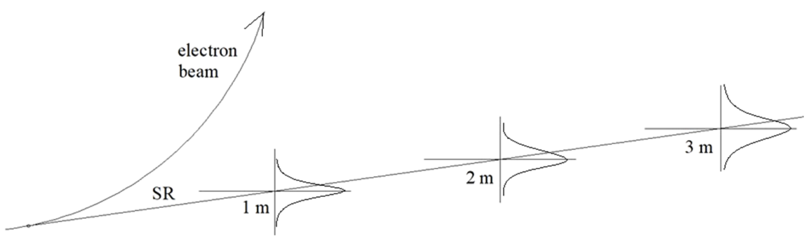

Figure 1. Scheme of registration of the flux of SR quanta

MATHEMATICAL MODEL

SR of a relativistic electron is characterized by a high degree of polarization [2]. In particular, in the ideal case, at zero angle $(\psi = 0)$ to the orbital plane, it is linearly polarized. The spectral-angular dependences of the flux of SR quanta of one electron in this case are calculated in accordance with the expressions that describe the flux density

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$w_\sigma(\psi)$ for the σ-component of polarization (in the plane of the orbit) and π-component $w_\pi(\psi)$ (perpendicular to the plane orbits)

$$w_\sigma(\psi) = \frac{8\pi e_0^2 R^2 f}{3c\hbar\lambda^3\gamma^4}(1+\gamma^2\psi^2)^2 K_{2/3}^2\left(\frac{\lambda_c}{2\lambda}(1+\gamma^2\psi^2)^{3/2}\right),$$
$$w_\pi(\psi) = \frac{8\pi e_0^2 R^2 f}{3c\hbar\lambda^3\gamma^4}\gamma^2\psi^2(1+\gamma^2\psi^2)\, K_{1/3}^2\left(\frac{\lambda_c}{2\lambda}(1+\gamma^2\psi^2)^{3/2}\right), \quad (1)$$

where $\gamma = E/E_0$ is the relativistic factor, E_0 is the electron rest energy, R is the orbital radius, f is the orbital frequency, $\lambda_c = 4\pi e_0^2 Rf/\sqrt{3}c\hbar\gamma^3$ is the critical radiation wavelength. The total angular density is: $w(\psi) = w_\sigma(\psi) + w_\pi(\psi)$.

The photon flux of each electron is characterized by the angular distribution, the axis of which coincides with the direction of motion of the particle, and the top of the distribution coincides with the place of emission. The electrons in the storage ring oscillate around the reference orbit. These oscillations are due to recoil during emission of SR quanta, as well as intrabeam scattering and scattering by residual gas particles. As a result, the beam particles are distributed around the reference orbit with the normal Gaussian law in the 6-dimensional space.

Let us consider the effect of the particle distribution on the properties of the flux of SR quanta. The distribution in the longitudinal direction does not affect the spectral-angular characteristics of the flux of SR quanta due to azimuthal symmetry. For the same reason, the radial distribution of particles also does not affect the characteristics of the SR flux. For the vertical distribution of particles, we use the formula:

$$\rho(y, y') = \frac{1}{2\pi\sigma_y\sigma_{y'}}\exp\left(-\frac{y^2}{2\sigma_y^2} - \frac{y'^2}{2\sigma_{y'}^2}\right), \quad (2)$$

where σ_y and $\sigma_{y'}$ are, respectively, the root-mean-square dimensions of the beam in y and y'. Bearing in mind (2), we consider the receiving plane at the base distance L that is perpendicular to the tangent of a circular orbit at the radiation emission point. The angle of emission of the quantum ψ, as well as the coordinates of emission (y, y') and reception h in the vertical direction, are related by the relation $h - \psi L = y + Ly'$. Therefore, for the angular distributions of the flux of SR quanta averaged over the beam, we obtain:

$$N_\sigma(\beta) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\psi \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dy'\, \rho(y, y')\ \delta(h - y - y'L - \psi L)\, w_\sigma(\psi),$$
$$N_\pi(\beta) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\psi \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dy'\, \rho(y, y')\ \delta(h - y - y'L - \psi L)\, w_\pi(\psi), \quad (3)$$

where $\beta = h/L$ and $\delta(.)$ is the Dirac delta function. Due to the Gaussian normal distribution of y and y' the random variable h is also Gaussian normal with the mathematical expectation ψL and the variance:

$$\sigma_L^2 = \sigma_y^2 + \sigma_{y'}^2 L^2. \quad (4)$$

Therefore, for the average angular distributions of the flux of SR quanta, we obtain:

$$N_\sigma(\beta) = L\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{d\psi}{\sqrt{2\pi}\sigma_L}\exp\left[-\frac{(\beta-\psi)^2 L^2}{2\sigma_L^2}\right] w_\sigma(\psi),$$
$$N_\pi(\beta) = L\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{d\psi}{\sqrt{2\pi}\sigma_L}\exp\left[-\frac{(\beta-\psi)^2 L^2}{2\sigma_L^2}\right] w_\pi(\psi). \quad (5)$$

It follows from (5) that the forming optical image is the convolution of the normal density of particles in the beam with the angular distribution that describes the emission of SR quanta. The dispersion of the resulting angular distribution $\langle(\beta\text{-}\psi)^2\rangle$ will decrease with increasing base L by $\sigma_L'^2 = \sigma_{y'}^2 + \sigma_y^2/L^2$. For sufficiently large L, it will be determined only by the distribution of particles along the directions of vertical oscillations. The distribution for the σ-component of polarization, due to its unimodality, is more resistant to this influence. The angular spectrum of the π-component of polarization has two symmetric maxima; therefore, its deformation and broadening by virtue of (5) turn out to be more noticeable. In this case, with an increase in the base distance L, the spatial picture along the vertical axis will also expand. The gradual distribution normalization for π-component of polarization will take place at $\sigma_{y'}^2 >> \langle\psi_\pi^2(\lambda)\rangle$, where $\langle\psi_\pi^2(\lambda)\rangle$ is the angular dispersion of π-component at the wavelength λ. Otherwise, when $\langle\psi_\pi^2(\lambda)\rangle >> \sigma_{y'}^2$, the angular distribution will be not normalized for any base L value.

NUMERICAL RESULTS

Based on expressions (4), the software was developed that makes it possible to calculate the necessary characteristics of the fluxes of SR quanta of both polarizations that have passed the selected base distance and to analyze the optical images formed in the given geometry of emission and observation. Let us present the results of calculations of the distribution of the optical image of the SR components in the "NESTOR" of electrons with an energy of E = 225 MeV [4-6]. Parameters for the "NESTOR" summarized in Table.

Table. The main parameters of the "NESTOR"

Parameter	Specification
Energy of electron beam E, MeV	40-225
Storage ring circumference, m	15.418
The maximum stored current I, mA	360
Number of electrons in orbit	1.1×10^{11}
Bending radius in magnets R, m	0.5
Magnetic field at maximum energy B, T	1.5
Electron beam size at radiation points σ_y, m (for the maximum energy of the electron beam)	2.08×10^{-4}
Electron beam divergence $\sigma_{y'}$ at radiation point (for the maximum energy of the electron beam)	1.51×10^{-4}

In Fig. 2 the family of angular distributions of flux densities for σ- and π-components of polarization calculated for one of the SR output channels in the generator, with $\sigma_y = 0.2$ mm and $\sigma_{y'} = 0.15$ mrad are shown. The dependences are presented in the form of two-dimensional histograms for different wavelengths $\lambda = 0.5\,\lambda_c$, $\lambda = \lambda_c$, $\lambda = 2\,\lambda_c$, as well as for different base lengths L, with $L_{\max}$=300 cm. For the chosen base and at $\lambda_c = 2.45\times10^{-6}$cm, the range of polar angle values β was $|\beta| \le 10$ mrad. It can be seen that with the base of $L_{\max}$=300 cm and the vertical size of the receiving window of H=6 cm, almost complete registration of quanta of the σ- and π-components at the selected SR wavelengths is provided. For large wavelengths of SR quanta, capture with a constant size of the receiving window H will be less effective due to the corresponding increase in of the root-mean-square angle of the distributions $w_\sigma(\psi)$ and $w_\pi(\psi)$ while the presence of two maxima in the distribution of the π-component becomes more evident.

Dependencies in Fig. 2 and Fig. 3 are given for the case when one electron turn around in the orbit.

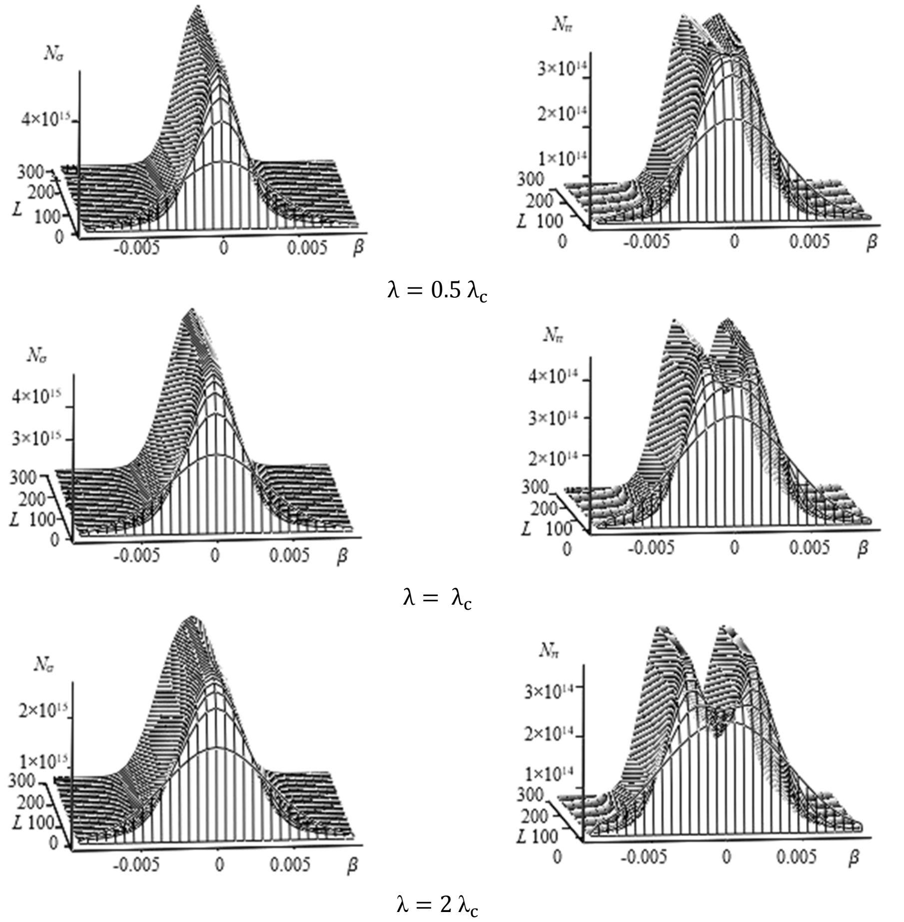

Figure 2. Family of angular distributions of the flux density of the σ-component (left) and the π-component (right) of the polarization of synchrotron radiation at an electron energy E=225 MeV and wavelengths, $\lambda = 0.5\,\lambda_c$, $\lambda = \lambda_c$, $\lambda = 2\,\lambda_c$, respectively

The rearrangement of optical images can be seen in Fig. 3. It shows the distributions that formed directly after the electron beam (L=5 cm) and distributions that formed on the basis of L=300 cm for different wavelengths of $\lambda = 0.5\,\lambda_c$, $\lambda = \lambda_c$, $\lambda = 2\,\lambda_c$ in addition to ideal angular distributions. From Fig. 3 one can see that at L=5 cm the angular distribution reflects the vertical imprint of the beam. At L=300 cm, the angular distribution is determined by the density of vertical oscillations of the electrons.

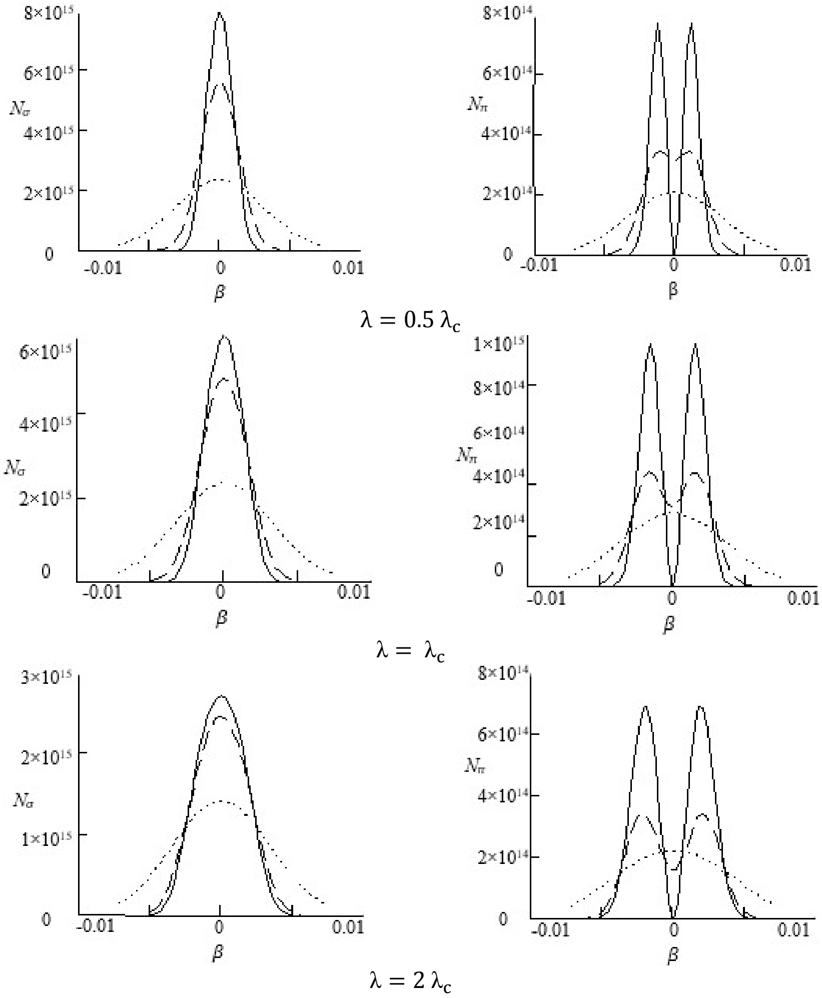

Figure 3. Family of angular distributions of flux density of the σ-component (left) and π-component (right) of the polarization of the SR at the electron energy E=225 MeV and wavelengths $\lambda = 0.5\,\lambda_c$, $\lambda = \lambda_c$, $\lambda = 2\,\lambda_c$, respectively. Line – ideal distribution; dotted line – distribution at L=5 cm, points – distribution at L=300 cm

CONCLUSION

The paper presents analytical expressions are obtained for the intensity of the flux of SR quanta of given wavelength for the selected registration geometry and algorithms for calculating the fluxes under consideration are proposed. It is shown that the forming optical image is the convolution of the normal density of particles in the beam with the angular distribution describing the emission of quanta. The dependences characterizing the intensity and spectral-angular properties of the SR photon flux are given. For the selected base distance and beam parameters with a vertical root-mean-square size σ_y and a root-mean-square size $\sigma_{y'}$ of vertical oscillations, the family of angular distributions is presented, which are presented in the form of two-dimensional histograms. The dimensions of the optical window are obtained, the value of which makes it possible to reliably register the flux of quanta of SR for the indicated registration characteristics. The paper presents the main characteristics of the angular distribution of the flux of SR quanta of relativistic electron beam in the storage ring of the "NESTOR" with a maximum electron energy of E_{max}=225 MeV.

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ФОРМУВАННЯ ОПТИЧНИХ ЗОБРАЖЕНЬ ПОТОКОМ КВАНТІВ СИНХРОТРОННОГО ВИПРОМІНЮВАННЯ РЕЛЯТИВІСТСЬКИХ ЕЛЕКТРОНІВ В РЕНТГЕНІВСЬКОМУ ГЕНЕРАТОРІ «НЕСТОР»

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При постановці фізичних експериментів, пов'язаних з використанням поляризаційних властивостей синхротронного випромінювання (СВ) або моноенергетичного пучка фотонів, представляє інтерес детальний розрахунок спектрального кутового розподілу СВ і його поляризаційних компонент. Урахування розмірів пучка показує, що в реальних умовах випромінювання, що поширюється в площині рівноважної орбіти, не буде повністю поляризованим, а форма і розміри кутового розподілу випромінювання будуть спотворені. Розглянуто рух електронів в однорідному магнітному полі і СВ пучка релятивістських частинок в накопичувачі "НЕСТОР". Проаналізовано вплив на формування зображень потоку квантів СВ розмірів пучка електронів з енергією E=225 МеВ в 6-вимірному просторі. Показано, що в формування зображень основний внесок вносить двовимірний розподіл часток по вертикалі і по вертикальним коливанням. Побудовано програмний засіб, використання якого дало можливість промоделювати процес формування оптичних зображень потоком квантів СВ. Розглянуто формування зображень випромінювання електронів з енергією E=225 МеВ при зміні поздовжньої відстані L до площини реєстрації. Визначено, що на малих поздовжніх відстанях основний внесок в зображенні вносить вертикальний розподіл часток в пучку. Зі збільшенням базової відстані L зростає внесок розподілу часток по вертикальним коливанням, який для великих L стає визначальним. Проведено чисельне моделювання формування зображень. Для базової відстані в 300 см і параметрів пучка з вертикальним середньоквадратичним розміром σ_y, що становить 0.2 мм, і середньоквадратичним розміром $\sigma_{y'}$ вертикальних коливань, що становить 0.15 мрад, наведено сімейство кутових розподілів, які оформлені у вигляді двовимірних гістограм для довжин хвиль $\lambda = 0.5\,\lambda_c$, $\lambda = \lambda_c$, $\lambda = 2\,\lambda_c$, де λ_c– критична довжина хвилі СВ. Отримано розміри оптичного люка, величина яких дозволяє гарантовано реєструвати весь потік квантів СВ для зазначених характеристик реєстрації.

Ключові слова: накопичувач електронів, електронний пучок, синхротронне випромінювання, кутовий розподіл, поляризація, σ-компонента, π-компонента, формування оптичного зображення.