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# TRANSFORMATION RATIO AT PLASMA WAKEFIELD EXCITATION BY LONG ELECTRON BUNCH WITH SHAPING OF ITS CHARGE ACCORDING COSINE

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Possibility of increase of the transformation ratio at plasma wakefield excitation by long relativistic electron bunch, which charge is shaped according to cosine, with purpose to provide the absence of oscillating (on length of bunch) decelerating wakefield, usually attainable, using bunch - precursor, is considered. Length of bunch is selected more larger than wavelength. It is shown analytically that at the choice of charge shaping of long bunch according to cosine distribution one can do without a bunch – precursor to eliminate oscillating decelerating wakefield. Thus the transformation ratio is just in one and a half times less than in the case with a precursor.

**KEY WORDS:** plasma wakefield, transformation ratio, relativistic electron bunch

**КОЭФФИЦИЕНТ ТРАНСФОРМАЦИИ ПРИ ВОЗБУЖДЕНИИ КИЛЬВАТЕРНОГО ПОЛЯ В ПЛАЗМЕ ДЛИННЫМ ЭЛЕКТРОННЫМ СГУСТКОМ С ПРОФИЛИРОВАНИЕМ ЕГО ЗАРЯДА ПО КОСИНУСУ**

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Рассматривается возможность увеличения коэффициента трансформации при возбуждении кильватерного поля в плазме профилированным по косинусу длинным релятивистским электронным сгустком с целью обеспечить отсутствие осциллирующего тормозящего кильватерного поля на длине сгустка, обычно достигаемое с использованием сгустка – предвестника. Длина сгустка выбирается значительно большей длины волны. Аналитически показано, что при выборе закона профилирования заряда длинного сгустка по распределению косинуса для устранеия осциллирующего тормозящего кильватерного поля можно обойтись без сгустка – предвестника. При этом коэффициент трансформации всего лишь в полтора раза меньше, чем в случае с предвестником.

**КЛЮЧЕВЫЕ СЛОВА:** кильватерное поле, коэффициент трансформации, релятивистский электронный сгусток

**КОЕФІЦІЄНТ ТРАНСФОРМАЦІЇ ПРИ ЗБУДЖЕННІ КІЛЬВАТЕРНОГО ПОЛЯ В ПЛАЗМІ ДОВГИМ ЕЛЕКТРОННИМ ЗГУСТКОМ З ПРОФІЛЮВАННЯМ ЙОГО ЗАРЯДУ ЗА КОСІНУСОМ**

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Розглядається можливість збільшення коефіцієнта трансформації при збудженні кільватерного поля в плазмі профільованим по косинусу довгим релятивістським електронним згустком з метою забезпечити відсутність осцилюючого гальмуючого кільватерного поля на довжині згустку, що зазвичай досягається з використанням згустку - передвісника. Довжина згустку вибирається значно більшій довжини хвилі. Аналітично показано, що при виборі закону профілювання заряду довгого згустку по розподілу косинуса для усунення осцилюючого гальмуючого кільватерного поля можна обійтися без згустку - передвісника. При цьому коефіцієнт трансформації усього лише в півтора рази менше, ніж у випадку з передвісником.

**КЛЮЧОВІ СЛОВА:** кільватерне поле, коефіцієнт трансформації, релятивістський електронний згусток

The transformation ratio $TR$, defined approximately as ratio $TR=E_2/E_1$ of the wakefield $E_2$, which is excited in plasma by driver-bunch, to the field $E_1$, in which an electron bunch is decelerated, determines the maximal energy $\varepsilon_{ac}$, to which the electrons are accelerated in the excited wakefield, for some energy of electron driver-bunch $\varepsilon_{dr}$ (see [1-12]). Surely that $\varepsilon_{ac}>>\varepsilon_{dr}$, if $TR \gg 1$. The large transformation ratio has been achieved at the linear driver-bunch shaping with the use of bunch-precursor before driver-bunch [3, 4]. This rectangular bunch-precursor of a small charge and of length, equal to one fourth of wavelength $\lambda/4$, is injected before the driver-bunch. Here $\lambda = 2\pi/k = 2\pi V_b/\omega_{pe}$, $V_b$ is the velocity of the driver-bunch, $\omega_{pe}$ is the electron plasma frequency. Because bunch-precursor is technologically difficult in experiments to prepare and place before driver-bunch, and natural in experiment longitudinal electron distribution is Gaussian, and cosine distribution is closed to Gaussian, we consider cosine distribution. We show analytically that approximately similar, as in [3, 4], transformation ratio can be derived simpler: at the use of driver-bunch, the charge of which is shaped along bunch accordingly to the shortened bell-shaped distribution (accordingly to the shortened half-cosine (Fig. 2 and (9)). The use of long electron bunch, shaped accordingly to half-cosine, has been discussed in [5].

The bunch length L is selected to be much more than wavelength L>>λ as in [3-5]. We show that the decelerating field in the area of the driver-bunch location is small and poorly inhomogeneous. The large transformation ratio TR=4N is achieved in the approximation of a given current of bunch, more precisely in the approximation of non-deformable bunch, N>>1 is the number of wavelengths on the length of driver-bunch. Optimal shortening of half-bell-shaped distribution is considered. Charge of this optimal bunch is shaped so that it is distributed accordingly to shortened on both-sides half-bell-shaped distribution (Fig. 2 and (9)).

We consider wakefield distribution in one-dimensional approximation, or more exactly wakefield distribution along axis without specifying the transverse structure of wakefield. The purpose of this paper is investigation of the possibility of the use of long driver-bunch of relativistic electrons, charge of which is shaped accordingly to half-cosine (see Fig. 1 and (1)) for achievement of the large transformation ratio at electron acceleration by wakefield, excited by this driver-bunch in plasma.

## TRANSFORMATION RATIO IN THE CASE OF LONG HALF-BELL-SHAPED DISTRIBUTION OF ELECTRONS

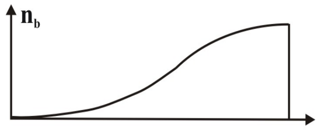


Fig. 1. The density distribution of bunch, shaped accordingly to half-bell-shaped distribution

We consider at first the current distribution of bunch electrons I(t) accordingly (similar to [5]) to half-bell-shaped distribution (Fig. 1)

$$I(t)=\frac{I_0}{2}\left[1-\cos\left(\frac{\pi\xi}{L}\right)\right],\ 0<\xi<L\,, \tag{1}$$

where $\xi = z - V_b t$. One can see that the current I and density $n_b$ of relativistic electrons equal zero (maximum) on the first (back) front of the driver-bunch.

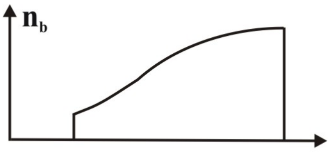


Fig. 2. The distribution of electron bunch density, shaped accordingly to the shortened on both-sides long half-bell-shaped distribution of electrons

Here $I_0$ is the current, corresponding to the linear density of charge $n_{b0}$. For analytical calculation of longitudinal on-axis electric wakefield, excited by long bunch, the charge of which is distributed accordingly to half-bell-shaped distribution, we use the expression, derived in [13], for longitudinal wakefield, excited by "point" bunch. Using this expression and integrating along the driver-bunch one can derive that this longitudinal wakefield $E_z$ is proportional into the driver-bunch to (Fig. 3)

$$E_z(\xi)\propto I_0\left(\frac{\lambda}{4L}\right)\left[1-\left(\frac{\lambda}{2L}\right)^2\right]^{-1}\left[\left(\frac{\lambda}{2L}\right)\sin(k\xi)+\sin\left(\frac{\xi\pi}{L}\right)\right]. \tag{2}$$

One can see that the field equals zero $E_z=0$ on the first front of the driver-bunch ξ=0 and approximately equals zero $E_z\approx 0$, in approximation $L \gg \lambda$ neglecting the first term in (2), on the back front of the driver-bunch ξ=L. The maximal decelerating wakefield into the bunch is small and at ξ=L/2 proportional to

$$E_z^{(\max)}(\xi)\propto I_0\left(\frac{\lambda}{4L}\right)\left[1-\left(\frac{\lambda}{2L}\right)^2\right]^{-1}. \tag{3}$$

Using the expression, derived in [13], for longitudinal wakefield, excited by "point" bunch, and integrating along the driver-bunch one can derive that the wakefield after ξ>L the bunch is proportional to (Fig. 3)

$$E_z(\xi)\propto I_0\left[1-\left(\frac{\lambda}{2L}\right)^2\right]^{-1}\left[\left(\frac{\lambda^2}{8L^2}\right)\sin(k\xi)+\left(1-\frac{\lambda^2}{8L^2}\right)\sin\left(k(\xi-L)\right)\right].$$

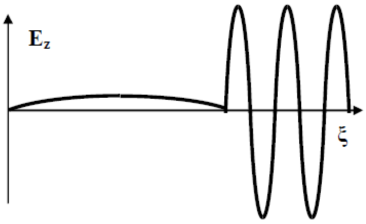


Fig. 3. The distribution of longitudinal on-axis electric wakefield, excited by long bunch, the charge of which is distributed accordingly to half-bell-shaped distribution, and corresponding to (4) and (2) in approximation $L \gg \lambda$ neglecting the first term (2)

For L/λ=p, p=1, 2, … we derive from the last expression

$$E_z(\xi) \propto I_0 \left[1-\left(\frac{\lambda}{2L}\right)^2\right]^{-1} \sin(k\xi) . \quad (4)$$

Thus, the amplitude of wakefield after ξ>L the shaped bunch is proportional to $I_0\left[1-(\lambda/2L)^2\right]^{-1}$. And the transformation ratio approximately at $L \gg \lambda$ equals

$$TR = \frac{4L}{\lambda} . \quad (5)$$

It is less in 1.57 times than the maximal known transformation ratio 2πL/λ in the case of triangle-bunch with a precursor [3, 4]. However it is more in 1.27 times, than the transformation ratio πL/λ in the case of triangle-bunch without a precursor.

## THE WAKEFIELD DISTRIBUTION IN THE CASE OF SHORTENED HALF-BELL-SHAPED LONG DISTRIBUTION OF ELECTRONS

We have shown that the field approximately equals zero $E_z \approx 0$ at the end of long bunch ξ=L, where the electron density of bunch is maximal. Hence these electrons do not excite the wakefield. Then it is useful to shorten the long half-bell-shaped distribution. We consider following distribution of electron current I(t)

$$I(t) = \frac{I_0}{2}\left[1-\cos\left(\frac{\pi\xi}{L}\right)\right], 0<\xi<X, X<L . \quad (6)$$

One can derive similar to (2) that the longitudinal on-axis electric wakefield into the bunch is proportional to

$$E_z(\xi) \propto \left(\frac{I_0}{2}\right)\left(\frac{\lambda}{2L}\right)\left[1-\left(\frac{\lambda}{2L}\right)^2\right]^{-1}\left[\left(\frac{\lambda}{2L}\right)\sin(k\xi)+\sin\left(\frac{\xi\pi}{L}\right)\right] . \quad (7)$$

The field equals zero $E_z=0$ at ξ=0 and does not equal zero $E_z \neq 0$ at ξ=X, because X<L. The maximal decelerating field equals (3).

One can derive similar to (4) that the wakefield after the bunch ξ>X is proportional to

$$E_z(\xi) \propto -\left(\frac{I_0}{2}\right)\left(\frac{k}{2}\right)\left\{\frac{2}{k}\left[\sin(k\xi)-\sin\left(k\xi-\frac{2\pi X}{\lambda}\right)\right]+\right.$$
$$\left.+\left(-\frac{\pi}{L}+k\right)^{-1}\left[\sin\left(k\xi+\frac{\pi X}{L}-\frac{2\pi X}{\lambda}\right)-\sin(k\xi)\right]-\left(\frac{\pi}{L}+k\right)^{-1}\left[\sin(k\xi)+\sin\left(k\xi-\frac{\pi X}{L}-\frac{2\pi X}{\lambda}\right)\right]\right\} . \quad (8)$$

## THE WAKEFIELD DISTRIBUTION IN THE CASE OF SHORTENED ON BOTH-SIDES LONG HALF-BELL-SHAPED DISTRIBUTION OF ELECTRONS

We consider the following electron current distribution I(t)

$$I(t) = \frac{I_0}{2}\left[1-\cos\left(\frac{\pi\xi}{L}\right)\right], Y<\xi<X, X<L, Y>0 . \quad (9)$$

It corresponds to shortened on both-sides long half-bell-shaped distribution of electrons. Thus shortening from one side is performed more, than on a half. I.e. the distribution is a trapezoid with smooth one side (see Fig. 2).

One can derive that the wakefield into the bunch is proportional to

$$E_z(\xi) \propto \left(\frac{I_0}{2}\right)\left(\frac{k}{2}\right)\left(\frac{\pi}{L}+k\right)^{-1}\left(-\frac{\pi}{L}+k\right)^{-1}\left\{\frac{2}{k}\left(\frac{\pi}{L}+k\right)\left(-\frac{\pi}{L}+k\right)\sin\left[k(\xi-Y)\right]+\frac{2\pi}{L}\sin\left(\frac{\xi\pi}{L}\right)-\right.$$
$$\left.-\left(\frac{\pi}{L}+k\right)\sin\left(k\xi-kY+\frac{\pi Y}{L}\right)-\left(-\frac{\pi}{L}+k\right)\sin\left(k\xi-kY-\frac{\pi Y}{L}\right)\right\} .$$

At $Y \approx k^{-1} << L$ we have approximately

$$E_z(\xi) \propto \left(\frac{I_0}{2}\right)\left(\frac{\lambda}{2L}\right)\left\{-\left(\frac{\lambda}{2L}\right)\sin\left[k(\xi - Y)\right] + \sin\left(\frac{\xi\pi}{L}\right)\right\}. \quad (10)$$

One can see that the oscillating field does not exceed the non-oscillating one and phase of oscillating field has been changed in comparison with phase of oscillating field in (2).

## CONCLUSIONS

So, it has been shown analytically that at the use of charge shaping of long driver-bunch accordingly to the shortened half-cosine distribution one can do without a bunch – precursor to eliminate oscillating decelerating wakefield. Thus the transformation ratio is only in one and a half times less, than in the case of linear shaping with a bunch-precursor.

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