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FORMATION OF ELEMENTARY CONVECTIVE CELL IN HORIZONTAL LAYER OF VISCOUS INCOMPRESSIBLE FLUID

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It is experimentally shown that with the increase of temperature of the vessel bottom with horizontal oil layer, separately placed convective cells appear in the quantity from one-two at the beginning of the process and till full fill-up of the oil volume at the end. On the basis of the obtained experimental data the theoretical model describing a cylindrical convective cell is offered, for which the structure of space speed distribution and temperature is calculated. Energy substantiation of the principle of pavement (coating) of the liquid surface with cylindrical convective cells is presented.
KEYWORDS: elementary convective cell, convective processes, transfer of heat, temperature gradient, velocity of mass transfer

ФОРМИРОВАНИЕ ЭЛЕМЕНТАРНОЙ КОНВЕКТИВНОЙ ЯЧЕЙКИ В ГОРИЗОНТАЛЬНОМ СЛОЕ ВЯЗКОЙ НЕСЖИМАЕМОЙ ЖИДКОСТИ

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Экспериментально показано, что с повышением температуры дна емкости с горизонтальным слоем масла возникают отдельно расположенные цилиндрические конвективные ячейки в количестве от одной-двух, в начале процесса, и до полного заполнения ими объема масла - в конце. На основе полученных экспериментальных данных предложена теоретическая модель, описывающая цилиндрическую конвективную ячейку для которой рассчитана структура пространственного распределения скорости массопереноса и температуры. Сформулирован энергетический принцип формирования конвективных структур в слое вязкой, подогреваемой снизу несжимаемой жидкости. Дано энергетическое обоснование принципа замощения (покрытия) поверхности жидкости цилиндрическими конвективными ячейками.
КЛЮЧЕВЫЕ СЛОВА: элементарная конвективная ячейка, конвективные процессы, перенос тепла, температурный градиент, скорость массопереноса

ФОРМУВАННЯ ЕЛЕМЕНТАРНОЇ КОНВЕКТИВНОЇ КОМІРКИ В ГОРИЗОНТАЛЬНОМУ ШАРІ В'ЯЗКОЇ НЕСТИСЛОВОЇ РІДИНИ

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Експериментально показано, що з підвищенням температури дна ємності з горизонтальним шаром масла виникають окремо розташовані циліндричні конвективні комірки в кількості від однієї-двох, на початку процесу, і до повного заповнення ними обсягу масла - у кінці. На основі отриманих експериментальних даних запропонована теоретична модель, що описує циліндричну конвективну комірку, для якої розрахована структура просторового розподілу швидкості масопереносу і температури. Сформульований енергетичний принцип формування конвективних структур у шарі в'язкої, що підігрівається знизу, нестисливої рідини. Дано енергетичне обгрунтування принципу замощення (покриття) поверхні рідини циліндричними конвективними комірками.
КЛЮЧОВІ СЛОВА: елементарна конвективна комірка, конвективні процеси, перенесення тепла, температурний градієнт, швидкість масопереносу

The French physicist Henri Bernard conducted the experiment on the research of mass-transfer in the heated from below layer of spermaceti oil, where appearance of hexagonal cells was observed in 1900. Subsequently such type of a cell began to call by his name [1; 2], although as Relay mentioned in [3], it looks like Hendri Bernard was not acquainted with James Thomson paper [4], where he describes similar structures in the layers of soap water, cooled from above (honeycomb structure in emulsions).

Most often the ordered structures in the form of Bernard cells are met in nature. It can be for example multi-layer systems of granulation on the Sun [5], cells on clouds [6], one-layer structures on the surface of saline lakes [7], hexahedral columns on islands and in caves [8], structures of ice peaks and cavities under uneven melting of soils [9], ordered convective cells in the upper mantle of the Earth. [10].

As far as the granulation on the Sun, it is observed in convective zone on the distance approximately 2×10^5 km, forming 20-27 layers, consisting of Bernard cells [11]. Proceeding from observations, there are three different scales of granulation: granulation (with typical diameter 150 - 2500 km), mezogranulation $(5.0 - 10.0)\times10^3$ km) and super-granulation (more than 2×10^4 km). Lower layers of granulation of convective zone can be characterized as super-granulation, since the granule size changes within the limits $(2 - 4)\times10^4$ km, and thickness - $(3 - 8)\times10^3$ km [12, 13]. Life period of granules depends on the size and placement in the convective zone: the biggest (super-granules) exist about 20-36 hours, the smallest, which are located in the outer layers of the zone, about 2 minutes. Speed of the fluxes on the axis of such cells is directed for the Sun center. Value of typical horizontal speed in cells, measured on Doppler-grams, is determined by value v_g = 205 m/s. In other publications its value is in the interval from 170 m/s to 360 m/s. [14]. Average density of the substance, which forms the convective cells on the Sun in convective layer is of order $\rho = 1.4\times10^3$ kg/m^3, and distribution of the temperature in the layer and characteristic coefficients have the following meanings [12-14]: $T_2 = 2\times10^6, T_1 = 5\times10^3\,K$ – temperature of the upper and lower layer, correspondingly; $g^* = 273.1$ m/s^2 acceleration of gravity; $\nu = 635$ m/s^2 - kinematic viscosity; $\chi = 705.6$ m/s^2 – thermal conductivity factor; $\beta = 3.42\times10^{-3}$ K^{-1} – volume thermal expansion factor.

It is known that solutions describing horizontal and vertical speed components of the speed inside of the cell, as a result of geometrical transformations, can form such structures, as convective horizontal shafts, square convective cells and regular polygons (triangles and hexagons) [3, 15 - 17]. These structures are fully filling the volume of convective layer and thus provide maximal transfer of heat between the layer boundaries.

However, in our opinion, main principle of forming the polygonal convective structures in temperature-tensed area should be not the geometric constructions but energy principle. This principle is visually demonstrated on the appearance of Bernard cell in the layer of viscous fluid: with the increase of vessel bottom temperature, progressive increase of quantity of convective cells till the point of full filling of the whole volume is observed.

In other words, this principle reads: the more the temperature of the vessel bottom under correspondent temperature gradient, the more convective cells appear, form of which, while vessel volume is filling, is reached to polygonal (including hexagonal), and the more effective heat exchange between the heated lower border of the environment with upper takes place.

For realization of such energy principle of formation of convective cells introduction of idea of elementary convective cell is needed, from a large number of which, under their tight packing, polygonal convective structures can form [18, 19]. Thus for example, selection of cubic convective cell with free boundaries as the elementary cell turned to be productive under description of processes of Langmuir circulations (LC) [20].

Purpose of this paper is experimental observation and mathematical description of elementary convective cell in the layer of viscous incompressible fluid, determination of its characteristic physical parameters.

EXPERIMENTAL DATA ON PARAMETERS OF BENARD CELLS

One of the questions arising during the research of Bernard cells is the question on the value of aspect number, which determines ratio of Bernard cells diameter to the depth of liquid depth layer. Experimental data of these values, taken from different scientific publications and obtained by authors in laboratory conditions are presented in Table 1, where $A_n = d/h$ local aspect number, equal to ratio of cell diameter *D* to the layer thickness *h*.

The error of the presented experimental data constitutes the value 5 - 10 %.

Table 1.

Geometrical sizes of Bernard cells

Source	Diameter *D*, mm	Thickness *h*, mm	A_n
M. Van – Dyke [21], Koschmieder E.L. [22]	6.25	1.9	3.3
E. D. Eidelman [23]	3.0	1	3.0
Experimental dada [18]	3.2	1.0	3.2

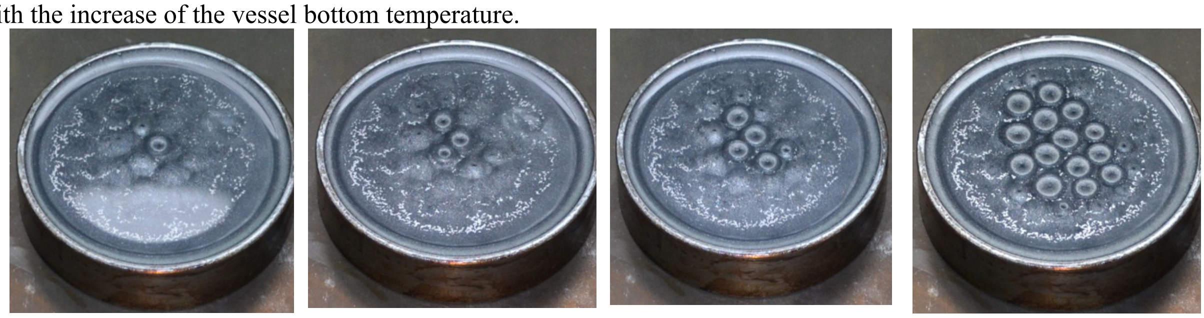
EXPERIMENTAL TECHNIQUE

Experimental data, presented in Table 1, are obtained with use of vacuum oil BM - 5 (2 ml) with addition of small quantity of aluminum powder (0.056 g). Conditions of experiment were the following. Oil with thickness layer about 1 mm was pored into cylindrical vessel with radius 26 mm and height 2.5 mm. Heating of the vessel was conducted from below from electric furnace, temperature of oil on the vessel bottom was on the level 120 ± 1 °C. Kinematic viscosity of oil under mentioned temperatures was estimated in the value 10 mm^2/s [24].

It was shown in the experiments, that when reaching the temperature of the vessel bottom $T \approx 120 \pm 1$ % °C, in vacuum oil with layer thickness about 1.0 mm Bernard cells appeared with diameter 3.2 mm.

Dependence of quantity of convective cells in the liquid layer on the temperature vessel bottom temperature. In order to research the dependence of quantity of convective cells on the temperature value of the vessel bottom, a number of experiments was conducted, conditions of which correspond to those which are described in experiments on determination of local aspect number of Bernard cell (see Table 1).

Fig.1 shows photos of cylindrical convective cells, number of which increases with the increase of vessel bottom temperature from 44 °C to 120 °C starting from one – two cells and ending with their tight packing of oil surface with appearance of polygons including hexagons. Temperature gradient dT/dz remained on the same level 10 -13 ^{0}C/mm with the increase of the vessel bottom temperature.



а) T_2=44 ºC T_1=34 ºC б) T_2=45 ºC T_1=35 ºC в) T_2=50 ºC T_1=40 ºC г) T_2=56 ºC T_1=43 ºC

Fig.1. Change of number of cylindrical convective cells depending on the vessel temperature T_2 change (T_1 – temperature of the upper oil layer). Local aspect number of cells about $A_n = 3.2$.

Proceeding from results of the experiments, presented on Fig. 1, it can be concluded that with the increase of the vessel bottom temperature, number of convective cells increases under insignificant increase of temperature gradient. The cells observed in experiment until full filling of the oil value with them, create circles with diameter about 3.2 mm on the upper surface of the oil and are characterized with separate mutual position.

MATHEMATICAL MODEL OF DESCRIPTION OF CONVECTIVE PROCESSE IN VISCOUS ENVIRONMENTS WITH FREE BOUNDARIES

In Rayleigh problem [3, 17, 18] infinite in the direction of axes x and y layer of viscous liquid with thickness h is studied. z axis is directed upwards, perpendicular to the borders of the layer $z = 0$ and $z = h$. Distribution of temperature inside the layer $T_0(z)$ is given so that the temperature of the lower border is more than the temperature of the upper border: $T_0(0) = T_2$, $T_0(h) = T_1$ ($T_2 > T_1$). We assume that in condition of equilibrium, distribution of temperature in the layer is described with linear function from the coordinate z :

$$\vec{\nabla} T_0(z) = -\frac{\Theta}{h}\vec{e}_z \tag{1}$$

where $\Theta = T_2 - T_1$ - difference of temperatures between lower and upper planes, $\vec{e}_z$ - unit vector, directed along the axis z .

The initial system of equations for dimensionless disturbances constitutes Navier-Stokes equation in the Boissinesq approximation and has the view: [15 - 17]:

$$\frac{\partial \vec{\mathrm{v}}}{\partial t} = -\vec{\nabla} p + \Delta\vec{\mathrm{v}} + RT\vec{e}_z \tag{2}$$

$$P\frac{\partial T}{\partial t} - \mathrm{v}_z = \Delta T \tag{3}$$

$$\mathrm{div}(\vec{\mathrm{v}}) = 0, \tag{4}$$

where Δ - Laplace operator, $\vec{\nabla}$ - gradient operator a, $R = g\beta h^3\Theta/(\nu\chi)$ - Rayleigh number, g – acceleration of gravity, directed against axis z , $P = \nu/\chi$ - Prandtl number, ν and χ - coefficients of kinematic viscosity and thermal diffusivity of the liquid correspondingly, β - coefficient of volume thermal expansion of the liquid, $\vec{\mathrm{v}} \equiv (\mathrm{v}_x, \mathrm{v}_y, \mathrm{v}_z), p, T$ - velocity, pressure and temperature disturbance, correspondingly.

While diverting the system of equations (2) - (4) to dimensionless form the following measurement units were used: length unit – thickness of the layer h ; time unit - $\tau = h^2\nu^{-1}$; temperature unit - Θ.

In order to determine “normal” disturbances in viscous heated from below liquid layer, equation system (2) - (4) can be used, on condition that it should be filled with boundary conditions.

The task with free boundary conditions when the tangent tensions and temperature disturbances on the layer boundaries $z=0$ and $z=1$ are equal to zero:

$$\mathrm{v}_z = 0, \frac{\partial^2 \mathrm{v}_z}{\partial z^2} = 0, T = 0. \tag{5}$$

System of equations (2) - (4) and boundary conditions (5) determine identically the spectrum of characteristic equilibrium conditions of the viscous, incompressible, and heated from below liquid, which can form such structures as convective shafts and regular polygons [15 - 17].

It should be noted that observed data on the Sun granulation, apparently indicate that the equations system, describing convective mass-transfer of the solar matter in gravitation and temperature fields [14], in linear approximation, neglecting rotation of the Sun and generation of magnetic field, can be brought to equation system (2) - (4) and boundary conditions (5).

Thus Navier-Stokes equations system in Boissinesq approximation (system of equations (2) - (4) with boundary conditions (5)) can be used for research of spectrums of convective disturbances and conditions of stable existence of layer of viscous, incompressible fluid, including solar substance, earth clouds, frozen ground and Earth mantle.

THEORY OF ELEMENTARY CONVECTIVE CELL WITH FREE BORDERS

Reference equations and boundary conditions. Taking into account the fact that view of the presented on Fig.1 convective cells has cylindrical form, we can find the solution of the task on the stability of equilibrium of heated from below layer of viscous, incompressible fluid in cylindrical geometry, assuming the boundary conditions are free.

Free boundary conditions in the cylindrical coordinate system (r,ϕ,z) correspond to the demand of absence of the layer on borders under $z=0$ and $z=1$ tangential stresses, i.e. $\partial \mathrm{v}_r / \partial z = 0$. This demand follows from boundary conditions in rectangular coordinate system $\partial \mathrm{v}_x / \partial z = 0$ and $\partial \mathrm{v}_y / \partial z = 0$, where variable z, after becoming non-dimensional on the layer depth h, changes in the limits from 0 to 1.

It is easy to show that for cylindrical cell boundary conditions coincide with conditions (5).

Based on experimental results presented on Fig.1, it can be concluded that structure of convective cells does not depend on azimuth angle ϕ. That is why we will study all disturbed values in axial-symmetrical approximation, which is we assume everywhere $\partial / \partial \phi = 0$.

In this case for layer with free plane boundaries, after action of the operator $\mathrm{rot}(\mathrm{rot}(...))$ on equation (2) and its record of z - projection, equations (2) - (3) take the view:

$$\frac{\partial}{\partial t} \Delta \mathrm{v}_z = \Delta\Delta \mathrm{v}_z + R \Delta_\perp T \tag{6}$$

$$P \frac{\partial T}{\partial t} = \Delta T + \mathrm{v}_z \tag{7}$$

where $\Delta = \Delta_\perp + \frac{\partial^2}{\partial z^2}$, $\Delta_\perp = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r} \right)$ - transverse Laplacian.

Solution of initial system of equations. The equations (6), (7) have particular solutions, which describe the time dynamics of disturbances of vertical speed and temperature in axially-symmetric cylindrical cell:

$$\mathrm{v}_z(r,z,t) = \mathrm{v}(z) J_0(k_r r) \exp(-\lambda t) \tag{8}$$

$$T(r,z,t) = \vartheta(z) J_0(k_r r) \exp(-\lambda t) \tag{9}$$

where λ - eigenvalues, which characterize attenuation ($\lambda > 0$), rise ($\lambda < 0$) or stationary condition ($\lambda = 0$) of disturbances (5), (6); $\mathrm{v}(z)$ and $\vartheta(z)$ - are amplitudes of disturbances of vertical speed and temperature correspondingly; $J_0(x)$ - Bessel functions of the first type of zero order from argument x; k_r - radial wave number, which characterizes dependence of disturbances on transverse coordinate r.

For free boundary conditions in cylindrical cells, determined by expression:

$$\mathrm{v}(0) = \mathrm{v}(1) = 0,\ \partial^2 \mathrm{v}(0)/\partial z^2 = \partial^2 \mathrm{v}(1)/\partial z^2 = 0,\ \vartheta(0) = \vartheta(1) = 0, \tag{10}$$

values of disturbance amplitude of vertical speed $\mathrm{v}(z)$ and temperature $\vartheta(z)$ can be presented by simple harmonics [15, 16], and the solutions can be presented in the following way:

$$\begin{aligned} \mathrm{v}_z(r,z,t) &= A \cdot sin(n\pi z) J_0(k_r r) \exp(-\lambda t), \\ T(r,z,t) &= B \cdot sin(n\pi z) J_0(k_r r) \exp(-\lambda t) \end{aligned} \tag{11}$$

where $n = 1,2,3,...$ - integral numbers, A and B - constant coefficient.

Expression for radial liquid speed in convective cell $\mathrm{v}_r(r,z,t)$ can be obtained from the condition of its incompressibility (equation(4)):

$$\mathrm{v}_r\left(r,z,t\right)=-An\pi k_r^{-1}\cos\left(n\pi z\right)J_1\left(k_r r\right)\exp\left(-\lambda t\right) \tag{12}$$

Thus, parameters of cylindrical convective cell with free boundaries are determined in this part.

Determination of radial wave number of perturbed velocity. Solution (12) corresponds to physically substantiated boundary conditions: on cell axis ($r=0$) and on its external border ($r=R_c$) – radial speed should be equal to zero. Radial wave number value can be determined from this:

$$k_{r,i}=\sigma_{1,i}R_c^{-1}, \tag{13}$$

where R_c - radius of convective cell, $\sigma_{1,i}$ - i - zero of Bessel function of the first type of the first order ($J_1\left(\sigma_{1,i}\right)=0$), $i=1,2,3,....$. In particular, first five Bessel function zeros have the following meanings [25]: $\sigma_{1,1}=3.832$; $\sigma_{1,2}=7.016$; $\sigma_{1,3}=10.173$; $\sigma_{1,4}=13.324$; $\sigma_{1,5}=16.471$.

It should be noted that solution of the type (8) is presented in [20, 26, 27] where on its basis current lines were built and axial-symmetric location of concentric convective shafts was analyzed. However usage of physically substantiated boundary conditions, which determined radial wave number of the obtained solutions is not mentioned in these papers.

Fig. 2 shows isolines (relative unit) of velocities projections in convective cells, described by solutions (8), (9) taking into account (11) under $n=1$. As it follows from figures, despite the cell of rectangular geometry [15], maximal meaning of module of vertical speed of the cylindrical cell $\left|\mathrm{v}_z\left(r,z,t\right)\right|$ on the axis is more of its meaning on the outer border, and maximums of radial speed $\left|\mathrm{v}_r\left(r,z,t\right)\right|$ on the lower and upper borders of the cell are equal and are equally dislodged to its axis. Calculations show that isolines of distribution over coordinates of the relative temperature of the cylindrical cell *T/B* correspond to the distribution of relative vertical speed with opposite sign.

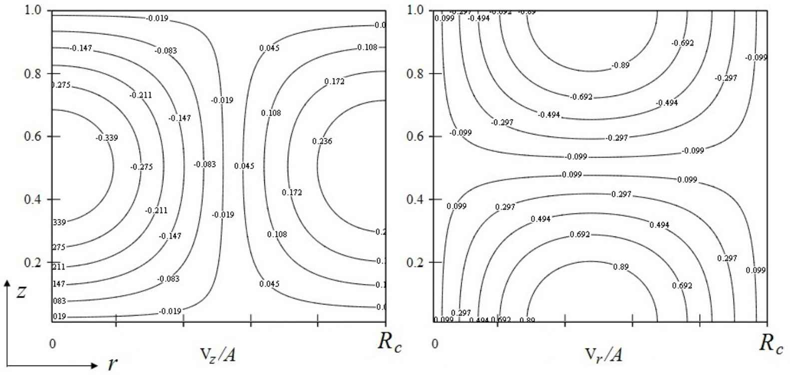


Fig. 2. Isolines of relative vertical and horizontal projections of speeds in convective cell.

Determination of the eigenvalues of cylindrical convective cell. In order to determine the spectrum of the eigenvalues for solutions (8), (9), their placement into input equations (2), (3) is needed. Such placement results in two equations, connecting the amplitudes of disturbances A and B:

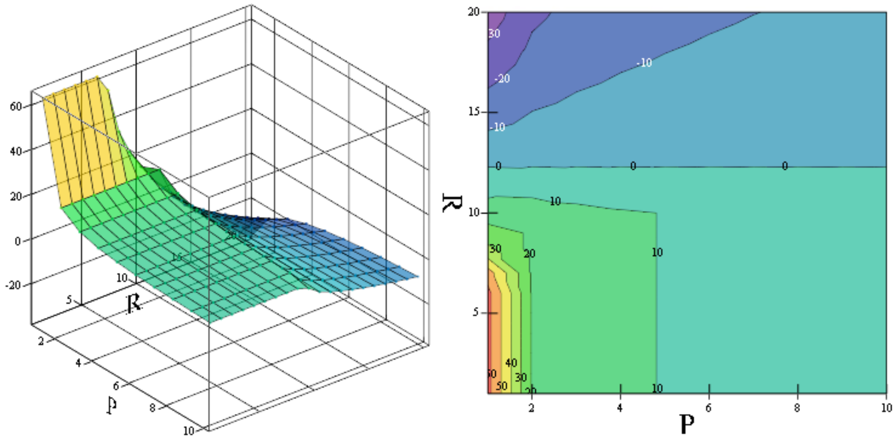
$$\begin{aligned}&\left(\pi^2n^2+k_{r,i}^2\right)\left[\lambda-\left(\pi^2n^2+k_{r,i}^2\right)\right]A+Rk_{r,i}^2B=0,\\&A+\left[\lambda P-\left(\pi^2n^2+k_{r,i}^2\right)\right]B=0.\end{aligned} \tag{14}$$

For existence of nontrivial solution of system of equations (14) its determinant should be compared to zero. As a result of this we will get the square equation relatively to the proper numbers λ. Roots of this equation $\lambda_{n,i}^{\pm}$ are determined with following expressions:

$$\lambda_{n,i}^{\pm}=\frac{P+1}{2P}\left(n^2\pi^2+k_{r,i}^2\right)\pm\left(\left(\frac{P-1}{2P}\right)^2\left(n^2\pi^2+k_{r,i}^2\right)^2+\frac{Rk_{r,i}^2}{P\left(n^2\pi^2+k_{r,i}^2\right)}\right)^{\frac{1}{2}}. \tag{15}$$

Spectrum of the eigenvalues (15) turns to be discrete not only on the mode of disturbances n, but also on radial wave of the cylindrical cell $k_{r,i}$, which receives discreet meanings, depending on the radius of cell R_c and roots of Bessel function $\sigma_{1,i}$.

Fig 3.a,b shows diagram of the surfaces, describing dependencies of real parts of the eigenvalues $\lambda_{n,i}^{\pm}$ on Rayleigh number R, Prandtl number P for the main mode $n=1$ at $i=1$ and $R_c=3$. While constructing the surface, the following change ranges R and P: $0.1\le P\le 3.0$; $-5000\le R\le 5000$ were set, and cell radius meaning R_c was selected in accordance with experimental data, presented in Table 1. On Fig. 3 to the left from diagram of surface of dependency of real parts of the eigenvalues from Rayleigh numbers and R Prandtl numbers P their levels lines are shown.



a b

Fig. 3. Dependence of real parts of the eigenvalues $\mathrm{Re}\left(\lambda_{1,1}^{-}\right)$ on the Rayleigh numbers R and Prandtl numbers P.

a – overall view, b – top view

It should be noted that imaginary values of solutions (15) provide oscillations of solutions (8), (9) which do not result in growth of their amplitudes.

Calculations show that real part of the eigenvalues $\mathrm{Re}\left(\lambda_{1,1}^{+}\right)$ is positive. For the such the eigenvalues, solutions (8), (9) attenuate exponentially.

Real part of other type of solutions $\mathrm{Re}\left(\lambda_{1,1}^{-}\right)$ (see Fig. 3) has areas either of positive or of negative meanings. As it follows from this there are meanings equal to zero in the spectrum of the eigenvalues $\mathrm{Re}\left(\lambda_{1,1}^{-}\right)$ This fact means that under $\mathrm{Re}\left(\lambda_{1,1}^{-}\right)=0$ in the cell, described by solutions (8), (9), stable condition of convective mass transfer is observed.

Determination of radius of cylindrical convective cell. Due to the fact that, for researched in the paper oil, the characteristic time turns to be about $\tau = 0.1$ s, then based on proportionality of disturbed vertical speed to the exponential multiplier $\exp\left(-\lambda\left(t/\tau\right)\right)$, stable existence of cylindrical cell is only possible at $\mathrm{Re}\left(\lambda_{1,1}^{-}\right)\approx 0$. From thism for the main mode ($n=1$), incomplete cubical equation can be written for determination of cell radius R_c :

$$f\left(x\right)\equiv x^3 - Rx + R\pi^2 = 0, \tag{16}$$

where $x = \pi^2 + \left(\sigma_{1,i}/R_c\right)^2$.

We will study only real solutions of equation (16). Under $R \ge 27\pi^4/4$ equation has three real solutions, two of which under $R \equiv R_{\min} = 27\pi^4/4$ are equal to [28]:

$$x_{1,2}\left(R\right) = -\left(a_+\left(R\right)+a_-\left(R\right)\right)/2 \mp i\sqrt{3}\left(a_+\left(R\right)-a_-\left(R\right)\right)/2$$

$$x_3\left(R\right) = a_+\left(R\right)+a_-\left(R\right)$$

$$\text{где } a_\pm\left(R\right)=\sqrt[3]{-\pi^2 R/2 \pm \sqrt{Q\left(R\right)}},\ Q\left(R\right) = -\left(R-27\pi^4/4\right)R^2/27.$$

Radius of convective cell, based on solutions of equation (16) can be determined by one of the following expressions:

$$\left(R_c\right)_k = \sigma_{1,i}\sqrt{R/\left(x_k\left(R\right)\right)^3} \tag{17}$$

where $k = 1,2,3$, $x_k\left(R\right)$ -solution of equation (16).

In order to select from expressions (17) of the physically substantiated meaning of convective cell radius we will proceed from the following experimental fact: with the increase of oil layer thickness cell radius in increasing. That is why as the results of numerical calculations show, in the expression (17), solution $x_1\left(R\right)$ should be used.

Qualitative estimation of heat power of the convective cell W under $z=0.5, n=1, \lambda=0$ shows proportionality to the radius R_c :

$$W = \alpha AC\rho S_{\Sigma}\left(T_2 - T_1\right), \tag{18}$$

where α - coefficient of proportionality is about one, C - specific heat capacity of the liquid, $S_{\Sigma} = 2\pi R_c \Delta$ - area of down flow liquid flux; $\Delta << R_c$ - thickness of liquid boundary layer, transferring the heat.

As the cell radius corresponds to the thickness of the layer (solution $x_1(R)$ is selected on this principle), expression for the power (18) can be brought to the value, which is proportional $\left(x_1(R)\right)^3$.

For analyses of conditions of appearance of convective cell we will study the dependency of solution $x_1(R)$ on the R number. This solution has maximal meaning $x_1(R) = \sqrt{R/3}$ at $R_{\min} = 27\cdot 4^{-1}\pi^4$ and with growth R is decreasing. Lets note that minimal meaning of Rayleigh number corresponds to minimal Rayleigh number of the main mode [13].

As it follows from solutions (17) and (18), with the increase of Rayleigh number from $R_{\min}$ to infinity, the radius of elementary convective cell in increasing, and heat power is decreasing $\propto \left(x_1(R)\right)^3$. Thus maximal power of the cell is reached under its minimal radius. Value of minimal radius can be determined by substitution of the obtained values $x_1(R_{\min})$ and $R_{\min}$ into expression (17):

$$R_c = \sigma_{1,i} 2^{\frac{1}{2}} \pi^{-1} \approx 0.45\sigma_{1,i}. \tag{19}$$

For the first zero of Bessel function $\sigma_{1,1} \approx 3.83$ cell radius is equal to $R_c \approx 1.72$, which if calculated on diameter, quantitatively corresponds to experimental data, presented in Table 1.

Thus for main disturbances mode ($n = 1$), maximal power of the cell is reached under minimal radius value. Radius value of cylindrical convective cell can receive discrete values, determined by zeroes of Bessel function of the first type of the first order ($J_1\left(\sigma_{1,i}\right) = 0$).

Substantiation of the cell size and forming of polygonal structures. As it follows from the described experiments (see Fig.1), with the increase of temperature of the vessel bottom, number of cylindrical convective cells of minimal diameter is increasing from one-two till filling of the whole surface of the vessel.

On our opinion, phenomenon of initiation of one cell and further appearance of the same second is stipulated by the fact that, they transfer more heat in the time unit between the liquid borders, than in case of elementary increase of cell diameter to the value, determined by second zero of Bessel function $\sigma_{1,2}$. In the second case, as it is shown in [22, 26 ,27], concentric convective shafts rotating towards can be created. However for creation of such structure, provision of special boundary conditions is needed, moreover they are unstable to discretization on hexagonal convective cells of smaller diameter [15].

Advantage of appearance of two similar cells is confirmed by the fact that power of convective liquid flux (18) is proportional to the radius either as to the perimeter of the cell. Consequently, forming of two similar cylindrical cells is energetically more profitable, than of one cell, but with larger radius. Energetic advantage is confirmed by correspondent inequality: $2\sigma_{1,1} = 7.664 > \sigma_{1,2} = 7.016$.

Based on the presented above statement, principle of pavement (coating) of the liquid surface with polygonal structures [29] can be formulated. As the increase of temperature of vessel bottom results in growth of a number of elementary cylindrical cells, there should be a moment when they start contacting each other and in the result of dense packing will start creating polygons, which cover (without gaps and coverings) the liquid surface. At this, the polygons structure with minimal radiuses of convective cells, and correspondingly with large total cells perimeter, provides maximal transfer of heat. In ideal case such polygons are hexagons. Further increase of temperature of the vessel bottom, as experiments show, will destroy the existing order and liquid layer is transferred into the stage of random boiling.

CONCLUSIONS

The paper is directed on make up for this deficiency. Paper demonstrates that with the increase of temperature of the vessel bottom with horizontal oil layer 1 mm, appearance of cylindrical convective cells in the quantity from one - two till full filling of the oil volume is observed. At this, the difference of temperatures between the oil borders remains on the level 10 – 13 °C/mm.

Cells appearing during the heating of the vessel bottom are characterized by distinct mutual position. On the upper surface of the oil they create similar circle with diameter about 3.2 mm. For free boundary conditions solutions were obtained which describe cylindrical convective cell. For such cell space distribution of the perturbed velocity and temperature was determined, and also its diameter was calculated, the value of which corresponds to experimental data. Based on the results of experiment and also based on the properties of the obtained solutions, cylindrical convective cell can be considered as elementary, from all large number of which convective structures are formed.

Energy principle of appearance of convective structures in the layer of viscous, incompressible liquid heated from below was formed. Energy substantiation of the principle of pavement (coating) of the liquid surface with cylindrical cells was presented.

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