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## COMPUTER SIMULATION OF EFFICIENCY ENHANCEMENT THROUGH PROFILING GUIDING MAGNETIC FIELD AT COAXIAL GYRO-BWO

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The gyro-BWO is a UHF powerful oscillator for cm and mm band of wavelength in which relativistic electrons beam is used for coupling with a backward wave on normal Doppler effect. The first research of gyro-devices was published in 60th. Possible applications of gyro-devices UHF radiation are the followings: electron cyclotron resonance heating of plasma for controlled fusion, communication, spectroscopic researches, high-resolution radars etc. Investigations of gyro-BWO confirm that an essential limitation of the considered device is the small efficiency. One of possibilities for efficiency enhancing is to use profiling guiding magnetic field along an interaction region. We investigated dependence of efficiency increasing under using optimal profiling guiding magnetic field by special law. As a result of effective process bunch formation under special conditions most electrons can be confined in the energy-losing phase HF field. Efficiency enhancement takes place from initial value  $\eta \sim 0.1$  for homogenous guiding field to  $\eta \sim 0.3$  for profiling one.

**KEY WORDS:** microwaves, gyro-devices, backward wave oscillators, guiding magnetic field, efficiency, computer simulation.

### КОМПЬЮТЕРНОЕ МОДЕЛИРОВАНИЕ УВЕЛИЧЕНИЯ КПД ПРИ ПРОФИЛИРОВАНИИ ВЕДУЩЕГО МАГНИТНОГО ПОЛЯ В КОАКСИАЛЬНОЙ ГИРО-ЛОВ

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Гиро-ЛОВ это мощный СВЧ генератор в сантиметровом и миллиметровом диапазоне длин волн, в котором релятивистский пучок электронов взаимодействует с обратной волной на нормальном эффекте Доплера. Первые исследования по гиро-приборам были опубликованы в 60-х годах. Возможные применения СВЧ излучения гиро-приборов следующие – это предварительный нагрев плазмы на эффекте циклотронного резонанса для управляемого термоядерного синтеза, для спектроскопических исследований, в радарх высокого разрешения и др. Исследование гиро-ЛОВ показали, что существенным ограничением для рассматриваемого генератора является малый КПД. Одной из возможностей увеличения КПД является профилирование ведущего магнитного поля в области взаимодействия. Мы исследовали зависимость увеличения КПД при использовании профилирования ведущего магнитного поля по определенному закону. При этих условиях эффективный процесс формирования банча приводил к тому, что большинство электронов оказывались в энергосоздающей фазе ВЧ поля. Имело место увеличение КПД от начального значения  $\eta \sim 0,1$  для однородного ведущего магнитного поля до значения  $\eta \sim 0,3$  для профилированного магнитного поля.

**КЛЮЧЕВЫЕ СЛОВА:** микроволны, гиро-приборы, генератор на обратной волне, ведущее магнитное поле, КПД, компьютерное моделирование.

### КОМП'ЮТЕРНЕ МОДЕЛЮВАННЯ ЗБІЛЬШЕННЯ ККД ПРИ ПРОФІЛЮВАННІ ВЕДУЧОГО МАГНІТНОГО ПОЛЯ В КОАКСІАЛЬНОЇ ГИРО-ЛЗХ

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Гиро-ЛЗХ- це потужний НВЧ генератор у см і мм діапазоні довжин хвиль, у якому релятивістський пучок електронів взаємодіє зі зворотною хвилею на нормальному ефекті Доплера. Перші дослідження з гиро-приладів були опубліковані в 60-х роках. Можливі застосування НВЧ випромінювання гиро-приладів – це попереднє нагрівання плазми на ефекті циклотронного резонансу для керованого термоядерного синтезу, спектроскопічних дослідженнях, радарх високого вирішення та ін. Дослідження гиро-ЛЗХ показали, що істотним обмеженням для розглянутого генератора є малий ККД. Однієї з можливостей збільшення ККД є профілювання ведучого магнітного поля в області взаємодії. Ми досліджували залежність зростання ККД при використанні профілювання ведучого магнітного поля по визначеному закону. При цих умовах ефективний процес формування банча приводив до того, що більшість електронів опинялись в енерговитратній фазі ВЧ поля. В остаточному підсумку мало місце збільшення ККД від початкового значення  $\eta \sim 0,1$  для однорідного ведучого магнітного поля до значення  $\eta \sim 0,3$  для профілюваного магнітного поля.

**КЛЮЧОВІ СЛОВА:** мікрохвилі, гиро-прилади, генератор на зворотній хвилі, магнітне поле, що веде, коефіцієнт корисної дії (ККД), комп'ютерне моделювання.

The gyro-BWO is a HF powerful oscillator for cm and mm band of wavelength in which relativistic electrons beam (REB) is used for coupling with a backward wave on normal Doppler effect. The first research of gyro-devices was published in 60<sup>th</sup> [1]. The state of the art of gyro-BWO program is represented in Ref. [2], [3]. Possible applications

of the HF radiation of obtained power levels are the followings: electron cyclotron resonance heating (ECRH) of plasma for controlled fusion, communication, spectroscopic researches, high-resolution radars etc.

In our paper the case of coaxial waveguide is investigated for gyro-BWO elaboration. The choice of coaxial waveguide conditioned by greater value of vacuum limiting current of REB for one comparatively to the other types of waveguide. Results of the linear and non-linear analytical investigation of coaxial gyro-BWO operation are presented in Ref. [4, 5].

An electron beam and waveguide support the oscillations with circular frequency  $\omega$ , which can be described by the expressions for normal Doppler effect, accordingly

$$\omega = k_z V_z + n\Omega_H / \gamma_0 \quad (1)$$

$$\omega^2 = c^2 k_\perp^2 + c^2 k_z^2, \quad (2)$$

where  $\Omega_H = \frac{eH_z^g}{mc}$  is non-relativistic gyro-frequency of electrons with energy  $W = m_0 c^2(\gamma_0 - 1)$ ,  $H_z^g$  -guiding magnetic field,  $\gamma_0$  -relativistic factor,  $k_z, V_z$  -longitudinal wave number and velocity,  $n=0, \pm 1, \pm 2, \dots$ . An operating mode for gyro-BWO is near to interception of a straight line (1) and hyperbola (2) in coordinate plane  $(\omega, k_z)$  (for gyro-BWO the longitudinal wave number  $k_z < 0$ ). An ordinary efficiency value for coaxial gyro-BWO is  $\sim 10\%$  for homogenous guiding magnetic field  $H_z^g$ .

The major attractive feature of the gyro-BWO is frequency tunability, which can be achieved by management the magnetic field or beam voltage. However, the efficiency of the gyro-BWO is relatively lower than one of other gyro-devices. Investigation results of efficiency enhancement in gyro-BWO were reported in Ref. [6-15]. In Ref. [6-9], the efficiency of the gyro-BWO has been found to be significantly improved by tapering the magnetic field. Results found revealed that the magnetic field tapering with a positive gradient tended to increase the initial frequency mismatch leading to the efficiency enhancement.

In Ref. [10-13] a tapered interaction structure (the reduction of the waveguide radius along the interaction region) was proposed and used in the experiment. The gyro-BWO with a tapered magnetic field and waveguide wall radius was analyzed in Ref. [14, 15].

The aim of our paper is enhancement of efficiency in coaxial gyro-BWO through profiling of guiding magnetic field.

### HIGH EFFICIENCY AND BUNCHED BEAM FORMATION

Optimal conditions for this process determine further process of reduction total beam energy converting to radiation. It follows that greater electrons can be located in the energy-losing phase as a result. The bunched beam formation process in gyro-BWO has difference from standard bunching process because of amplitude of EM wave depends on longitudinal coordinate. That reduction of EM amplitude might be compensated by changing position electron bunch as a whole one relatively phase of EM wave. It is possible to do through local variation of guiding magnetic field for compensating of EM field amplitude reduction along longitudinal coordinate during formation process only by changing total (helical) phase, including EM wave phase plus helical phase of guiding magnetic field

$$\Psi = k_z z - \omega t - \Omega_H t / \gamma.$$

Confinement of as many electrons as possible takes place in energy-losing phase due to phase shift under optimal bunched beam formation. As a result phase shift of bunched beam's formation in energy-losing phase takes place.

### COMPUTER SIMULATION

We investigated in our paper efficiency enhancement in coaxial gyro-BWO through profiling of guiding magnetic field  $H_z^g(z)$  at longitudinal direction  $z$  as

$$H_z^g(\xi) = H_{z_0}^g (1 + \alpha(\xi/\bar{L})^j \cos^m(\pi\xi/2\bar{L}))^{1/2}, \quad (3)$$

comparatively to homogenous case  $H_z^g = H_{z_0}^g$ , where  $\alpha$  is non-homogeneity amplitude,  $\xi = z\omega/c$  is normalizing longitudinal coordinate,  $\bar{L} = L\omega/c$  is normalizing waveguide length,  $\xi/\bar{L} = z/L$ ,  $j > 0$ ,  $m > 0$ . A corresponding transversal component one is

$$H_r^g(z) = -\frac{r}{2} \frac{\partial H_z^g}{\partial z},$$

where  $r$  is transversal coordinate.

We considered waveguide exciting mode  $TE_{01}$  with components of an electromagnetic field  $E_\varphi, H_r, H_z$  under satisfy conditions (1, 2). For computer simulation we used equations for electrons motion and exciting field  $TE_{01}$  from Ref. [5]. We investigated coaxial gyro-BWO with oscillation frequency  $f_0 = 7.7 \text{ GHz}$  for satisfying expressions (1, 2),

homogenous guiding magnetic field  $H_{z0}^g = 6.1kOe$ , inner radius of the coaxial waveguide gyro-BWO  $b = 3cm$ , outer radius one is  $a = 5cm$ , inner beam radius is  $r_b = 3.9cm$ , outer beam radius is  $r_a = 4.1cm$ , energy of injected electron beam is  $W_0 = 511keV$  ( $\gamma_0 = 2$ ), an initial ratio transversal momentum to longitudinal one  $\mu = 1$ , length of system is  $L = 60cm$ , cut off frequency  $f_c = 7.5GHz$ , starting current  $I_{st} = 3.7A$ , limiting vacuum current  $I_{lim} = 6.6kA$  for coaxial waveguide. Maximal efficiency  $\bar{\eta}_{max} \approx 0.11$  is under input beam current  $I_b = 0.6kA$  for homogenous guiding field and cited above gyro-BWO parameters [5]. Computer simulation of optimal regime gyro-BWO performance time averaged efficiency  $\bar{\eta}$  was carried out for determination values  $\alpha, m$  and  $j$  of profiling guiding magnetic field (3) under the same input electron beam current  $I_b = 0.6kA$ .

First, we have to determine location of guiding field maximum relatively longitudinal coordinate  $\xi$  under  $j=1$  and  $H_{z0}^g = 6.1kOe$ . Results of that investigation are presented in Table 1 under  $I_b = 0.6kA$ .

Table 1

$\bar{\eta}$	0.19	0.27	0.29	0.32	0.29	0.26
$m$	0	2	4	6	7	9
$\alpha$	0.69	1.6	2.4	2.9	3.2	3.7
$\xi_0 / \bar{L}$	-	0.41	0.3	0.25	0.24	0.21
$\frac{H_{zmax}^g(\xi_0)}{H_{z0}^g}$	-	1.2	1.2	1.2	1.2	1.2

We can see from Table 1 maximal value of time averaged efficiency  $\bar{\eta}_{max} = 0.32$  is under  $m=6$ ,  $\alpha = 2.9$  and  $\xi_0 / \bar{L} = z_0 / L = 0.25$ . The location of maximum one  $\xi_0 / \bar{L}$  can be determined analytically from expression (3)

$$mx_0 \operatorname{tg}(x_0) = j,$$

where  $x_0 = \pi z_0 / (2L)$ , or approximately for  $x_0 \ll 1$

$$\xi_0 / \bar{L} = z_0 / L \approx (2\sqrt{j/m}) / \pi, \quad (m \neq 0). \quad (4)$$

Second step is determining optimal amplitude of additional guiding magnetic field under cited above fixed  $m=6$ ,  $\xi_0 / \bar{L} = 0.25$  and  $j=1$ . An amplitude variation takes place due to variation of parameter  $\alpha$  from (3).

Table 2

$\bar{\eta}$	0.22	0.32	0.28
$(H_z^g)_{max}(\xi_0) / H_{z0}^g$	1.19	1.2	1.22
$\alpha$	2.7	2.9	3.1

We can see from Table 2 maximal efficiency  $\bar{\eta}_{max} = 0.32$  takes place under  $(H_z^g)_{max} / H_{z0}^g = 1.2$  ( $\alpha = 2.9$ ) in our case.

Finally, third step is determining optimal width  $\Delta \xi_1 / \bar{L}$  of profiling guiding magnetic field (3) under fixed  $\xi_0 / \bar{L} = 0.25$  and  $(H_z^g)_{max} / H_{z0}^g = 1.2$ . A width variation takes place due to level variation of parameter  $j$  from (3) under  $m/j = \text{const}$  (4) for fixed maximum location  $\xi_0 / \bar{L} = 0.25$ . Widening of guiding magnetic field takes place under  $1 > j > 0$ , narrowing of one takes place under  $j > 1$  relatively width of field distribution under  $j=1$ .

Table 3

$\bar{\eta}$	0.144	0.32	0.18
$J$	0.5	1	2
$M$	3	6	12
$\alpha$	1.14	2.9	18.5
$\Delta \xi_1 / \bar{L}$	0.53	0.41	0.29

In our case maximal efficiency  $\bar{\eta}_{\max} = 0.32$  (see Table 3) takes place under  $j=1$  ( $m=6, \alpha = 2.9$ ) and width  $\Delta\xi_1 / \bar{L} = 0.41$  for  $H_z^g(\xi_1) = H_z^g(\xi_1 + \Delta\xi_1) = 1.1H_{z0}^g$ .

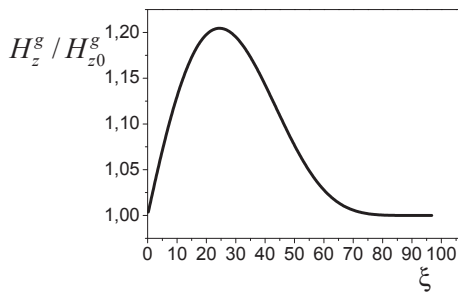


Fig.1. Dependence normalized magnetic field amplitude  $H_z^g(\xi)/H_{z0}^g$  on dimensionless longitudinal coordinate  $\xi$ .

Hence, we determined location, amplitude and width values of profiling guiding magnetic field for our gyro-BWO parameters.

In our case optimal process for bunching of input electron beam takes place under

$$m=6, \alpha = 2.9, j=1, \tag{5}$$

for expression (3). In fig.1 we can see longitudinal distribution of guiding magnetic field for parameters (5).

The new profile of guiding magnetic field changes spatial distribution of EM wave amplitude  $C_\varphi = eE_\varphi/(mc\omega)$  along longitudinal coordinate. A spatial distribution of normalized amplitude on longitudinal coordinate  $\xi$  practically haven't local maxima (see Fig.2) homogenous one.

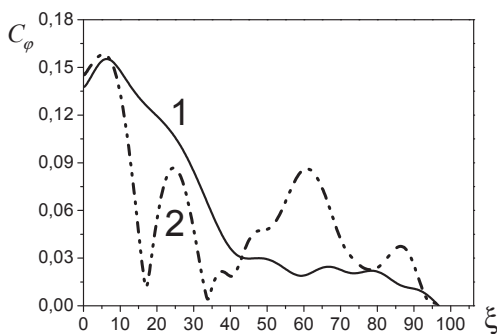


Fig.2. Dependence of normalized amplitude  $C_\varphi$  on dimensionless longitudinal coordinate  $\xi$  (curve 1 is for profiling guiding magnetic field (3), curve 2 is for homogenous one under  $H_z^g(z) = H_{z0}^g$ ). Normalized observation time  $\tau_0 = k_z ct = 1500$ .

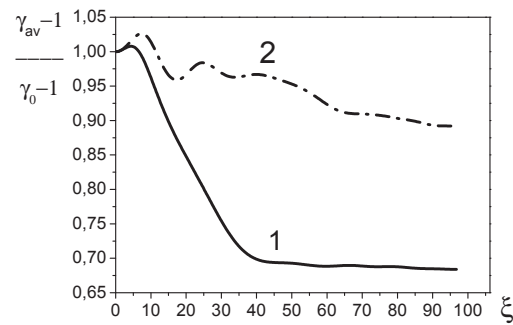


Fig.3. Dependence normalized average bunch energy  $(\gamma_{av}-1)/(\gamma_0-1)$  on dimensionless longitudinal coordinate  $\xi$  (curve 1 is for profiling guiding magnetic field, curve 2 is for homogenous one). Normalized observation time  $\tau_0 = k_z ct = 1500$ .

The dependence of normalized average bunch energy  $(\gamma_{av}-1)/(\gamma_0-1)$  on dimensionless longitudinal coordinate  $\xi$  has a monotone character in contrast to homogenous distribution (fig.3). The energy loss has maximal value at the first half of interaction region.

For determining frequency characteristics of output signal spectrum analysis was obtained for normalized spectrum density  $S(f - f_0)$

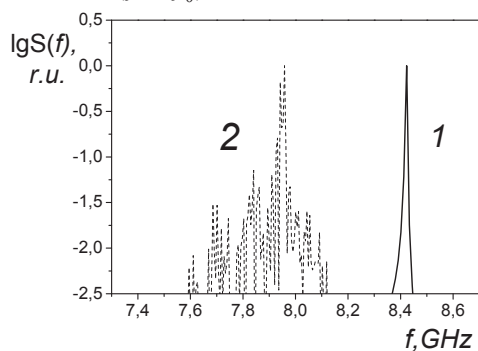


Fig.4. Dependence normalized spectrum density  $S(f)$  on frequency  $f$  (curve 1 is for profiling guiding magnetic field (3), curve 2 is for homogenous one under  $H_z^g(z) = H_{z0}^g$ ).

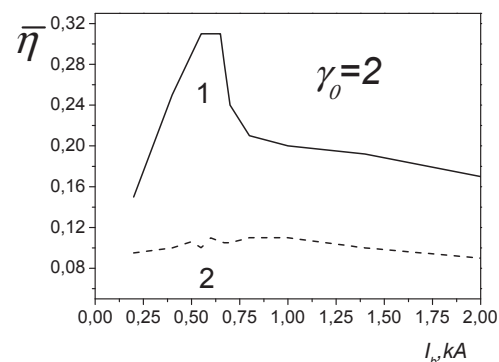


Fig.5. Dependence time averaged efficiency  $\bar{\eta}$  from injection beam current  $I_b$  (curve 1 is for profiling guiding magnetic field (3), curve 2 is for homogenous one  $H_z^g(z) = H_{z0}^g$ ).

Dependence normalized spectrum density  $S(f)$  on frequency  $f$  for homogenous guiding magnetic field (stochastic oscillations) and for profiling one (stationary oscillations for the same remaining conditions) is shown in fig.4. We can see in fig.4 conversion stochastic oscillations under homogenous guiding magnetic field to stationary oscillations for profiling one.

Then we used computer simulation for investigation efficiency  $\bar{\eta}$  dependence on injection beam current  $I_b$  under fixed parameters of guiding field  $\alpha$ ,  $m$  and  $j$  [see eq. (5)]. For given injection energy  $\gamma_0 = 2$  efficiency  $\bar{\eta} \geq 0.25$  is for  $0.7kA \geq I_b \geq 0.4kA$  (see fig. 5). Efficiency  $\bar{\eta}$  for profiling guiding field has essentially greater value than  $\bar{\eta}$  for homogenous case. Average output longitudinal energy  $(\gamma_{av,z} - 1)/(\gamma_{0,z} - 1)$  of a bunch increases by 10% for homogenous guiding magnetic field and by 15 % for profiling one under  $\gamma_0 = 2$  and  $I_b = 0.6kA$ .

The main reason for relatively lower gyro-BWO efficiency is its spatial distribution of the wave power that reaches a maximum near the entrance of the electron beam and decays along the propagation direction of electrons. This power profile leads to non-optimal formation of bunched electron beam at the beginning of gyro-BWO under homogenous distribution of guiding magnetic field.

We suggested non-homogenous distribution (3) for creation the most optimal conditions during process bunch formation of input electron beam. In fig.6-8 you can see difference between bunch formation for homogenous guiding magnetic field and non-homogenous one on phase plane energy-phase for various fixed values of longitudinal coordinate  $\xi$  ( $I_b = 0.6kA$ ,  $\gamma_0 = 2$ ). For all of figures circles correspond to homogenous case, black points correspond to non-homogenous one.

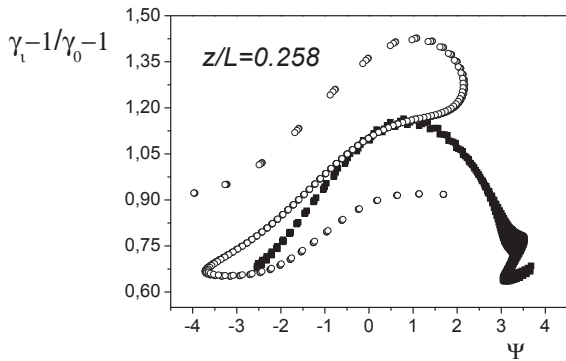


Fig.6. Dependence normalized energy of the particles beam  $\gamma_t - 1/\gamma_0 - 1$  on helical (total) phase  $\Psi$  for  $z/L=0.258$  (black points correspond profiling field, circles correspond homogenous field).

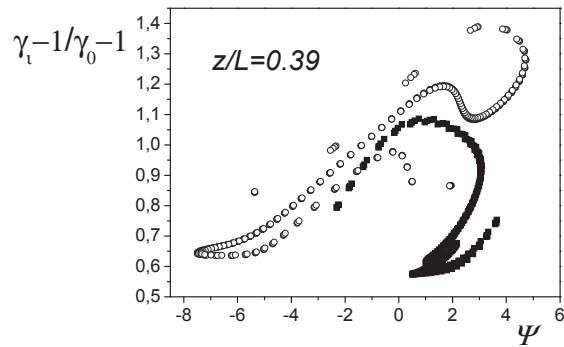


Fig.7. Dependence normalized energy of the particle beam  $\gamma_t - 1/\gamma_0 - 1$  on helical (total) phase  $\Psi$  for  $z/L=0.39$ .

The majority particles for profiling guiding magnetic field have energy-losing phase from the beginning bunch formation process comparatively homogenous one (see fig. 6). An effective bunch formation process takes place along longitudinal coordinate (fig.7, 8).

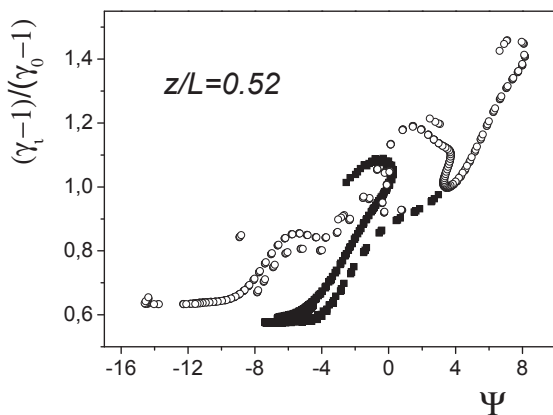


Fig.8. Dependence normalized energy of the particles beam  $\gamma_t - 1/\gamma_0 - 1$  on helical (total) phase  $\Psi$  for  $z/L=0.52$ .

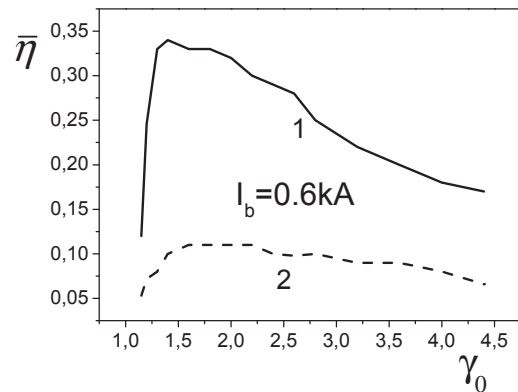


Fig.9. Dependence time averaged efficiency  $\bar{\eta}$  on relativistic factor  $\gamma_0$  (curve 1 is for profiling guiding magnetic field (3), curve 2 is for homogenous one under  $H_z^g(z) = H_{z0}^g$ ).

As result phase portrait of bunched beam along longitudinal coordinate is more compact in comparison with homogenous guiding magnetic field. An effective formation of bunched beam in energy-losing phase leads to most electrons can be confined in the losing energy phase even after completion a compact bunching process (fig.8).

We obtained enhancing of gyro-BWO's efficiency for given injection beam energy  $W_0 = 511 keV$  ( $\gamma_0 = 2$ ) as indicated above. Then we investigated dependence  $\bar{\eta}$  on injection energy  $W_0$  under condition  $H_{z0}^g/\gamma_0 \approx const.$  Dependence time averaged efficiency  $\bar{\eta}$  on relativistic factor  $\gamma_0$  under injection beam  $I_b = 0.6 kA$  is shown in fig.9. Let's consider more detailed energy range close to  $\bar{\eta}$  peak in Fig.9 for profiling field.

Table 4

$W_0, keV$	102	153	307	613	715
$\gamma_0$	1.2	1.3	1.6	2.2	2.4
$H_{z0}^g, kOe$	3.5	3.9	4.8	6.7	7.4
$\bar{\eta}$	0.25	0.33	0.33	0.3	0.29

High efficiency value  $\bar{\eta} \approx 0.3$  takes place not only for injection  $\gamma_0 = 2$ , but for sufficiently wide range energy  $\gamma_0 = (1.2 \div 2.4)$  too (see Table 4).

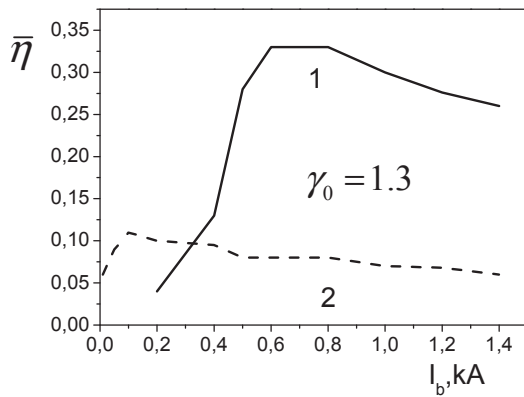


Fig.10. Dependence time averaged efficiency  $\bar{\eta}$  from injection beam current  $I_b$  (curve 1 is for profiling guiding magnetic field (3), curve 2 is for homogenous one  $H_z^g(z) = H_{z0}^g$ )

Finally, under fixed parameters of guiding field  $\alpha$ ,  $m$  and  $j$  (3) we considered dependence of time averaged efficiency  $\bar{\eta}$  on injection beam current  $I_b$  under new injection beam energy  $W_0 = 153 keV$  ( $\gamma_0 = 1.3$ ). For given injection energy  $\gamma_0 = 1.3$  efficiency  $\bar{\eta} \geq 0.25$  is for  $1.4 kA \geq I_b \geq 0.5 kA$  (see Fig.10). Maximal efficiency  $\bar{\eta} \approx 0.3$  is close to injection beam current  $I_b \approx 0.6 kA$  the same as for injection energy  $\gamma_0 = 2$ . Increasing of starting current takes place under profiling guiding magnetic field takes place from  $I_{st} = 3.7 A$  (homogenous magnetic field) to  $I_{st} = 17 A$  for profiling one under  $\gamma_0 = 2$ . Oscillations of competition mode  $TE_{11}$   $f=4$  GHz under given resonance conditions (1) and (2) are unlikely event. The  $TE_{11}$  mode is separated far enough from the operating  $TE_{01}$  mode under the given conditions. Resonant frequency of the  $TE_{11}$  mode is  $f = 4 GHz$ , the frequency of the  $TE_{01}$  mode is  $f = 7.7 GHz$  in our case.

## CONCLUSIONS

In our paper we obtained enhancement of gyro-BWO's efficiency from 11% (homogenous distribution of guiding magnetic field) up to 32% (non-homogenous one) through profiling of magnetic field (3). Oscillation frequency has fixed value under satisfying equations (1), (2). As a result of effective process of bunch formation under special conditions most electrons can be confined in the energy-losing phase. Results of our investigations haven't static character for other gyro-BWO's parameters or other gyro-devices. In every case parameter values of profiling guiding magnetic field (maximum field coordinate, amplitude and width of like-bell guiding field distribution) are subject of investigations.

The obtained efficiency is closely to gyrotron's efficiency (without single stage depressed collector (SDC) for energy recovery). The current mechanism can also be applied to interpret the efficiency enhancement in other gyrotron oscillators (for example, cyclotron autoresonance maser in Ref. [16]) with profiling guiding magnetic field.

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