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ENERGETIC CHARACTERISTICS OF THE SURFACE ELECTROMAGNETIC WAVE AT PLASMA-PLASMA INTERFACE IN MAGNETIC FIELD

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Energetic characteristics of a surface wave at a plane plasma-plasma interface in an external magnetic field are studied. The dependencies of energy flow density and energy density on wave vector for different magnitudes of external magnetic field are analyzed. The sign of the Poynting vector in each plasma region is investigated. It is shown that the total energy flux of the surface wave is directed along the wave vector. The velocity of energy propagation is found. The equivalence between energy velocity and group velocity of the wave is demonstrated. The results obtained in the paper may be useful for analysis of transmission of electromagnetic waves through two-layer plasma structures in a magnetic field.
KEY WORDS: surface waves, Voigt geometry, Poynting vector, energy velocity, energy density

ЕНЕРГЕТИЧНІ ХАРАКТЕРИСТИКИ ПОВЕРХНЕВОЇ ЕЛЕКТРОМАГНІТНОЇ ХВИЛІ НА МЕЖІ ПОДІЛУ ПЛАЗМА-ПЛАЗМА В МАГНІТНОМУ ПОЛІ

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Вивчаються енергетичні характеристики поверхневої хвилі на пласкій межі плазма-плазма в зовнішньому магнітному полі. Аналізуються залежності густини потоку енергії та густини енергії від хвилевого вектора для різних значень зовнішнього магнітного поля. Досліджується знак вектора Пойнтінга в кожній плазмовій області. Показується, що повний потік енергії поверхневої хвилі спрямований вздовж хвилевого вектора. Знайдено швидкість поширення енергії. Продемонстровано рівність між енергетичною та груповою швидкістю хвилі. Результати, що були отримані в статті, можуть бути використані для аналізу проходження електромагнітної хвилі крізь двошарову плазмову структуру в магнітному полі.
КЛЮЧОВІ СЛОВА: поверхневі хвилі, геометрія Фойгта, вектор Пойнтінга, енергетична швидкість, густина енергії

ЭНЕРГЕТИЧЕСКИЕ ХАРАКТЕРИСТИКИ ПОВЕРХНОСТНОЙ ЭЛЕКТРОМАГНИТНОЙ ВОЛНЫ НА ГРАНИЦЕ ПЛАЗМА-ПЛАЗМА В МАГНИТНОМ ПОЛЕ

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Изучаются энергетические характеристики поверхностной волны на плоской границе плазма-плазма во внешнем магнитном поле. Анализируются зависимости плотности потока энергии и плотности энергии от волнового вектора для разных значений внешнего магнитного поля. Исследуется знак вектора Пойнтинга в каждой плазменной области. Показывается, что полный поток энергии поверхностной волны направлен вдоль волнового вектора. Найдена скорость распространения энергии. Продемонстрировано равенство между энергетической и групповой скоростью волны. Результаты, полученные в статье, могут быть использованы для анализа прохождения электромагнитной волны через двухслойную плазменную структуру в магнитном поле.
КЛЮЧЕВЫЕ СЛОВА: поверхностные волны, геометрия Фойгта, вектор Пойнтинга, энергетическая скорость, плотность энергии

Propagation of electromagnetic waves in plasma has been extensively studied in various contexts for many years [1–3]. Researchers are still interested in the interaction of the electromagnetic radiation with overdense plasma. Transmission and absorption of electromagnetic waves that propagate through layered plasma structures, in particular, are studied due to practical applications. The supersonic aircrafts are surrounded by the layer of dense plasma, created by the heat from the compression of the atmosphere. The plasma layer is not transparent to radio waves that causes radio communications blackout. Different solutions [4,5] are proposed to this problem. Effective absorption of electromagnetic energy is important in plasma generation and heating. Enhanced absorption was observed in plasmas with steep electron density profiles. The increased absorption was explained by surface wave excitation [6,7]. Another way to increase absorption of the incident wave is to place subwavelength diffraction grating in front of plasma [8].

The effect of the transmission assisted by the surface waves occurs also in plasma-like media such as metals in optical range. It was shown both experimentally and theoretically, that three-layer structure composed of metal sandwiched between two layers of dielectric is absolutely transparent for incident *p*-polarized electromagnetic wave with certain wave vector and frequency [9]. Lately it was noticed that similar effect can be achieved by adding only one layer of dielectric [10]. One more way to achieve transparency for metallic film is to perforate it with a periodic array of subwavelength-sized holes [11]. Perforation is needed to transform incident propagating wave into non-propagating that couples with surface mode at vacuum-metal interface. Resonant nature of a transparency in that type of structures can be used to build various kinds of tunable filters and spatial and spectral multiplexors [12,13].

The conditions of total transparency for two- and three-layer structures were studied in [14]. For two-layer structure it was found that the condition for total transparency coincides with dispersion relation for surface waves at the interface separating layers. The method of finding resonant conditions for multilayer structures with arbitrary number of layers was proposed in [15]. The role of dissipation effects in transmission through the structure composed of two layers of warm plasma was studied in [16].

Propagation of energy in case of total transmission was also considered. It was demonstrated that finite energy flux in direction perpendicular to the interface is possible if width of the layers is finite and there is a finite phase shift between the amplitudes of the growing and decaying part of the evanescent wave [14,16]. In [17] tunneling time and velocity of electromagnetic wave propagating through the barrier was studied. It was compared two velocities of wave propagation: energy velocity and group delay velocity that are equal only in case of total transmission.

In our work [18] we studied the influence of external magnetic field on transparency of *p*-polarized electromagnetic wave through two-layer plasma structure. We found condition for absolute transparency and showed that it coincides with dispersion relation for surface waves only if layers are sufficiently thick. Later [19] we investigated the influence of inhomogeneity of one of the layers on wave transmission.

In this paper we study energetic properties of the electromagnetic wave propagating in a two-layer structure in external magnetic field. We consider infinitely thick layers because resonance condition in this case has simple form. The normal component of the time-averaged energy flux turns to zero but we still can obtain information about the tangential component that gives us approximation for the case of layers of finite thickness. We derive and analyze expressions for group velocity, time-averaged Poynting vector and energy density. The special focus is given to the direction of energy propagation.

MAIN EQUATIONS

Consider an electromagnetic surface wave (SW) propagating at a plane interface between two semi-infinite plasma regions. It is assumed that plasma in each region is uniform, cold, collisionless and consists of electrons and ions. The wave frequency ω is assumed to be larger than the ion plasma frequency ω_{pi}, therefore, the effect of ions on the wave may be neglected. The less dense plasma with electron plasma frequency ω_{p1} occupies the region $x < 0$, while the more dense plasma with electron plasma frequency $\omega_{p2}(> \omega_{p1})$ is located at $x > 0$. The plasma system is immersed in an external magnetic field $\mathbf{H}_0$ directed along *z*-axis parallel to the plasma-plasma interface.

We assume the surface wave propagates in *y*-direction along the interface perpendicularly to the magnetic field. The wave is assumed to be *p*-polarized, and it is characterized by the wave vector $\vec{k}_y$directed along *y*-axis. The SW amplitude decays from the boundary $x = 0$. The electromagnetic field of the surface wave has the following components

$$\mathbf{E} = \left(E_x(x), E_y(x), 0\right)\exp\left(ik_y y - i\omega t\right),$$
$$\mathbf{H} = \left(0,0, H_z(x)\right)\exp\left(ik_y y - i\omega t\right).$$

The amplitudesE_x, E_y,H_z can be found from the following expressions [3]

$$E_x(x) = -\frac{1}{k_0(\varepsilon^2 - g^2)}\left(k_y\varepsilon H_z + g\frac{dH_z}{dx}\right), \tag{1}$$
$$E_y(x) = -\frac{i}{k_0(\varepsilon^2 - g^2)}\left(k_y g H_z + \varepsilon\frac{dH_z}{dx}\right), \tag{2}$$
$$\frac{d^2H_z}{dx^2} + \kappa^2 H_z = 0, \tag{3}$$

where $\kappa = \sqrt{k_y^2 - \varepsilon_V k_0^2}$ is the decay constant and $k_0 = \omega/c$. We introduce the Voigt dielectric permittivity$\varepsilon_V = \varepsilon - g^2/\varepsilon$, where ε and g are the components of the dielectric tensor $\epsilon_{ij}$for cold magnetoactive plasma neglecting ion motion and particle collisions:

$$\varepsilon \equiv \epsilon_{11} = \epsilon_{22} = 1 - \frac{\omega_p^2}{\omega^2 - \omega_c^2},$$
$$g \equiv -i\epsilon_{12} = i\epsilon_{21} = \frac{\omega_c\omega_p^2}{\omega(\omega^2 - \omega_c^2)},$$

where $\omega_c > 0$ is the electron cyclotron frequency. It is assumed that the component of tensor ε_1 for the region $x < 0$is positive, while the component ε_2 for the region $x > 0$ is negative. Here, indexes 1 and 2 correspond to the less and more dense plasma regions, respectively.

The flux of energy carried by the electromagnetic wave is described by the Poynting vector

$$\mathbf{S} = \frac{c}{4\pi}[\mathbf{EH}].$$

The Poynting vector of a *p*-polarized surface wave has two components: S_x and S_y.The time-averaged flux in *x*-

direction is zero $\langle S_x \rangle = 0$. Thus, there is only one non-zero time-averaged component of the energy flux density

$$\langle S_y \rangle = -\frac{c}{8\pi} Re[E_x H_z^*].$$

Using the expression for $S_y$and the expressions for wave field components (1) – (3), one gets the time-averaged energy flux density in the first and second plasma regions, correspondingly

$$\langle S_{y1} \rangle = \frac{c}{8\pi k_0}\left(\frac{k_y\varepsilon_1 + \kappa_1 g_1}{\varepsilon_1^2 - g_1^2}\right)|H_{z0}|^2 \exp(2\kappa_1 x), \tag{4}$$

$$\langle S_{y2} \rangle = \frac{c}{8\pi k_0}\left(\frac{k_y\varepsilon_2 - \kappa_2 g_2}{\varepsilon_2^2 - g_2^2}\right)|H_{z0}|^2 \exp(-2\kappa_2 x), \tag{5}$$

where $|H_{z0}|$ is the amplitude of magnetic field of the surface wave at the interface. The energy flux density per unit area of the interface can be obtained by integrating Eqs.(4) - (5) along *x*-axis:

$$S_{y1} = \int_{-\infty}^{0} \langle S_{y1} \rangle dx = \frac{S_0}{\kappa_1}\left(\frac{k_y\varepsilon_1 + \kappa_1 g_1}{\varepsilon_1^2 - g_1^2}\right), \tag{6}$$

$$S_{y2} = \int_{0}^{+\infty} \langle S_{y2} \rangle dx = \frac{S_0}{\kappa_2}\left(\frac{k_y\varepsilon_2 - \kappa_2 g_2}{\varepsilon_2^2 - g_2^2}\right), \tag{7}$$

where we introduce the unit of energy flux density $S_0 = c|H_{z0}|^2/16\pi k_0$. The total energy flux density carried by the surface wave is a sum of energy fluxes in each media:

$$S = S_{y1} + S_{y2} = S_0\left(\frac{g_1}{\varepsilon_1^2 - g_1^2} - \frac{g_2}{\varepsilon_2^2 - g_2^2} + k_y\left(\frac{1}{\kappa_1\varepsilon_{V1}} + \frac{1}{\kappa_2\varepsilon_{V2}}\right)\right). \tag{8}$$

We can simplify this equation by making use of the SW dispersion equation. The dispersion equation derived by matching tangential electric and magnetic fields of the wave at the interface is:

$$\frac{k_y g_1 + \varepsilon_1\kappa_1}{\varepsilon_1^2 - g_1^2} - \frac{k_y g_2 - \varepsilon_2\kappa_2}{\varepsilon_2^2 - g_2^2} = 0. \tag{9}$$

Equation (9) can be rewritten in the following form

$$\frac{g_1}{\varepsilon_1^2 - g_1^2} - \frac{g_2}{\varepsilon_2^2 - g_2^2} = -\frac{1}{k_y}\left(\frac{\kappa_1}{\varepsilon_{V1}} + \frac{\kappa_2}{\varepsilon_{V2}}\right). \tag{10}$$

Combining Eq.(10) with Eq.(8), after some algebraic manipulations one gets the expression for SW time-averaged energy flux density per unit area

$$S = S_0\frac{k_0^2}{k_y}\left(\frac{1}{\kappa_1} + \frac{1}{\kappa_2}\right). \tag{11}$$

Since $\kappa_1 > 0$ and $\kappa_2 > 0$, the sign of total energy flux density$S$ is determined by the sign of wave vector k_y. Thus, the wave energy propagates in the same direction as the wave phase.

The energy flux density (11)can be expressed through the derivative of the dispersion equation with respect to wave vector k_y.

Using Eqs. (6) and (7), the total energy flux density per unit area can be presented in the different form:

$$S = \frac{|E_{y0}|^2}{16\pi}\omega\left(\frac{1}{\kappa_1}\frac{\hat{E}_{x1}}{\hat{E}_{y1}^2} + \frac{1}{\kappa_2}\frac{\hat{E}_{x2}}{\hat{E}_{y2}^2}\right), \tag{12}$$

where $|E_{y0}| = |E_y(0)|$ is the tangential component of the wave electric field at the interface.

$$\hat{E}_{x1} = \frac{k_y\varepsilon_1 + \kappa_1 g_1}{\varepsilon_1^2 - g_1^2}, \quad \hat{E}_{x2} = \frac{k_y\varepsilon_2 - \kappa_2 g_2}{\varepsilon_2^2 - g_2^2}, \quad \hat{E}_{y1} = \frac{k_y g_1 + \kappa_1\varepsilon_1}{\varepsilon_1^2 - g_1^2}, \quad \hat{E}_{y2} = \frac{k_y g_2 - \kappa_2\varepsilon_2}{\varepsilon_2^2 - g_2^2}. \tag{13}$$

Note that

$$\hat{E}_i = \frac{k_0|E_i(0)|}{|H_{z0}|}, \tag{14}$$

where $E_i(0)$ is the *i*-th component of electric field at the interface $x = 0$.

Now, the dispersion equation (9) can be presented in the form

$$D = \frac{1}{\hat{E}_{y1}} - \frac{1}{\hat{E}_{y2}} = 0. \tag{15}$$

Taking derivative from *D* on k_y, one obtains

$$\frac{\partial D}{\partial k_y} = -\frac{1}{\hat{E}_{y1}^2(\varepsilon_1^2 - g_1^2)}\left(g_1 + \varepsilon_1\frac{k_y}{\kappa_1}\right) - \frac{1}{\hat{E}_{y2}^2(\varepsilon_2^2 - g_2^2)}\left(g_2 - \varepsilon_2\frac{k_y}{\kappa_2}\right) = -\left(\frac{1}{\kappa_1}\frac{\hat{E}_{x1}}{\hat{E}_{y1}^2} + \frac{1}{\kappa_2}\frac{\hat{E}_{x2}}{\hat{E}_{y2}^2}\right).$$

That is exactly up to a sign the expression in parentheses in Eq. (12). Finally, one can write

$$S = -\frac{|E_{y0}|^2}{16\pi}\omega\frac{\partial D}{\partial k_y}. \tag{16}$$

The time-averaged energy density in media with time dispersion can be found from the formula [20]

$$w = \frac{1}{16\pi}\left(\frac{\partial}{\partial\omega}(\omega\epsilon_{ij})E_iE_j^* + \frac{\partial}{\partial\omega}(\omega\mu_{ij})H_iH_j^*\right). \quad (17)$$

Taking into account the form of the tensor of dielectric permittivity and the wave field components, it follows from Eq. (17) that for the waves considered here

$$w = \frac{1}{16\pi}\left(\frac{\partial\omega\varepsilon}{\partial\omega}(|E_x|^2 + |E_y|^2) + 2i\frac{\partial\omega g}{\partial\omega}E_xE_y^* + |H_z|^2\right),$$

where

$$\frac{\partial\omega\varepsilon}{\partial\omega} = 1 + \frac{\omega_p^2(\omega^2+\omega_c^2)}{(\omega^2-\omega_c^2)^2},$$

$$\frac{\partial\omega g}{\partial\omega} = -\frac{2\omega\omega_c\omega_p^2}{(\omega^2-\omega_c^2)^2}.$$

Using Eq. (14), the expressions for time-averaged energy densities per unit area for the first and second plasma areas can be presented in the following form

$$W_1 = \frac{W_0}{2\kappa_1 k_0}\left(k_0^2 + \frac{\partial\omega\varepsilon_1}{\partial\omega}(\hat{E}_{x1}^2 + \hat{E}_{y1}^2) - 2\frac{\partial\omega g_1}{\partial\omega}\hat{E}_{x1}\hat{E}_{y1}\right), \quad (18)$$

$$W_2 = \frac{W_0}{2\kappa_2 k_0}\left(k_0^2 + \frac{\partial\omega\varepsilon_2}{\partial\omega}(\hat{E}_{x2}^2 + \hat{E}_{y2}^2) - 2\frac{\partial\omega g_2}{\partial\omega}\hat{E}_{x2}\hat{E}_{y2}\right), \quad (19)$$

respectively. Here, $W_0 = |H_{z0}|^2/16\pi k_0$.

The total energy density of a surface wave ($W = W_1 + W_2$) can be expressed through the frequency derivative of dispersion relation. Unfortunately, the expression for frequency derivative of D in the presence of an external magnetic field is very complicated. Therefore, we consider here only the case of large values of the wave vector $|k_y| \gg k_0\sqrt{\varepsilon_V}$, $|k_y| \gg k_0$. For large k_y and $k_y > 0$, $\kappa_{1,2} \approx k_y$ and expressions (13) can be simplified:

$$\hat{E}_{x1} = \hat{E}_{y1} = \frac{|k_y|}{\varepsilon_1 - g_1}, \qquad \hat{E}_{x2} = -\hat{E}_{y2} = \frac{|k_y|}{\varepsilon_2 + g_2}.$$

Substituting the expressions for normalized field components into Eqs. (18) and (19), one obtains

$$W_1 = \frac{|H_{z0}|^2}{32\pi\kappa_1 k_0^2}\left(k_0^2 + 2\hat{E}_{y1}^2\left(\frac{\partial\omega\varepsilon_1}{\partial\omega} - \frac{\partial\omega g_1}{\partial\omega}\right)\right), \quad (20)$$

$$W_2 = \frac{|H_{z0}|^2}{32\pi\kappa_2 k_0^2}\left(k_0^2 + 2\hat{E}_{y2}^2\left(\frac{\partial\omega\varepsilon_2}{\partial\omega} + \frac{\partial\omega g_2}{\partial\omega}\right)\right). \quad (21)$$

Since $|k_y| \gg k_0$ and $\kappa_{1,2} \approx k_y$, it follows from Eqs. (20) and (21) that

$$W_1 = \frac{|E_{y0}|^2}{16\pi}\frac{\partial}{\partial\omega}\left(\omega\frac{\varepsilon_1 - g_1}{|k_y|}\right);$$

$$W_2 = \frac{|E_{y0}|^2}{16\pi}\frac{\partial}{\partial\omega}\left(\omega\frac{\varepsilon_2 + g_2}{|k_y|}\right).$$

In this case, dispersion relation (15) can be presented in the following form

$$D = \frac{\varepsilon_1 - g_1}{|k_y|} + \frac{\varepsilon_2 + g_2}{|k_y|} = 0.$$

From the expressions for D, W_1 and W_2, it follows that

$$W = W_1 + W_2 = \frac{|E_{y0}|^2}{16\pi}\frac{\partial}{\partial\omega}(\omega D). \quad (22)$$

Since $\partial_\omega(\omega D) = \omega\,\partial_\omega D$, dividing (16) by (22), one obtains the expression for velocity of SW energy propagation

$$v_{en} = \frac{S}{W} = -\frac{\frac{\partial D}{\partial k_y}}{\frac{\partial D}{\partial\omega}} = \frac{\partial\omega}{\partial k_y} = v_{gr}. \quad (23)$$

Thus, the SW energy propagates with the group velocity v_{gr}. For arbitrary values of k_y, this result can be obtained numerically if $\mathbf{H}_0 \neq 0$ and analytically when the external magnetic field is absent ($\mathbf{H}_0 = 0$).

ANALYSIS

Now, study the dependence of SW energetic characteristics (the energy flux density, the energy density and the energy propagation velocity) on the wave vector k_y and the external magnetic field. We specify frequency ranges where the surface waves may exist. In the presence of magnetic field, the surface waves propagating in positive direction, i.e. with $k_y > 0$, and the waves propagating in negative direction ($k_y < 0$) have different dispersion. Thus, we have to consider these two cases separately. We narrow down our study to the surface waves with frequencies larger than electron cyclotron frequency ($\omega > \omega_c$), that gives $g_1 > 0$ and $g_2 > 0$. Moreover, it is assumed that $\varepsilon_1 > 0$ and $\varepsilon_2 < 0$.

Waves propagating in positive direction

Let us start with the surface waves propagating in positive direction. In our previous work [18], we found that the lower frequency limit for the waves is determined by equation $\kappa_1 = 0$ if $\omega_{V1}^{+} < \omega_{V2}^{-}$, and $\kappa_2 = 0$ if $\omega_{V1}^{+} > \omega_{V2}^{-}$. Here, $\omega_{V1}^{+} = \frac{1}{2}\left(\sqrt{\omega_c^2 + 4\omega_{p1}^2} + \omega_c\right)$, $\omega_{V2}^{-} = \frac{1}{2}\left(\sqrt{\omega_c^2 + 4\omega_{p2}^2} - \omega_c\right)$ are the frequencies at which $\varepsilon_{V1} = 0$ and $\varepsilon_{V2} = 0$ correspondingly. First, consider the case when $\omega_{V1}^{+} < \omega_{V2}^{-}$, that imposes the following condition on the magnitudes of external magnetic field and plasma densities:

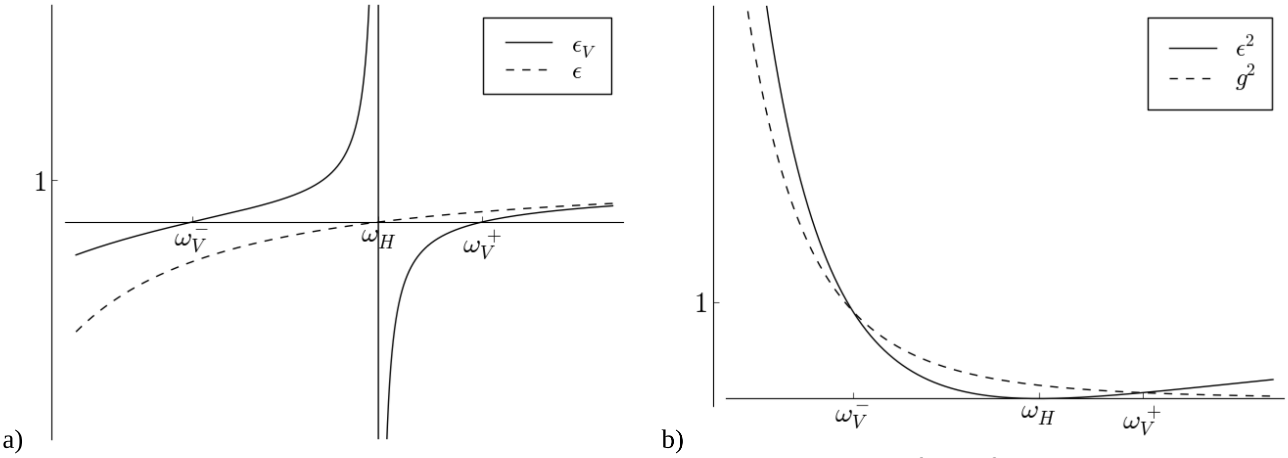
$$\omega_c < \frac{\omega_{p2}^2 - \omega_{p1}^2}{\sqrt{2(\omega_{p2}^2 + \omega_{p1}^2)}}.$$

In this case, the upper frequency limit ω_{inf} is determined by the following equation [18]

$$g_1 - \varepsilon_1 = g_2 + \varepsilon_2$$

The frequency corresponding to the low frequency limit ω_0 satisfies the following condition $\omega_{V1}^{+} \leq \omega_0$. By comparing the boundaries of the SW existence domain $[\omega_0, \omega_{inf}]$ with ω_{V1}^{+} and ω_{V2}^{-}, we obtained that $\omega_{V1}^{+} \leq \omega_0 < \omega_{inf} < \omega_{V2}^{-}$. In the SW existence domain,$\varepsilon_1 > 0$ because $\omega_{H1} < \omega_{V1}^{+}$ [see Fig.1(a)], thus, the both terms in the numerator of Eq.(6) are positive. And since $|\varepsilon_1| > |g_1|$ [see Fig. 1 (b)], the denominator is also positive. As a result, the energy flux in the first plasma layer is directed along $k_y (S_1 > 0)$, while the energy flux in the second layer is directed in opposite direction ($S_2 < 0$), because $\varepsilon_2 < 0$ and $|\varepsilon_1| > |g_1|$ (see Fig.1). Meantime, the total energy flux is directed along k_y (Eq. (11)) and, therefore, $|S_1| > |S_2|$.

If the wave frequency is close to $\omega_{V1}^{+} (\kappa_1 \to 0)$, the energy flux density and energy density in the low density plasma region are essentially larger than those in the dense plasma region (see Eqs. (11), (20) and (21)). In this case,



a) b)

Fig.1. (a)Typical dependences of ε and ε_V on thewave frequency. (b) Typical dependences of ε^2 and g^2 on the wave frequency.

$$\hat{E}_{x1} = \frac{k_y \varepsilon_1}{\varepsilon_1^2 - g_1^2}, \qquad \hat{E}_{y1} = \frac{k_y g_1}{\varepsilon_1^2 - g_1^2},$$

and from Eqs. (12) and (18) one obtains

$$S \approx S_1 \approx S_0 \frac{k_y}{\varepsilon_{V1} \kappa_1}; \tag{24}$$

$$W \approx W_1 \approx \frac{W_0}{2k_0 \kappa_1}\left(k_0^2 + \frac{k_y^2}{(\varepsilon_1^2 - g_1^2)^2}\left(\frac{\partial \omega \varepsilon_1}{\partial \omega}(\varepsilon_1^2 + g_1^2) - 2\frac{\partial \omega g_1}{\partial \omega}\varepsilon_1 g_1\right)\right). \tag{25}$$

At the lower frequency boundary, $S \to \infty$ and $W \to \infty$ [see Eqs. (24) and (25) and Fig.2], but the energy propogation velocity

$$v_{en} = \frac{S_1}{W_1} = \frac{2c}{\sqrt{\varepsilon_{V1}}}\left(2 + \frac{\omega}{\varepsilon_{V1}}\left(\frac{\partial \varepsilon_1}{\partial \omega} + \frac{g_1^2}{\varepsilon_1^2}\frac{\partial \varepsilon_1}{\partial \omega} - \frac{2g_1}{\varepsilon_1}\frac{\partial g_1}{\partial \omega}\right)\right)^{-1} \tag{26}$$

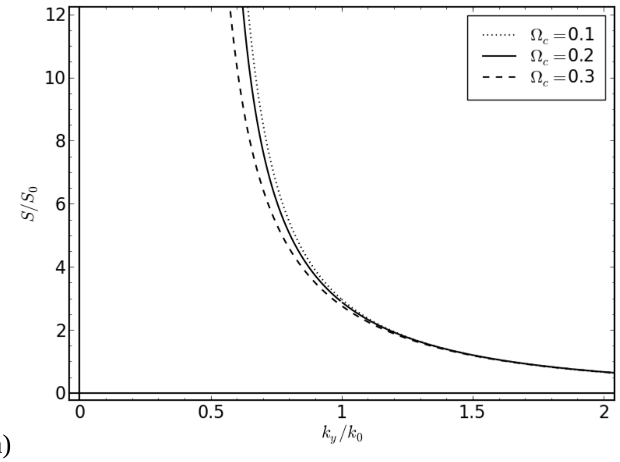
is finite [see Fig. 2(c)].

Taking derivative of ε_{V1} with respect to ω

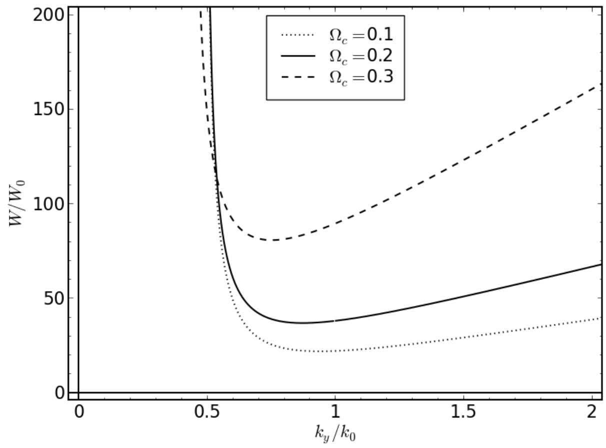
$$\frac{\partial \varepsilon_{V1}}{\partial \omega} = \frac{\partial \varepsilon_1}{\partial \omega} + \frac{g_1^2}{\varepsilon_1^2}\frac{\partial \varepsilon_1}{\partial \omega} - \frac{2g_1}{\varepsilon_1}\frac{\partial g_1}{\partial \omega},$$

we can write Eq. (26) in the following form

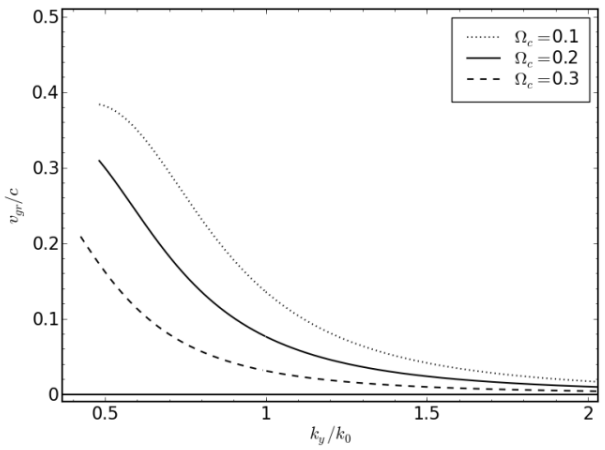
$$v_{en} = c\left(\sqrt{\varepsilon_{V1}} + \frac{\omega}{2\sqrt{\varepsilon_{V1}}}\frac{\partial \varepsilon_{V1}}{\partial \omega}\right)^{-1}. \tag{27}$$



a)



b)



c)

Fig.2. The dependences of normalized energy flux density (a), normalized wave energy (b) and group velocity (c) on the normalized wave vector for $k_y > 0$.

Taking into account that $v_{en} = v_{gr}$, one could get Eq. (27) by differentiating the dispersion equation, having in this limiting case the form

$$D = k_y - \frac{\omega}{c}\sqrt{\varepsilon_{V1}} = 0.$$

Using this expression for *D*, one obtains the derivatives:

$$\frac{\partial D}{\partial k_y} = 1, \quad \frac{\partial D}{\partial \omega} = -\frac{1}{c}\left(\sqrt{\varepsilon_{V1}} + \frac{\omega}{2\sqrt{\varepsilon_{V1}}}\frac{\partial \varepsilon_{V1}}{\partial \omega}\right).$$

Then, from Eq. (23) and the expressions for derivatives of *D*, one obtains Eq. (27).

At the upper boundary of the frequency domain that is determined by equation

$$g_1 - \varepsilon_1 = g_2 + \varepsilon_2,$$

the wave vector is large ($k_y \gg k_0$), and, therefore, $\kappa_1 \approx \kappa_2 \approx |k_y|$. Thus, the energy flow densities in the first and second plasma regions are

$$S_1 \approx \frac{S_0}{\varepsilon_1 - g_1}, \; S_2 \approx \frac{S_0}{\varepsilon_2 + g_2}, \text{ and } S_1 = -S_2.$$

Hence, the total energy flow density and energy propagation velocity are vanishing at the upper boundary (see Figs. 2 (a) and 2 (c)). At the boundary, the energy densities in both plasma regions are equal and are going to infinity proportionally to k_y(it follows from the expressions for W_1 and W_2 presented after Eq. (21)):

$$W_1 = \frac{W_0 k_y}{k_0(\varepsilon_1 - g_1)^2}\frac{\partial \omega(\varepsilon_1 - g_1)}{\partial \omega}, \quad (12)$$

$$W_2 = \frac{W_0 k_y}{k_0(\varepsilon_2 + g_2)^2}\frac{\partial \omega(\varepsilon_2 + g_2)}{\partial \omega}. \quad (13)$$

In Fig.2, energetic characteristics of the wave for different values of magnetic field are shown. One can see that with the increase of magnetic field the energy flux density decreases, while the energy density increases. As a result, the velocity of energy propagation decreases with growth of external magnetic field.

Waves propagating in negative direction

Consider surface waves propagating in negative direction. The frequency domain for these waves is bounded by the hybrid frequencies: $\omega_{H1} < \omega < \omega_{H2}$. We determine the direction of energy flow in each plasma region. Let us start from the energy flow in the first region S_1. We divide aforementioned interval in two parts: $\omega_{H1} < \omega < \omega_{V1}^{+}$ and

$\omega > \omega_{V1}^{+}$. For the first part $\varepsilon_1^2 < g_1^2$, thus the denominator of Eq.(6) is negative, and the terms in the numerator have opposite signs, because $\varepsilon_1 > 0$ and $k_y < 0$. Therefore, to determine the sign of the expression we need to compare these terms by magnitude. From inequalities $\varepsilon_1^2 < g_1^2$ and $\varepsilon_1 > 0$, it follows that the Voigt dielectric permittivity is negative: $\varepsilon_{V1} = (\varepsilon_1^2 - g_1^2)/\varepsilon_1 < 0$, and thus $\kappa_1 = \sqrt{k_y^2 - \varepsilon_{V1} k_0^2} > |k_y|$. Taking into account that $|\varepsilon_1| < |g_1|$, one comes to the conclusion that $|k_y \varepsilon_1| < |\kappa_1 g_1|$, i.e. the numerator of Eq.(6) is positive and the expression as whole is negative. For the frequencies ω larger than ω_{V1}^{+}, the following inequalities hold true $\varepsilon_1 > 0$, $\varepsilon_1^2 > g_1^2$, $\varepsilon_{V1} > 0$, meaning that $\kappa_1 < |k_y|$ and $|k_y \varepsilon_1| > |\kappa_1 g_1|$, thus the numerator of Eq. (6) is positive but the denominator is negative. As a result, the sign of Eq. (6) is also negative in the frequency range considered. Therefore, for $k_y < 0$ the energy flow in the first plasma region is directed along the wave vector ($S_1 < 0$).

Determine direction of the Poynting vector in the second plasma region. Since $\omega_c < \omega < \omega_{H2}$, the dielectric permittivity ε_2 is negative and $g_2 > 0$, and for $k_y < 0$ we have $k_y \varepsilon_2 > 0$, meaning that the terms in the numerator of Eq. (7) have different signs. Depending on the magnitude of wave vector k_y, the difference $k_y \varepsilon_2 - \kappa_2 g_2$ can be positive or negative. The difference is zero if

$$k_y = k_{S0} = -\frac{g_2}{\sqrt{-\varepsilon_2}} k_0.$$

The difference $k_y \varepsilon_2 - \kappa_2 g_2$ is negative for $|k_y| < |k_{S0}|$, and it is positive if $|k_y| > |k_{S0}|$.
Substituting $k_y = k_{S0}$ into dispersion equation, we find the corresponding zero frequency

$$\omega_{S0}^2 = \frac{1}{2}\left(\omega_{H1}^2 + \sqrt{\omega_{H1}^4 + 4\omega_c^2(\omega_{p2}^2 - \omega_{p1}^2)}\right) > \omega_{H1}^2.$$

Note that the total energy flow density is always directed along the wave vector (see Eq.(11)). Near the lower frequency limit ($\omega = \omega_{H1}$), where $\varepsilon_1 \to 0$ and consequently $\kappa_1 \to \infty$, the expression for total energy flow density may be simplified (see Eq.(11)):

$$S = \frac{S_0 k_0^2}{k_y \kappa_2}.$$

As a result, at $\omega = \omega_{H1}$ the total energy flow density is finite [see the region of small wave vectors in Fig. 3 (a)]. Meanwhile, the total energy density goes to infinity (see Fig. 3(b)). This conclusion follows from Eq. (18) if one takes $\kappa_1 \to \infty$:

$$W \approx W_1 \approx \frac{W_0 \kappa_1}{2k_0}\left(\frac{\varepsilon_1^2 + g_1^2}{(\varepsilon_1^2 - g_1^2)^2}\frac{\partial \omega \varepsilon_1}{\partial \omega} + \frac{2\varepsilon_1 g_1}{(\varepsilon_1^2 - g_1^2)^2}\frac{\partial \omega g_1}{\partial \omega}\right).$$

Taking into account that at low frequencies $\kappa_1 \approx k_0 g_1/\sqrt{\varepsilon_1}$ (because $\varepsilon_1 \to 0$), one concludes that

$$W \approx \frac{W_0}{2g_1\sqrt{\varepsilon_1}}\frac{\partial \omega \varepsilon_1}{\partial \omega}.$$

Since the total energy density is finite and $W \to \infty$, one obtains that the energy propagation velocity goes to zero at the low frequency limit [Fig. 3(c)].

At the upper frequency limit determined by equation

$$g_1 + \varepsilon_1 = g_2 - \varepsilon_2,$$

the energy flows in the first and the second plasma regions are equal on absolute magnitude but have different directions:

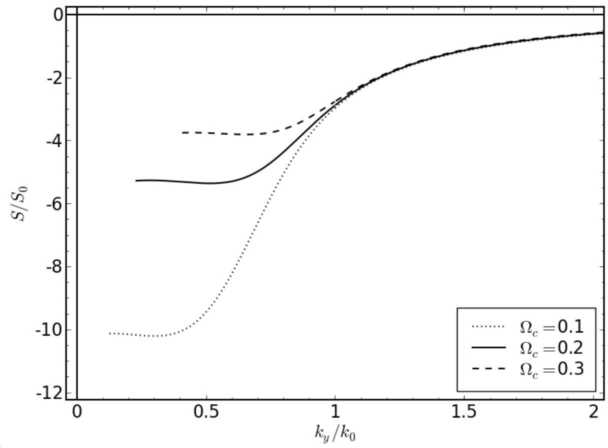
$$S_1 \approx -\frac{S_0}{\varepsilon_1 + g_1}, \qquad S_2 \approx \frac{S_0}{g_2 - \varepsilon_2}.$$

Thus, the total energy flux goes to zero for $k_y^2 \gg k_0^2$ (see Fig. 3(a)). If $k_y \to \infty$, the total energy density goes to infinity:

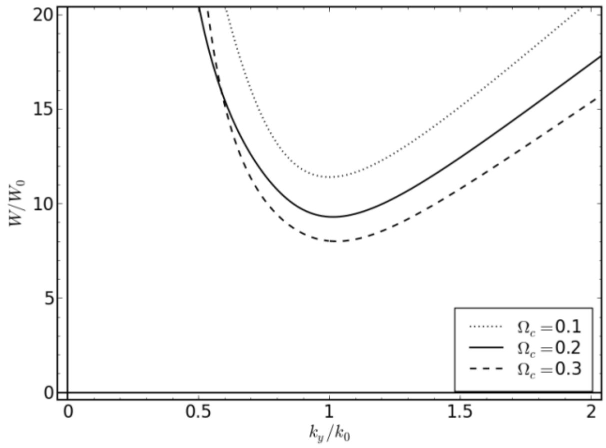
$$W_1 = \frac{W_0 k_y}{k_0(\varepsilon_1 + g_1)^2}\frac{\partial \omega(\varepsilon_1 + g_1)}{\partial \omega}, \quad W_2 = \frac{W_0 k_y}{k_0(\varepsilon_2 - g_2)^2}\frac{\partial \omega(\varepsilon_2 - g_2)}{\partial \omega}.$$

Note that the expressions for W_1 and W_2 are similar to Eqs. (28) and (29), describing the energy densities when the waves propagate in positive direction.

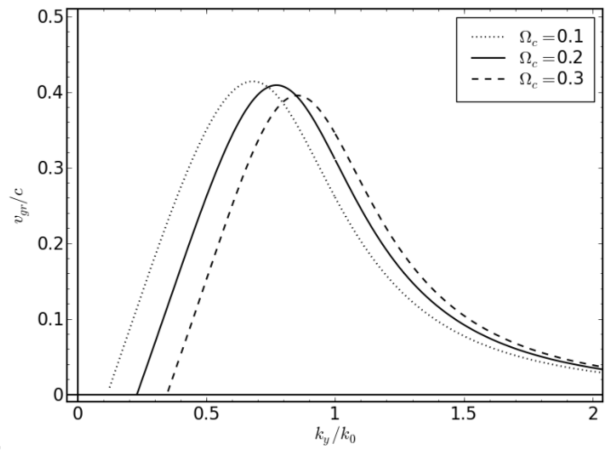
With the growth of magnetic field the energy flux density decreases by magnitude and the energy also decreases (see Fig.3). For higher values of magnetic field the maximum of group velocity decreases and shifts to the larger magnitudes of the wave vector.



a)



b)



c)

Fig.3. The dependences of normalized energy flux density (a), normalized wave energy (b) and group velocity (c) on the normalized wave vector for $k_y < 0$.

CONCLUSIONS

We have studied the energetic characteristics of the surface waves propagating along the plane plasma-plasma interface immersed in an external magnetic field. The expressions for time-averaged energy flow density and time averaged energy density of the waves in each plasma region have been obtained. We have analyzed the dependency of these characteristics on the wave vector magnitude and direction for different values of magnetic field.

The direction of Poynting vector has been investigated. For the waves with positive k_y, the energy fluxes in the first and the second plasma regions are always opposite in direction. Meanwhile, for the waves with negative wave number, the energy flux in the second plasma region may be directed along the flux in the first region, as well as in opposite direction. The wave vector and corresponding wave frequency for the case when the flux in the second region is absent have been found. Thus, for a certain range of wave vectors, the fluxes in both plasma regions are in same direction, what is impossible in the absence of an external magnetic field. Meantime, the total time-averaged energy flux of the waves is always directed along the wave vector.

The expression for the velocity of wave energy propagation has been obtained. We have showed that the velocity of SW energy propagation is equal to the group velocity of the wave. We have studied the dependency of group velocity on the wave vector and magnitude of external magnetic field. It has been found that the SW energy flux and energy velocity are vanishing when the wave vector is large.

At the lower frequency limit, the energy propagation velocity depends on direction of wave vector. If the wave vector is directed in negative direction, the group velocity goes to zero, while for positive k_y it reaches a finite value. With an increase of the external magnetic field the maximum of the group velocity decreases.

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