UDC 539

STRANGE QUARK MATTER IN A STRONG MAGNETIC FIELD

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Received 12 October 2012, accepted 14 January 2013

Thermodynamic properties of strange quark matter are considered in strong magnetic fields up to 10^{20} G. It is shown that the appearance of the longitudinal (along the magnetic field) instability beyond some critical magnetic field precludes the formation of fully polarized quark states in strange quark matter as well as prevents a significant drop of strangeness which, otherwise, could happen in such ultrastrong magnetic fields.
KEY WORDS: strange quark matter, strong magnetic field, pressure anisotropy, longitudinal instability

ДИВНА КВАРКОВА МАТЕРІЯ В СИЛЬНОМУ МАГНІТНОМУ ПОЛІ

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Розглядаються термодинамічні властивості дивної кваркової матерії в сильних магнітних полях до 10^{20} Гс. Показане, що поява повздовжньої (вздовж магнітного поля) нестійкості в полях більш за критичного перешкоджає виникненню повністю поляризованих кваркових станів у дивної кваркової матерії, а також запобігає значному зменшенню дивини, які, інакше, могли б відбутися за такими сильними магнітними полями.
КЛЮЧОВІ СЛОВА: дивна кваркова матерія, сильне магнітне поле, анізотропія тиску, повздовжня нестійкість

СТРАННАЯ КВАРКОВАЯ МАТЕРИЯ В СИЛЬНОМ МАГНИТНОМ ПОЛЕ

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Рассматриваются термодинамические свойства странной кварковой материи в сильных магнитных полях до 10^{20} Гс. Показано, что появление продольной (вдоль магнитного поля) неустойчивости в полях больше критического препятствует возникновению полностью поляризованных кварковых состояний в странной кварковой материи, а также предотвращает значительное уменьшение странности, которые, иначе, могли бы произойти в таких сильных магнитных полях.
КЛЮЧЕВЫЕ СЛОВА: странная кварковая материя, сильное магнитное поле, анизотропия давления, продольная неустойчивость

The study of Quantum Chromodynamics (QCD) phase diagram under extreme conditions of temperature and/or density is currently a hot research topic. In particular, when the baryon density is essentially larger than the nuclear saturation density (equal approximately to 0.16 fm^{-3}), quarks are expected to be liberated from the nucleon bags. It was suggested [1-3] that strange quark matter (SQM), composed of deconfined u, d and s quarks, can be the true ground state of matter. If this conjecture will be confirmed, it would have important astrophysical implications. In particular, SQM can form strange quark stars self-bound by strong interactions [4]. Also, if SQM is metastable at zero pressure, it can appear in the high-density core of a neutron star as a result of the deconfinement phase transition. In this case, the stability of SQM is provided by the gravitational pressure from the outer hadronic layers. Then a relevant astrophysical object is a hybrid star having a quark core and the crust of hadronic matter.

Another important aspect related to the physics of compact stars is that they are endowed with the magnetic field. For magnetars, the field strength at the surface can reach the values of about 10^{14}-10^{15} G. In the interior of a magnetar the magnetic field strength can reach even larger values of about 10^{20} G [5]. In such ultrastrong magnetic fields, the effects of the $O(3)$ rotational symmetry breaking by the magnetic field become important [5-8]. In particular, the longitudinal (along the magnetic field) pressure is less than the transverse pressure resulting in the appearance of the longitudinal instability of the star's matter if the magnetic field exceeds some critical value. The effects of the pressure anisotropy should be accounted for in the consistent investigation of structural and polarization properties of a strongly magnetized stellar object. The aim of this research is to study the effects of the pressure anisotropy in SQM under the presence of a strong magnetic field within the framework of the Massachusetts Institute of Technology (MIT) bag model.

BASIC EQUATIONS

In the simplest version of the MIT bag model, quarks are considered as free fermions moving inside a finite region of space called a "bag". The effects of the confinement are accomplished by endowing the finite region with a constant energy per unit volume, the bag constant B. The energy spectrum of free relativistic fermions (u, d, s quarks and electrons) in an external magnetic field has the form

$$\varepsilon_\nu^i = \sqrt{k_z^2 + m_i^2 + 2\nu\,|q_i|\,H}, \quad \nu = n + \frac{1}{2} - \frac{s}{2}\mathrm{sgn}(q_i), \quad i = u,d,s,e,$$

where $\nu = 0,1,2...$ enumerates the Landau levels, n is the principal quantum number, $s = +1$ corresponds to a fermion with spin up, and $s = -1$ to a fermion with spin down. The lowest Landau level with $\nu = 0$ is single degenerate and other levels with $\nu > 0$ are double degenerate.

Further we will consider magnetized SQM at zero temperature. In the zero temperature case, the thermodynamic potential for an ideal gas of relativistic fermions of i th species in the external magnetic field reads [9]

$$\Omega_i = -\frac{|q_i|\,g_i H}{4\pi^2}\sum_{\nu=0}^{\nu_{\max}^i}(2-\delta_{\nu,0})\left\{\mu_i k_{F,\nu}^i - \bar{m}_{i,\nu}^2 \ln\left|\frac{k_{F,\nu}^i + \mu_i}{\bar{m}_{i,\nu}}\right|\right\}, \tag{1}$$

where the factor $(2-\delta_{\nu,0})$ takes into account the spin degeneracy of Landau levels, g_i is the remaining degeneracy factor [$g_f = 3$ for quarks (number of colors), and $g_e = 1$ for electrons], μ_i is the chemical potential, and

$$\bar{m}_{i,\nu} = \sqrt{m_i^2 + 2\nu\,|q_i|\,H}, \quad k_{F,\nu}^i = \sqrt{\mu_i^2 - \bar{m}_{i,\nu}^2}\,.$$

In Eq. (1), summation runs up to $\nu_{\max}^i = I\left[\dfrac{\mu_i^2 - m_i^2}{2\,|q_i|\,H}\right]$, *I*[...] being an integer part of the value in the brackets. The number density $\varrho_i = -\left(\dfrac{\partial\Omega_i}{\partial\mu_i}\right)_T$ of fermions of i th species is given by

$$\rho_i = \frac{|q_i|\,g_i H}{2\pi^2}\sum_{\nu=0}^{\nu_{\max}^i}(2-\delta_{\nu,0})k_{F,\nu}^i\,. \tag{2}$$

The sum in Eq. (2) can be split into two parts representing the fermion number densities with spin up and spin down. As explained earlier, the only difference between the two sums is in the term with $\nu = 0$, corresponding to spin-up fermions if they are positively charged, and to spin-down fermions, if they are negatively charged. Then the zero temperature expression for the spin polarization parameter of the i th species subsystem reads:

$$\Pi_i \equiv \frac{\varrho_i^{\uparrow} - \varrho_i^{\downarrow}}{\varrho_i} = \frac{q_i g_i H}{2\pi^2 \varrho_i}\sqrt{\mu_i^2 - m_i^2}\,. \tag{3}$$

In a strong enough magnetic field, when only a lowest Landau level is occupied by fermions of i th species, a full polarization occurs with $|\Pi_i| = 1$.

In order to find the chemical potentials of all fermion species, we will use the following constraints:

$$\frac{1}{3}(\rho_u + \rho_d + \rho_s) = \rho_B\,, \tag{4}$$

$$2\rho_u - \rho_d - \rho_s - 3\rho_{e^-} = 0\,, \tag{5}$$

$$\mu_d = \mu_u + \mu_{e^-}\,, \tag{6}$$

$$\mu_d = \mu_s\,, \tag{7}$$

being the conditions of the total baryon number conservation, Eq. (4) (ϱ_B is the total baryon number density), charge neutrality, Eq. (5), and chemical equilibrium, Eqs. (6), (7), with respect to the weak processes

$$d \to u + e^- + \bar{\nu}_e, \quad u + e^- \to d + \nu_e, \tag{8}$$

$$s \to u + e^- + \bar{\nu}_e, \quad u + e^- \to s + \nu_e, \tag{9}$$

$$s + u \leftrightarrow d + u, \tag{10}$$

occurring in the quark core of a neutron star [4].

At zero temperature, the energy density $E_i = \Omega_i + \mu_i \varrho_i$ for fermions of i th species reads

$$E_i = \frac{|q_i| g_i H}{4\pi^2} \sum_{\nu=0}^{\nu_{\max}^i} (2 - \delta_{\nu,0}) \left\{ \mu_i k_{F,\nu}^i + \bar{m}_{i,\nu}^2 \ln \left| \frac{k_{F,\nu}^i + \mu_i}{\bar{m}_{i,\nu}} \right| \right\}. \tag{11}$$

In the MIT bag model, the total energy density E, longitudinal p_l and transverse p_t pressures in quark matter are given by [5]

$$E = \sum_i E_i + \frac{H^2}{8\pi} + B, \tag{12}$$

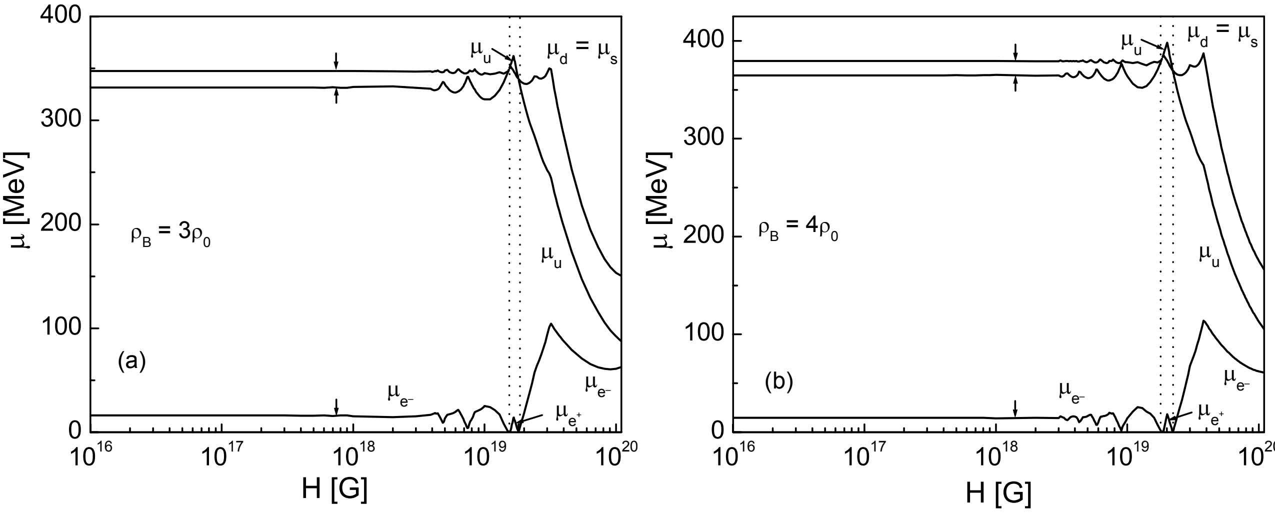
$$p_l = -\sum_i \Omega_i - \frac{H^2}{8\pi} - B, \quad p_t = -\sum_i \Omega_i - HM + \frac{H^2}{8\pi} - B, \tag{13}$$

where B is the bag constant, and $M = \sum_i M_i = -\sum_i \left(\frac{\partial \Omega_i}{\partial H} \right)_{\mu_i}$ is the total magnetization. It is seen that the magnetic field strength enters differently to the longitudinal and transverse pressures that reflects the breaking of the $O(3)$ rotational symmetry in a magnetic field. In a strong enough magnetic field, the quadratic on the magnetic field strength term (the Maxwell term) will be dominating, leading to increasing the transverse pressure and to decreasing the longitudinal pressure. Hence, there exists a critical magnetic field H_c, at which the longitudinal pressure vanishes, resulting in the longitudinal instability of SQM. In the astrophysical context, this means that in the magnetic fields $H \geqslant H_c$ a neutron star with the quark core will be subject to the gravitational collapse along the magnetic field.

NUMERICAL RESULTS AND CONCLUSIONS

As was mentioned in Introduction, SQM can be in absolutely stable state (strange quark stars), or in metastable state, which can be stabilized by high enough external pressure (hybrid stars). Note that the analysis of the absolute stability window in the parameter space for magnetized superconducting color-flavor-locked strange matter [10] shows that the maximum allowed bag pressure decreases with the magnetic field strength (see Eq. (32) of that work). The same holds true for magnetized nonsuperconducting SQM because the arguments of Ref. [10] can be reiterated in the given case with the only change that in Eq. (32) of Ref. [10] one should use the potential $\Omega = \sum_i \Omega_i$ with Ω_i given by Eq. (1) of the present study. In numerical calculations, we adopt two values of the bag constant, $B = 100$ MeV/fm^3 and $B = 120$ MeV/fm^3, which are slightly larger than the upper bound $B \simeq 90$ MeV/fm^3 from the absolute stability window at zero magnetic field strength [3]. The core densities corresponding to these bag pressures are chosen equal to $\varrho_B = 3\varrho_0$ and $\varrho_B = 4\varrho_0$, respectively, which are, in principle, sufficient to produce deconfinement ($\varrho_0 = 0.16$ fm^{-3} being the nuclear saturation density). Therefore, in the astrophysical context, we assume a scenario in which SQM can be formed in the core of a strongly magnetized neutron star. For the quark masses, we use the values $m_u = m_d = 5$ MeV, and $m_s = 150$ MeV [9]. The value of the strange quark mass m_s is an important issue because it substantially affects the SQM equation of state [11]. Here we do not study the impact of varying strange quark mass m_s on the critical magnetic field H_c. This can be done analogously to that in Ref. [3] where such an impact on the energy per baryon of nonmagnetized SQM was investigated.

Fig. 1 shows the chemical potentials of all fermion species as functions of the magnetic field strength. It is seen that the chemical potentials of fermions, first, stay practically constant under increasing the magnetic field, with d and s quark chemical potentials being somewhat larger (on the value of $\mu_{e^-} \sim 14 - 16$ MeV) than the u quark chemical potential. The apparent Landau oscillations of the chemical potentials appear beginning from $H \sim 3{\cdot}10^{18} - 4{\cdot}10^{18}$ G, depending on the total baryon number density. At $H \gtrsim 4{\cdot}10^{19}$ G, the quark chemical potentials decrease with the magnetic field. An interesting peculiarity occurs in a narrow interval near $H \sim 2{\cdot}10^{19}$ G, marked by the vertical dotted lines. Namely, for magnetic field strengths from that interval the u quark chemical potential is larger than the d and s quark chemical potentials, $\mu_u > \mu_d = \mu_s$.



a) b)

Fig. 1. Various fermion species chemical potentials as functions of the magnetic field strength at zero temperature for the total baryon number density a) $\varrho_B = 3\varrho_0$ and b) $\varrho_B = 4\varrho_0$.

The chemical potentials of quarks and positrons are shown by the curves between the vertical dotted lines. The vertical arrows indicate the points corresponding to the critical field H_c ; see further details in the text.

Hence, for such magnetic fields, according to Eq. (6), the electron chemical potential would be negative, $\mu_{e^-} < 0$. If to recall the finite temperature expression for the electron number density [9]:

$$\varrho_{e^-} = \frac{|q_e|H}{2\pi^2}\sum_{\nu=0}^{\infty}(2-\delta_{\nu,0})\int_0^{\infty} dk_z \left(\frac{1}{e^{\beta(\varepsilon_\nu^e-\mu_{e^-})}+1} - \frac{1}{e^{\beta(\varepsilon_\nu^e+\mu_{e^-})}+1}\right),$$

its zero temperature limit at $\mu_{e^-} < 0$ is, formally, negative, contrary to the constraint $\varrho_{e^-} \geqslant 0$. In fact, this means that in this interval on H electrons are missing and, hence, the weak β^- processes (8), (9) are impossible. However, for such magnetic fields, the following weak β^+ processes become allowable

$$u \to d + e^+ + \nu_e, \quad d + e^+ \to u + \bar{\nu}_e,$$
$$u \to s + e^+ + \nu_e, \quad s + e^+ \to u + \bar{\nu}_e.$$

Hence, for this specific range of the magnetic field strengths, the charge neutrality and chemical equilibrium conditions should read

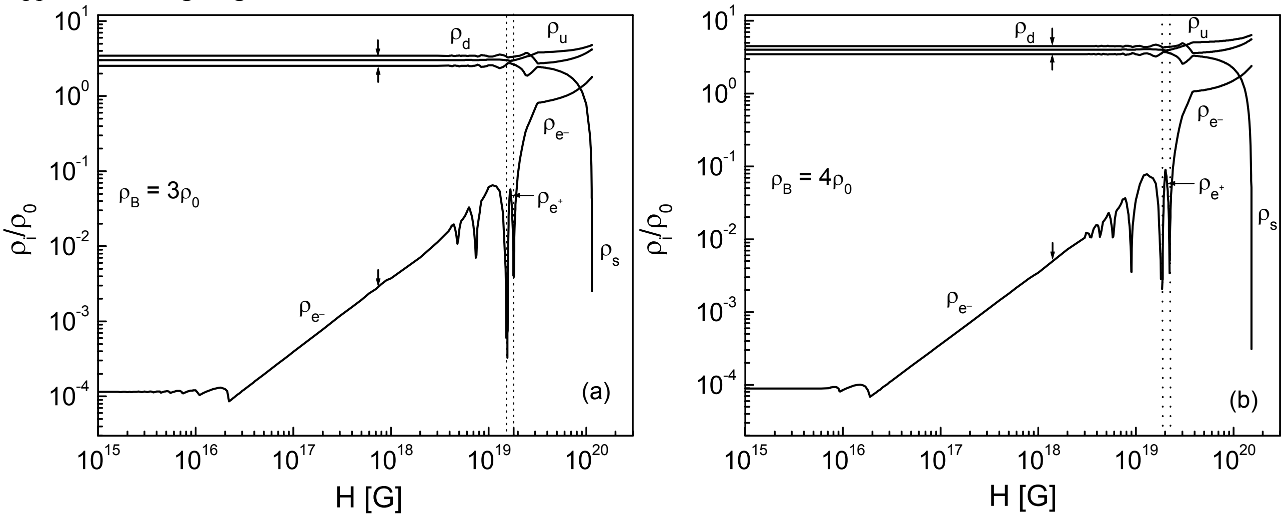
$$2\varrho_u - \varrho_d - \varrho_s + 3\varrho_{e^+} = 0, \tag{14}$$

$$\mu_u = \mu_d + \mu_{e^+}, \quad \mu_d = \mu_s, \tag{15}$$

which should be solved jointly with the condition of the total baryon number conservation, Eq. (4). The quark and positron chemical potentials obtained as solutions of these equations are shown graphically in Fig. 1 as the corresponding curves between the vertical dotted lines. With increasing the core density, the width of the interval on H , where positrons appear, increases slightly as well (cf. the ranges $1.56\cdot10^{19}$ G-$1.80\cdot10^{19}$ G at $\varrho_B = 3\varrho_0$ and $1.86\cdot10^{19}$ - $2.21\cdot10^{19}$ G at $\varrho_B = 4\varrho_0$). Thus, as a matter of principle, in strongly magnetized strange quark matter at zero temperature, subject to the total baryon number conservation, charge neutrality and chemical equilibrium conditions, positrons can appear in a certain narrow interval of the magnetic field strengths, replacing electrons. In this case, strange quark matter will have negative hadronic electric charge.

Note that, according to Ref. [3], the contact of stable strange quark matter, having negative hadronic electric charge, with the ordinary matter would have the disastrous consequences for the latter, because positively charged nuclei would be attracted to strange quark matter and absorbed. However, the contact of metastable strange quark matter, having negative hadronic electric charge, with hadronic matter in the interior of a neutron star is possible, because the outer hadronic layer provides the necessary external pressure to stabilize strange quark matter in the core and cannot be completely depleted. Nevertheless, we should calculate the critical field H_c for the appearance of the longitudinal instability, which could prevent the occurrence of positrons in a certain range of magnetic field strengths with $H \gtrsim 10^{19}$ G. The meaning of the vertical arrows in Fig. 1 will be discussed later in the text.

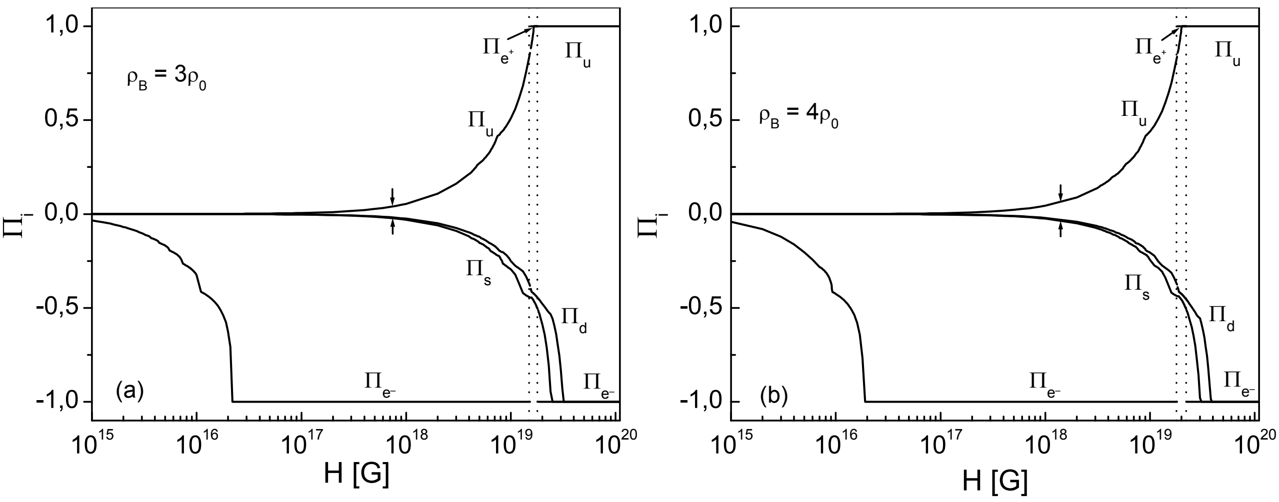
Fig. 2 shows the abundances of various fermion species as functions of the magnetic field strength. The number densities of u and d quarks are quite close to each other for all magnetic fields under consideration. The electron number density begins quite rapidly to increase at $H \approx 2.2{\cdot}10^{16}$ G for $\varrho_B = 3\varrho_0$ and at $H \approx 1.9{\cdot}10^{16}$ G for $\varrho_B = 4\varrho_0$. As noted earlier, in the narrow interval near $H \sim 2{\cdot}10^{19}$ G electrons are replaced by positrons, and beyond this interval electrons appear again with the number density increasing with H. The s quark content of strange quark matter stays practically constant till the field strength $H \approx 4.1{\cdot}10^{18}$ G at $\varrho_B = 3\varrho_0$ and $H \approx 3.8{\cdot}10^{18}$ G at $\varrho_B = 4\varrho_0$, beyond which the s quark number density experiences visible Landau oscillations. Then, beginning from the field strength $H \approx 3.2{\cdot}10^{19}$ G at $\varrho_B = 3\varrho_0$ and $H \approx 3.9{\cdot}10^{19}$ G at $\varrho_B = 4\varrho_0$, the s quark content rapidly decreases. Strange quark matter loses its strangeness and turns into two-flavor quark matter in the magnetic fields slightly larger than 10^{20} G. Again, we should determine the critical field H_c in order to check whether this significant drop of strangeness could happen in a strong magnetic field.



a) b)

Fig. 2. Same as in Fig. 1 but for the particle number densities ϱ_i / ϱ_0 of various fermion species.
a) $\varrho_B = 3\varrho_0$ and b) $\varrho_B = 4\varrho_0$.

Fig. 3 shows the spin polarization parameter Π_i for various fermion species, determined according to Eq. (3), as a function of the magnetic field strength. Spin polarization of u quarks is positive while for d,s quarks and electrons it is negative. The magnitude of the spin polarization parameter Π_i increases with H till it is saturated at the respective saturation field H_s^i. At $H = H_s^i$, the corresponding ith fermion species becomes fully spin polarized. The respective values of the saturation field are: $H_s^e \approx 2.2{\cdot}10^{16}$ G for electrons, $H_s^u \approx 1.7{\cdot}10^{19}$ G for u quarks, $H_s^s \approx 2.5{\cdot}10^{19}$ G for s quarks and $H_s^d \approx 3.2{\cdot}10^{19}$ G for d quarks at $\varrho_B = 3\varrho_0$, and $H_s^e \approx 1.9{\cdot}10^{16}$ G for electrons, $H_s^u \approx 2.0{\cdot}10^{19}$ G for u quarks, $H_s^s \approx 3.1{\cdot}10^{19}$ G for s quarks and $H_s^d \approx 3.9{\cdot}10^{19}$ G for d quarks at $\varrho_B = 4\varrho_0$. Note that quite a rapid increase of the electron number density with the magnetic field (cf. Fig. 2) begins just at the saturation field H_s^e, and, hence, this increase occurs when electrons become completely spin polarized. Further oscillations in the electron number density are, in fact, caused by the Landau oscillations of the quark number densities, which influence the electron population through the charge neutrality condition. Although the s-quark current mass is larger than that for d quark, $m_s > m_d$, s quarks become fully polarized at a smaller saturation field because their particle density is smaller than for d quarks, $\varrho_s < \varrho_d$. The spin polarization parameter of various fermion species in the magnetic field range, where positrons appear, is shown by the respective curves between the vertical dotted lines. It is seen that positrons occur already fully polarized, and u quarks become totally polarized just in this range of the magnetic field strengths. Nevertheless, as mentioned before, only after determining the critical field H_c for the appearance of the longitudinal instability, it would be possible to determine the degree of spin polarization which could be reached for each of the fermion species.



a) b)

Fig. 3. Same as in Fig. 1 but for the spin polarization parameter of various fermion species.
a) $\varrho_B = 3\varrho_0$ and b) $\varrho_B = 4\varrho_0$.

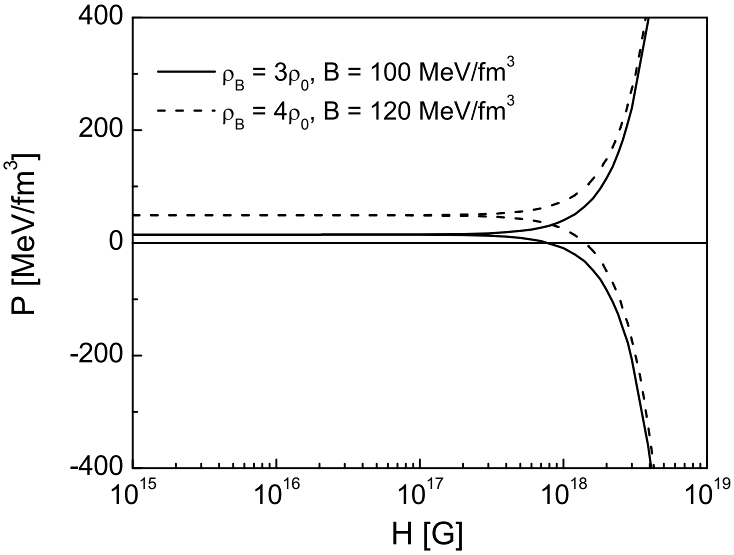


Fig. 4. Transverse (ascending branches) and longitudinal (descending branches) pressures in magnetized SQM at zero temperature as functions of the magnetic field strength for $\varrho_B = 3\varrho_0, B = 100$ MeV/fm^3 (solid lines) and $\varrho_B = 4\varrho_0, B = 120$ MeV/fm^3 (dashed lines).

Now we present the results of calculations of the longitudinal p_l and transverse p_t pressures. Fig. 4 shows the longitudinal p_l and transverse p_t pressures at zero temperature as functions of the magnetic field strength. It is seen that, first, the transverse and longitudinal pressures stay practically constant and indistinguishable from each other. This behavior of the pressures p_t and p_l corresponds to the isotropic regime. Beyond some threshold magnetic field H_{th}, the transverse pressure p_t increases with H while the longitudinal pressure p_l decreases with it, clearly reflecting the anisotropic nature of the total pressure in SQM in such strong magnetic fields (anisotropic regime). In the critical magnetic field H_c, the longitudinal pressure p_l vanishes. This happens at $H_c \approx 7.4{\cdot}10^{17}$ G for $\varrho_B = 3\varrho_0, B = 100$ MeV/fm^3, and at $H_c \approx 1.4{\cdot}10^{18}$ G for $\varrho_B = 4\varrho_0, B = 120$ MeV/fm^3. Above the critical magnetic field, the longitudinal pressure is negative leading to the longitudinal instability of SQM. Therefore, the thermodynamic properties of SQM should be considered in the magnetic fields $H < H_c$.

Now, in order to see, which of the discussed already features of strange quark matter at zero temperature in a strong magnetic field are preserved before the appearance of the longitudinal instability, we show in Figs. 1-3 by the vertical arrows the respective values of the physical quantities corresponding to the critical field H_c. Let us begin with Fig. 1 for the chemical potentials of various fermion species. It is seen that the chemical potentials of quarks and electrons stay practically unchanged before the appearance of the longitudinal instability. The significant changes in the chemical potentials occur only in the fields $H > H_c$. In particular, the longitudinal instability precludes the appearance of positrons for which the fields $H \gtrsim 10^{19}$ G are necessary.

Let us turn to Fig. 2 for the abundances of various fermion species. Till the critical field H_c, the content of quark species stays practically constant while the electron fraction remains quite small, $\varrho_{e^-} / \varrho_0 \lesssim 10^{-2}$. Also, there is no room for the significant drop of the strange quark content in a strong magnetic field which occurs in the fields $H \sim 10^{20}$ G. In fact, despite the presence of strong magnetic fields $H \sim 10^{18}$ G, strange quark matter has the same fraction of s quarks as in the field-free case.

Let us now consider Fig. 3 for spin polarizations of various fermion species. It is seen that the full polarization in a strong magnetic field can be achieved only for electrons. For various quark species, the spin polarization remains quite moderate up to the critical magnetic field H_c. E.g., at $\varrho_B = 4\varrho_0, H = H_c$ we have $\Pi_u \approx 0.06$, $\Pi_d \approx -0.03$, $\Pi_s \approx -0.04$; at $\varrho_B = 3\varrho_0, H = H_c$, the quark spin polarizations are similar to these values with the maximum magnitude of the spin polarization parameter for u quarks, $\Pi_u \approx 0.04$. Therefore, the occurrence of a field-induced fully polarized state in strange quark matter is prevented by the appearance of the longitudinal instability in the critical magnetic field.

In summary, we have considered the impact of strong magnetic fields up to 10^{20} G on the thermodynamic properties of strange quark matter at zero temperature under additional constraints of total baryon number conservation, charge neutrality and chemical equilibrium with respect to various weak processes occurring in the system. The study has been done within the framework of the MIT bag model with the finite current quark masses $m_u = m_d \neq m_s$. In the numerical calculations, we have adopted two sets of the total baryon number density and bag pressure, $\varrho_B = 3\varrho_0, B = 100$ MeV/fm^3 and $\varrho_B = 4\varrho_0, B = 120$ MeV/fm^3. It has been found that in strong magnetic fields up to 10^{20} G some interesting features in the chemical composition and spin structure of strange quark matter could occur:

(1) The content of strange quarks rapidly decreases in the fields somewhat larger than 10^{19} G and becomes negligible in the fields slightly exceeding 10^{20} G;

(2) For the magnetic field strengths in the quite narrow interval near $H \sim 2{\cdot}10^{19}$ G the constraints of total baryon number conservation, charge neutrality and chemical equilibrium can be satisfied only if positrons appear in various weak processes in that range of the field strengths (instead of electrons);

(3) Electrons occupy only the lowest Landau level and, hence, become completely spin polarized in the magnetic fields somewhat larger than 10^{16} G; u, s and d quarks become fully polarized in the fields somewhat larger than 10^{19} G (the recitation of the quark species is in the order in which they appear fully polarized under increasing H).

Nevertheless, under such strong magnetic fields, the total pressure containing also the magnetic field contribution, becomes anisotropic, and the effects of the pressure anisotropy change most of the above conclusions. Namely, the longitudinal (along the magnetic field) pressure decreases with the magnetic field (contrary to the transverse pressure increasing with H) and vanishes in the critical field H_c resulting in the longitudinal instability of strange quark matter. The value of the critical field H_c depends on the total baryon number density of strange quark matter and the bag pressure B, and it turns out to be somewhat less or larger than 10^{18} G for the two sets of the parameters, considered in the given study. Therefore, the appearance of the longitudinal instability in strong magnetic fields beyond the critical one precludes the features (1), (2) in the chemical composition of strongly magnetized strange quark matter. Concerning the conclusion (3), only electrons can reach the state of full polarization, that is not true for quarks of all flavors, whose polarization remains mild even for magnetic fields near H_c.

The obtained results can be important in the studies of structural and polarization properties of strongly magnetized neutron stars with quark cores. In particular, it is worth noting that, since the equation of state (EoS) of strange quark matter becomes highly anisotropic in an ultrastrong magnetic field, the usual scheme for finding the mass-radius relationship based on the Tolman-Oppenheimer-Volkoff (TOV) equations [12] for a spherically symmetric and static stellar object should be revised. Instead, the corresponding relationship should be found by the self-consistent treatment of the anisotropic EoS and axisymmetric TOV equations substituting the conventional TOV equations in the case of an axisymmetric neutron star with the quark core. The masses and radii of neutron stars are measurable quantities, and, hence, the relevance of the effects of the pressure anisotropy in a strong magnetic field can be directly tested for strongly magnetized compact stars.

The author would like to thank the Organizing Committee of the International Conference "Problems of Modern Physics", held in occasion of the 50th anniversary of the Physics and Technology Faculty at Kharkov National University, for kind invitation and possibility to present the results of this study at the meeting.

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