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HIGH-ENERGY WAVE PACKETS. 'HALF-BARE' ELECTRON

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The evolution in space and time of localized high-energy electromagnetic wave packets, which take place in processes of radiation by ultra relativistic electrons is considered. It is shown that high energies make stabilizing influence on the motion of such packets and that the lengths, within which their dispersion and reconstruction into the packets of diverging waves occurs, can be macroscopic. The electromagnetic field evolution in the process of ultra relativistic electron emission from substance into vacuum is considered. It is demonstrated, that in this case the electron can be in 'half-bare' state with considerably suppressed low frequency Fourier-components of the field around it during long period of time after the emission. It is shown that such state of electron can manifest itself in significant dependence of further ionization energy losses of the electron in thin plate situated in the direction of the particle motion on the distance between the plate and the scattering point.
KEY WORDS: electromagnetic wave packets, ionization energy losses, transition radiation, density effect, 'half-bare' electron

ВЫСОКОЭНЕРГЕТИЧЕСКИЕ ВОЛНОВЫЕ ПАКЕТЫ. «ПОЛУГОЛЫЙ» ЭЛЕКТРОН
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Рассмотрена пространственно-временная эволюция локализованных высокоэнергетических электромагнитных волновых пакетов, имеющих место в процессах излучения ультра релятивистскими электронами. Показано, что высокие энергии оказывают стабилизирующее влияние на движение таких пакетов и что длины, на которых происходит их расплывание и перестройка в пакеты расходящихся волн, могут иметь макроскопические размеры. Рассмотрена эволюция электромагнитного поля в пространстве при вылете ультра релятивистского электрона из вещества в вакуум. Показано, что в этом случае электрон в течение длительного промежутка времени после вылета из вещества может пребывать в «полуголом» состоянии с сильно подавленными низкочастотными компонентами Фурье в окружающем его поле. Также показано, что такое состояние электрона может проявляться в существенной зависимости его последующих ионизационных потерь энергии в тонкой пластинке, расположенной в направлении движения частицы, от расстояния между пластинкой и веществом.
КЛЮЧЕВЫЕ СЛОВА: электромагнитные волновые пакеты, ионизационные потери энергии, переходное излучение, эффект плотности, «полуголый» электрон

ВИСОКОЕНЕРГЕТИЧНІ ХВИЛЬОВІ ПАКЕТИ. «НАПІВГОЛИЙ» ЕЛЕКТРОН
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Розглянуто просторово-часову еволюцію локалізованих високоенергетичних електромагнітних хвильових пакетів, що мають місце у процесах випромінювання ультра релятивістськими електронами. Показано, що високі енергії мають стабілізуючий вплив на рух таких пакетів і що довжини, на яких відбувається їх розпливання і перебудова у пакети розбіжних хвиль, можуть мати макроскопічні розміри. Розглянуто еволюцію електромагнітного поля в просторі при вильоті ультра релятивістського електрона з речовини у вакуум. Показано, що в цьому випадку електрон протягом тривалого проміжку часу після вильоту з речовини може перебувати в «напівголому» стані з сильно заглушеними низькочастотними компонентами Фур'є в оточуючому полі. Також показано, що такий стан електрона може виявлятися в істотній залежності його подальших іонізаційних втрат енергії в тонкій пластинці, розташованій у напрямку руху частинки, від відстані між пластинкою і речовиною.
КЛЮЧОВІ СЛОВА: електромагнітні хвильові пакети, іонізаційні втрати енергії, перехідне випромінювання, ефект густини, «напівголий» електрон

A lot of high-energy physical processes develop within large domains of space along the direction of particle motion (see, for example monographs [1-4] and references in them). In the case of electromagnetic processes the size of these domains can substantially exceed sometimes not only interatomic distances of substance but the size of experimental facility (detectors) as well [1, 2, 4-13]. Essential in this case is the fact that interaction of particles with atoms and experimental facility situated within such domains and outside them can substantially differ. Such situation arises, for example, when considering long-wave radiation in processes of bremsstrahlung and transition radiation by ultra relativistic electrons [14-16]. Therefore, it is necessary to know what happens within such regions and what the peculiarities of evolution of such processes in space and time are. In the present paper we consider the evolution of transition radiation process during relativistic electron emission from dielectric substance into vacuum and manifestation of transformation of electromagnetic field around the particle in this process during further interaction of this electron with matter.

The considered problem is closely related to the problem of study of the behavior of localized high-energy wave packets. Therefore, firstly, we will consider some peculiarities of the behavior of such wave packets drawing special attention to the questions of their stability and reconstruction into the packets of diverging waves. Further we show that the discussed wave packets naturally arise in the process of relativistic electron emission from substance into vacuum.

The consideration is made on the basis of classical electrodynamics. In this case the moving electron is considered as a charge with its own Coulomb field moving together with it. During the electron traversal of the medium-vacuum interface the perturbation of this field occurs. This perturbation is treated here as appearance of a packet of free plane electromagnetic waves, which reconstructs then into a packet of diverging waves of transition radiation. For ultra relativistic particles, however, this does not happen at once. The distance from the interface, within which this process develops, has a name of the coherence length of the transition radiation process [1, 2]. It is $2\gamma^2$ times larger than the length λ of the considered radiated waves (γ is here the electron Lorentz-factor). We show that within this length the field around the electron in vacuum substantially differs from the Coulomb one and the particle exists in so called 'half-bare' state [17, 18] with suppressed low-frequency components of the field around it. It is possible to place thin dielectric plate within this distance from the substance in the direction of the electron motion and consider ionization energy losses of the particle in it. In the present paper it is shown that the electron energy losses in such plate significantly depend on the distance between the plate and the substance, from which the electron is emitted, and are defined by the magnitude of interference between the electron's own Coulomb field and the packet of free waves.

The aim of the paper is to investigate some peculiarities of space-time evolution of the field of high-energy electromagnetic wave packets and to consider the manifestation of such field evolution in the process of relativistic electron ionization energy losses.

HIGH-ENERGY WAVE PACKETS

The general solution of the wave equation can be presented in the form of a wave packet, which spatially disperses in course of time. In semiclassical approximation such packet does not disperse. It moves according to the laws of classical mechanics (see, for example [2, 19]). It is going beyond the semiclassical approximation that leads to the packet dispersion. The high-energy wave packets are of special interest because the speed of their dispersion decreases with the increase of their energy. Let us pay attention to some peculiarities of dispersion of such packets. Significant here is the fact that characteristic features of this dispersion are similar for all fields. Therefore it is sufficient to consider just scalar field.

The general solution of the wave equation

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2 + m^2\right)\varphi(\vec{r},t) = 0 \tag{1}$$

for a scalar particle with the mass m can be written in the following form of the expansion of the field $\varphi(\vec{r},t)$ over plane waves:

$$\varphi(\vec{r},t) = \int \frac{d^3\kappa}{(2\pi)^3} e^{i(\vec{\kappa}\vec{r}-\omega\, t)} C_{\vec{\kappa}} \,, \tag{2}$$

where $\omega = \sqrt{\vec{\kappa}^2 + m^2}$ and $C_{\vec{\kappa}}$ - are the expansion coefficients. Here and further we will use the system of units in which the speed of light c and the Plank constant $\hbar$ equal unit.

Let us consider the dispersion of the wave packet, which at the initial moment of time coincides with the Gaussian packet modulated by the plane wave with large value of the momentum $\vec{p}$ [2, 20]. Moreover we will assume that the initial widths of the packet $a_{\|}$ and $a_{\perp}$ parallel and perpendicular to the particle momentum $\vec{p}$ are different. For such packet at the initial moment of time the field $\varphi(\vec{r},t)$ has the following form:

$$\varphi(\vec{r},t) = \exp\left\{ i\vec{p}\vec{r} - \frac{z^2}{2a_{\|}^2} - \frac{\vec{\rho}^2}{2a_{\perp}^2} \right\}, \tag{3}$$

where z and $\vec{\rho}$ are the coordinates parallel and orthogonal to $\vec{p}$. At the moment of time t this packet will be defined by the relation (2) with

$$C_{\vec{\kappa}} = (2\pi)^{3/2} a_{\|} a_{\perp}^2 \exp\left\{ -\frac{(p-\kappa_z)^2 a_{\|}^2}{2} - \frac{\kappa_{\perp}^2 a_{\perp}^2}{2} \right\}. \tag{4}$$

We can write the obtained expression for the field $\varphi(\vec{r},t)$ in the form

$$\varphi(\vec{r},t) = Ae^{i(\vec{p}\vec{r}-\varepsilon\, t)} I(\vec{r},t)\,, \tag{5}$$

in which $A = a_{\|} a_{\perp}^2$, $\varepsilon = \sqrt{\vec{p}^2 + m^2}$ and

$$I(\vec{r},t) = (2\pi)^{-3/2} \int d^3\kappa \exp\left\{ i(\vec{\kappa} - \vec{p})\vec{r} - \frac{(p-\kappa_z)^2 a_{\|}^2}{2} - \frac{\kappa_{\perp}^2 a_{\perp}^2}{2} - i(\omega_k - \varepsilon)t \right\}. \tag{6}$$

Having made in this expression the variable substitution $\vec{\kappa} = \vec{p} + \vec{q}$ we find that

$$I(\vec{r},t) = (2\pi)^{-3/2} \int d^3q \exp\left\{ i\vec{q}\vec{r} - \frac{q_z^2 a_{\|}^2}{2} - \frac{q_{\perp}^2 a_{\perp}^2}{2} - i(\omega_{\vec{p}+\vec{q}} - \varepsilon)t \right\}. \tag{7}$$

In the case of large energies it is possible to make the expansion over $|\vec{q}|/p$ in the quantity $(\omega_{\vec{p}+\vec{q}} - \varepsilon)$ in (7). Having preserved the quadratic terms of expansion we obtain

$$\omega_{\vec{p}+\vec{q}} - \varepsilon \approx v q_z + \frac{q_z^2}{2\varepsilon\gamma^2} + \frac{q_{\perp}^2}{2\varepsilon}\,, \tag{8}$$

where $v = p/\varepsilon$ and $\gamma = (1-v^2)^{-1/2}$. Substituting this expression into (7) after simple calculations we obtain

$$I(\vec{r},t) = \frac{1}{\sqrt{a_{\|}^2 + i\dfrac{t}{\varepsilon\gamma^2}}} \frac{1}{a_{\perp}^2 + i\dfrac{t}{\varepsilon}} \exp\left\{ -\frac{(z-vt)^2}{2\left(a_{\|}^2 + i\dfrac{t}{\varepsilon\gamma^2}\right)} - \frac{\rho^2}{2\left(a_{\perp}^2 + i\dfrac{t}{\varepsilon}\right)} \right\}. \tag{9}$$

The formula (9) can be written in the following form as well:

$$I(\vec{r},t) = A(t) \exp\left\{ i\alpha(\vec{r},t) - \frac{(z-vt)^2}{2\Delta_{\|}^2(t)} - \frac{\rho^2}{2\Delta_{\perp}^2(t)} \right\}, \tag{10}$$

in which $A(t)$ is a slowly changing quantity

$$A(t) = \frac{1}{\left(a_{\perp}^2 + i\dfrac{t}{\varepsilon}\right)\sqrt{a_{\|}^2 + i\dfrac{t}{\varepsilon\gamma^2}}}\,, \tag{11}$$

$\alpha(\vec{r},t)$ is the real phase

$$\alpha(\vec{r},t) = \frac{(z-vt)^2}{2} \frac{t/\varepsilon\gamma^2}{a_{\|}^4 + \left(t/\varepsilon\gamma^2\right)^2} + \frac{\rho^2}{2} \frac{t/\varepsilon}{a_{\perp}^4 + \left(t/\varepsilon\right)^2}\,, \tag{12}$$

$\Delta_{\|}(t)$ and $\Delta_{\perp}(t)$ are the longitudinal and transverse widths of the packet at the moment of time t

$$\Delta_{\|}^2(t) = a_{\|}^2 + \left(\frac{t}{a_{\|}\varepsilon\gamma^2}\right)^2, \qquad \Delta_{\perp}^2(t) = a_{\perp}^2 + \left(\frac{t}{a_{\perp}\varepsilon}\right)^2. \tag{13}$$

In the case of $a_{\|} = a_{\perp}$ the obtained above formulae coincide with the corresponding result of the paper [20].

The formulae (13) show that in longitudinal and transverse directions the squares of the widths of the packet $\Delta_{\|}^2(t)$ and $\Delta_{\perp}^2(t)$ grow with time proportionally to $t^2 m^4/\varepsilon^6$ and t^2/ε^2. In nonrelativistic case these quantities do not depend on the particle energy ($t^2/\varepsilon^2 = t^2/m^2$). In relativistic case the quantities $t^2 m^4/\varepsilon^6$ and t^2/ε^2 are substantially smaller than the corresponding values for nonreativistic particles. Let us note that the additional factor m^4/ε^4 exists for longitudinal direction in $\Delta_{\parallel}^2(t)$. It leads to the substantial decrease of the speed of the packet dispersion in this direction compared to the speed of the packet dispersion in transverse direction. Thus the relativistic effects do the stabilizing influence upon the wave packets.

Now let us consider high energy packets of free electromagnetic waves. Scalar and vector potentials of such packets are the solutions of the wave equation (1) with $m=0$. Therefore in order to analyze the peculiarities of dispersion of such packets we can use the previous formulae assuming that all the terms in them containing the Lorentz-factor γ equal zero. In this case for scalar potential we find that

$$\varphi(\vec{r},t) = Ae^{i(\vec{k}\vec{r}-\omega t)}I(\vec{r},t)\,, \tag{14}$$

where $\vec{k}$ and ω are the wave vector and the frequency of the electromagnetic wave and

$$I(\vec{r},t) = A(t)\exp\left\{i\alpha_k(\vec{r},t) - \frac{(z-t)^2}{2\Delta_{\parallel}^2} - \frac{\rho^2}{2\Delta_{\perp}^2}\right\}. \tag{15}$$

Here

$$A(t) = \frac{1}{a_{\parallel}\left(a_{\perp}^2 + it/\omega\right)}\,, \qquad \alpha(\vec{r},t) = \frac{\rho^2}{2}\frac{t/\omega}{a_{\perp}^4 + \left(t/\omega\right)^2}\,, \qquad 16)$$

and

$$\Delta_{\parallel}^2(t) = a_{\parallel}^2\,, \qquad \Delta_{\perp}^2(t) = a_{\perp}^2 + \left(t/a_{\perp}\omega\right)^2.$$

The obtained formulae show that the initially Gaussian packet does not disperse in the direction parallel to the $\vec{k}$ vector. In transverse direction the square of the packet widths grows proportionally to $(t/\omega)^2$. Thus the speed of the packet dispersion decreases with the increase of the wave frequency ω.

When considering a process of radiation by relativistic electrons it is often necessary to deal with packets, which are constructed of plane waves with wave vectors, which directions are close to the direction of a given vector $\vec{k}$. Such wave packets differ somehow from the ones considered above. Let us consider some peculiarities of dispersion of such packets assuming for simplicity that at the initial moment of tome $t=0$ the distribution of the waves over the wave vectors is Gaussian relative to the given vector $\vec{k}$ [21]. For such distribution in the initial moment of time the scalar potential $\varphi_k(\vec{r},0)$ has the following form:

$$\varphi_k(\vec{r},0) = \frac{1}{\pi\overline{\Delta}_{\vartheta}^2}\int d^2\vartheta e^{-\vartheta^2/\overline{\Delta}_{\vartheta}^2}e^{i\vec{k}\vec{r}}\,, \tag{17}$$

where ϑ is the angle between the packet wave vector and the wave vector $\vec{k}$, $\overline{\Delta}_{\vartheta}^2$ is the average value of the square of the angle ϑ, $\overline{\Delta}_{\vartheta}^2 << 1$.

The coefficients $C_{\vec{q}}$ of the Fourier expansion (2) for such initial packet have the following form

$$C_{\vec{q}} = (2\pi)^3\int\frac{d^2\vartheta}{\pi\overline{\Delta}_{\vartheta}^2}e^{-\vartheta^2/\overline{\Delta}_{\vartheta}^2}\delta(\vec{k}-\vec{q})\,, \tag{18}$$

in which $\delta(\vec{k}-\vec{q})$ is the delta-function. As a result we come to the following expression for the scalar potential

$$\varphi_k(\vec{r},t) = \frac{1}{1+ikz\overline{\Delta}_{\vartheta}^2/2}e^{ik(z-t)-\frac{(k\rho/2)^2\overline{\Delta}_{\vartheta}^2}{1+ikz\overline{\Delta}_{\vartheta}^2/2}}\,, \tag{19}$$

where z and $\vec{\rho}$ are the coordinates parallel and orthogonal to $\vec{k}$.

The given expression for the wave packet has the same structure as the corresponding expression for the packet (14). If the substitutions $(t/a_{\perp}^2\varepsilon) \to (\omega\, z\overline{\Delta}_{\vartheta}^2/2)$ and $a_{\parallel}^2 \to \infty$ are made in the latter expression the both formulae for the wave packet will become identical.

The formula (19) shows that for $\omega\, z\overline{\Delta}_{\vartheta}^2/2 << 1$

$$\langle \varphi_k(\vec{r},t)\rangle \approx \exp\left\{ i\omega(z-t) - \left(\frac{\omega\rho}{2}\right)^2 \bar{\Delta}_g^2 \right\}, \tag{20}$$

and for $\omega\; z\bar{\Delta}_g^2/2 >> 1$

$$\langle \varphi_k(\vec{r},t)\rangle \approx -\frac{2i}{\omega\; z\bar{\Delta}_g^2}\exp\left\{ i\omega(z-t) + i\omega\frac{\rho^2}{2z} - \frac{\rho^2}{z^2\bar{\Delta}_g^2} \right\}. \tag{21}$$

For $z >> \rho$ the latter formula can be written in the form of a diverging wave

$$\langle \varphi_k(\vec{r},t)\rangle \approx -\frac{2i}{\omega\; r\bar{\Delta}_g^2}\exp\left\{ i\omega(r-t) - \frac{\rho^2}{z^2\bar{\Delta}_g^2} \right\}, \tag{22}$$

where $r = \sqrt{z^2+\rho^2} \approx z + \rho^2/2z$. Thus on distances z from the center of the initial packet, which satisfy the condition

$$\omega\; z\bar{\Delta}_g^2/2 << 1 \tag{23}$$

the form of the packet (19) coincides with the form of the packet at $t = 0$. Only on the distances, which satisfy the condition

$$\omega\; z\bar{\Delta}_g^2/2 >> 1 \tag{24}$$

the transformation of the packet (19) to the packet of spherical diverging waves occurs.

Let us note that in the theory of radiation of electromagnetic waves by a moving electron the spatial region in which the formation of spherical diverging waves occurs has a name of the wave zone (see for example [22]). In particular, for nonrelativistic charged particles the wave zone begins on distances from the radiation region, which exceed the length of the radiated wave λ . However, the condition (24) shows that for $\bar{\Delta}_g^2 << 1$ the wave zone formation occurs not on distances $z >> \lambda$ as in the case of a nonrelativistic particle but on distances

$$z >> 2\lambda/\bar{\Delta}_g^2, \tag{25}$$

which are much larger than the wave length $\lambda = 1/\omega$. For sufficiently small values of $\bar{\Delta}_g^2$ the length $z = 2\lambda/\bar{\Delta}_g^2$ can reach macroscopic size.

ELECTROMAGNETIC FIELD OF ELECTRON AFTER ITS EMISSION FROM SUBSTANCE

The electromagnetic wave packets similar to the ones considered above arise, for example, in the process of ultra relativistic electron traversal of boundary between two substances. In particular, let us consider some peculiarities of evolution of such packets during the electron emission from dielectric substance into vacuum.

During the motion of a charged particle in substance with dielectric permittivity ε_ω the screening of its own Coulomb field on large distances from the particle takes place. It happens due to polarization of the substance by the particle's field. In transverse direction with respect to the particle motion such screening occurs on distances $\rho \ge 1/\omega_p$, where ω_p is the plasma frequency. After the particle emission from the substance into vacuum the field around it is defined from the solution of Maxwell equations with corresponding boundary conditions for electric field and induction on the dielectric-vacuum interface. Taking into account these boundary conditions the electric field around the electron in vacuum can be presented as superposition of the electron's own Coulomb field in vacuum $\vec{E}^c(\vec{r},t)$ and the field of packet of free waves $\vec{E}^f(\vec{r},t)$: $\vec{E}(\vec{r},t) = \vec{E}^c(\vec{r},t) + \vec{E}^f(\vec{r},t)$. Significant here is the fact that considerable interference between these fields takes place, as a result of which on small distances from the interface the total field $\vec{E}(\vec{r},t)$ around the electron significantly differs from the Coulomb one. With the increase of distance between the particle and the interface such interference decreases and it is possible to consider the fields $\vec{E}^c$ and $\vec{E}^f$ as independent electron's own field and the field of radiation. For the Fourier-components of these fields with different frequencies such separation, however, occurs independently and on different distances. Therefore further we will consider the evolution of a single-frequency Fourier-component of the surrounding electron field.

In the considered ultra relativistic case the field around the electron can be considered as transversal to the direction of the particle motion (which is chosen to coincide with positive direction of z - axis), having just ρ - component. Taking into account the mentioned above boundary conditions, the Fourier-harmonic of frequency ω of the transversal component of the total field around the electron in vacuum can be presented in the next form (watch for example [1]):

$$E_\omega(\vec{\rho},z) = -\frac{ie}{\pi v}\frac{\vec{\rho}}{\rho}\int_0^\infty d^2q\vec{q}e^{i\vec{q}\vec{\rho}}\left\{ Q_f(q)e^{iz\sqrt{\omega^2-q^2}} + Q_c(q)e^{i\omega z/v}\right\}, \tag{26}$$

where

$$Q_f(q) = \frac{\sqrt{\omega^2-q^2}}{\sqrt{\omega^2\varepsilon-q^2}+\varepsilon\sqrt{\omega^2-q^2}}\left[\frac{1+\frac{v}{\omega}\sqrt{\omega^2\varepsilon-q^2}}{q^2+\frac{\omega^2}{v^2\gamma^2}-(\varepsilon-1)\omega^2} - \frac{\varepsilon+\frac{v}{\omega}\sqrt{\omega^2\varepsilon-q^2}}{q^2+\frac{\omega^2}{v^2\gamma^2}}\right],$$

$$Q_c(q) = \frac{1}{q^2+\frac{\omega^2}{v^2\gamma^2}}$$

and $\vec{q}$ is the orthogonal to z - axis component of the wave vector $\vec{k}$. The first term in (26) defines the packet of free waves in vacuum, which appears during the particle traversal of the substance-vacuum interface and gradually transforms into the field of transition radiation. It is a packet of plane waves with different directions of wave vector $\vec{k}$ ($|\vec{k}| = \omega$) of the similar type as the packet (19) (here we do not include the time-dependent factor $e^{-i\omega t}$ to the expression for E_ω). The second term in (26) defines the electron's own Coulomb field uniformly moving in vacuum.

After integration over the angle between $\vec{q}$ and $\vec{\rho}$ in (26) we receive:

$$E_\omega(\rho,z) = \frac{2e}{v}\int_0^\infty dqq^2J_1(q\rho)\left\{ Q_f(q)e^{iz\sqrt{\omega^2-q^2}} + Q_c(q)e^{i\omega z/v}\right\}, \tag{27}$$

where $J_1(x)$ is the Bessel function.

Let us note, that in the region $q > \omega$ the term in (27), which contains the quantity Q_f, exponentially decreases with the increase of z. Therefore it is possible to neglect the contribution of the region $q > \omega$ to the integral over q in this term.

In ultra relativistic case the frequencies ω, which make the main contribution to transition radiation spectrum, significantly exceed the plasma frequency of the substance ω_p. In this case we can use the following expression for dielectric permittivity of the substance, from which the electron is emitted:

$$\varepsilon(\omega) \approx 1 - \omega_p^2/\omega^2. \tag{28}$$

In the terms containing Q_f in (27) the main contribution to the integral over q is made by values $q \le \omega_p << \omega$ and the square roots in (27) can be expanded in parameter q^2/ω^2. At the same time due to convergence of the integral over q the integration in terms containing Q_f can be extended to infinity. As a result for the Fourier-component of the total field we obtain the expression

$$E_\omega(\rho,z) = \frac{2e}{v}e^{i\omega z}\int_0^\infty dqq^2J_1(q\rho)\left\{ \tilde{Q}_f(q)\exp\left(-i\frac{\omega z}{2v^2\gamma^2} - i\frac{q^2z}{2\omega}\right) + Q_c(q)\right\}, \tag{29}$$

with

$$\tilde{Q}_f = -\frac{\omega_p^2}{\left(q^2+\omega_p^2+\frac{\omega^2}{v^2\gamma^2}\right)\left(q^2+\frac{\omega^2}{v^2\gamma^2}\right)}.$$

On very small distances from the interface between the substance and the vacuum ($z \to 0$) the harmonic (29) of the total field around the electron nearly coincides with the harmonic of the particle's Coulomb field in the substance screened by polarization:

$$E_{\omega}(\rho,z)=\frac{2e}{v}\sqrt{\frac{\omega^2}{v^2\gamma^2}+\omega_p^2}\,K_1\left(\rho\sqrt{\frac{\omega^2}{v^2\gamma^2}+\omega_p^2}\right)e^{i\omega\ z}\,. \tag{30}$$

From (30) we can see that after the electron emission from substance the Fourier-components of frequencies $\omega \le \gamma\omega_p$ are suppressed in the field around the electron comparing to the corresponding Fourier components of electron's Coulomb field in vacuum, which are defined by the expression (30) with $\omega_p = 0$. The electron with such field is known as 'half-bare' electron [17, 18]. Such state of the particle also appears in the process of ultra relativistic electron scattering to a large angle and can manifest itself, for example, during further collisions of such electron with atoms of substance causing different effects of bremsstrahlung suppression (Landau-Pomeranchuk-Migdal effect [23, 24], the effect of radiation suppression in thin layer of substance (TSF-effect [25, 26]), etc.). The 'half-bare' state of the scattered electron should also manifest itself in the process of further transition radiation by such electron [14-16].

With the increase of distance between the electron and the substance the interference between the particle's own field and the packet of free waves decreases and the half-bare electron 'dresses' with its proper Coulomb field in vacuum. The distance z from the substance, on which the 'dressing' of the particle and reconstruction of the wave packet into the field of transition radiation occur is the coherence length of the radiation process. It is defined by the relation $l_c \approx 2\gamma^2/\omega$ and significantly exceeds the wavelength of the considered Fourier-component $\lambda = 2\pi/\omega$. In the considered case of ultra relativistic electron the transition radiation is mainly concentrated in the range of small angles ϑ between the radiation direction and the electron velocity [27]. For such angles in the region of distances from the substance $z \ge l_c$ the integral over q in the first item in (29) can be calculated with the use of stationary phase method. In this case the Fourier-harmonic of the total field naturally divides into the harmonic of electron's Coulomb field in vacuum and spherical diverging wave of transition radiation:

$$E_{\omega}(\rho,z)=\frac{2e}{v}\left\{\frac{\omega}{v\gamma}K_1\left(\frac{\omega\rho}{v\gamma}\right)e^{i\omega\ z/v}+\frac{e^{i\omega\ r}}{r}F(\vartheta)\right\}, \tag{31}$$

where

$$F(\vartheta)=\frac{\omega_p^2}{\omega^2}\frac{\vartheta}{(\vartheta^2+\omega_p^2/\omega^2+1/v^2\gamma^2)(\vartheta^2+1/v^2\gamma^2)}\,,$$

$\vartheta = \rho/z << 1$ and $r=\sqrt{\rho^2+z^2}\approx z+\rho^2/2z$.

The spectral-angular density of transition radiation, associated with the second item in (31) is given by the formula

$$\frac{d\mathcal{E}}{d\omega do}=\frac{r^2}{4\pi^2}\left|E_{\omega}^f(\vec{r})\right|^2=\frac{e^2}{\pi^2}F^2(\vartheta)$$

and coincides with the well-known expression of transition radiation theory [27].

IONIZATION ENERGY LOSSES OF 'HALF-BARE' ELECTRON IN THIN PLATE

Let us now consider how the reconstruction of the field around the electron after its emission from substance can influence on further interaction of the electron with matter. In particular, let us consider how the 'half-bare' state of the electron manifests itself in the particle ionization energy losses in thin dielectric plate situated close to the substance from which the electron is emitted.

A fast charged particle in substance loses its energy on excitation and ionization of the atoms of the substance. In the case of not very fast particles the value of such energy losses is defined by Bethe-Bloch formula [28, 29]. With the increase of the particle energy Fermi density effect [30] becomes essential, which leads to the decrease of energy losses in comparison with the correspondent result of Bethe and Bloch. This effect is caused by polarization of the atoms of the medium by the field of the particle which moves in substance. During the particle traversal of the border between two media due to reconstruction of the particle's field its ionization energy losses in the boundary region can significantly differ from the energy losses on large distances from the border. The possibility of such effect was indicated in the paper of G.M. Garibian [31], in which it was shown that in the case of passage of ultra relativistic electrons through sufficiently thin layer of substance (a layer of thickness less than $\delta = I/\omega_p^2$, where I is mean ionization potential of the atoms of substance and ω_p is the plasma frequency) Fermi density effect is absent and ionization energy losses are defined by Bethe-Bloch formula even in the case of very large particle energies. Experimentally such effect was observed in the works [32,33]. Let us note that some deviation from Fermi density effect was observed in the work [34] for ionization energy losses of ultra high energy electrons in relatively thick plates of substance as well.

Significant role in the process of ionization of the atoms of substance is played by Fourier-components of the particle's field with the frequencies close to the own frequencies of the atoms of substance. At the same time the characteristic distances from the boundary between two media, on which the reconstruction of the field around the electron occurs, are defined by the lengths of absorption of these Fourier-components in substance. By the order of the magnitude these lengths correspond to thicknesses of the targets, for which the Garibian effect in ionization energy losses takes place.

Let us consider now the ionization energy losses of the electron in thin dielectric plate, situated on distance z from the point of the electron emission from substance (see Fig.). The peculiarity of the considered process is in the fact that the reconstruction of the field around the electron after its emission occurs in vacuum in the absence of any absorption. Hence in this process the effects associated with reconstruction of the field around the electron can be observed in the particle ionization energy losses on large distances between the plate and substance, which significantly exceed the distances, on which the effect of Garibian takes place.

The consideration of the given process is carried out on the basis of Bohr's method [35, 36] of description of fast particle ionization energy losses in substance associated with separate consideration of the process in the region of small and large values of impact parameters. We draw the special attention in this case to the processes associated with large impact parameters. In this case the particle interaction with electron subsystem of the plate is considered in Fermi model of atoms of the medium [37], in which the bound atomic electrons are represented by a set of harmonic oscillators. In the region of large values of impact parameters the excitation of the atoms of the plate by incident electron is caused mostly by the Fourier-harmonics of transversal component of the electromagnetic field around the electron inside the plate, with frequencies ω of the order of characteristic frequencies of electron vibration in atoms ω_0. At the same time if the plate thickness is less than the absorption length of electromagnetic waves with frequency $\omega \approx \omega_0$ in substance, the energy transmitted to the electron subsystem of the plate will be defined by the Fourier-component (27) with the frequency $\omega \approx \omega_0$ of the field falling on the plate. In the simplest model of atoms of the medium, in which the own frequencies of electron oscillators ω_0 are taken to equal mean value of atomic ionization potential of the substance, the energy transfer to each atomic electron is defined by the relation [38]:

$$\Delta\mathcal{E} = \frac{e^2}{2m}\left|E_{\omega_0}\right|^2,$$

where e and m are charge and mass of the electron and E_{ω_0} – Fourier-component of the field around the incident electron transverse to the direction of the particle motion. Let us note that the electric field E_{ω_0} includes both electron's own Coulomb field in vacuum and the packet of free waves, which appeared as a result of electron emission from substance.

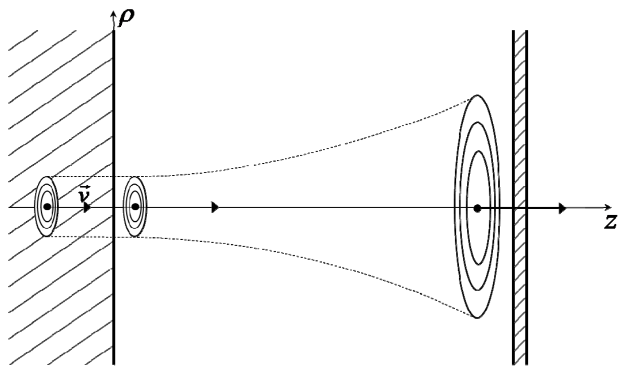

Fig. Traversal of thin plate by an electron after its emission from substance

The energy transmitted to the electron subsystem of the plate per unit path of the incident electron inside the plate is defined then by the following expression:

$$\frac{d\mathcal{E}}{dz} = \frac{\pi e^2 n}{m}\int_0^\infty d\rho\rho\left|E_{\omega_0}(\rho,z)\right|^2, \quad (32)$$

where n is the electron density in the plate. If the condition $\omega_0 >> \omega_p$ is fulfilled, for calculation of the energy losses it is possible to use the expression (28) for dielectric permittivity of the substance, from which the electron is emitted. In this case the considered Fourier-harmonic of the field around the electron is defined by the expression (29) with $\omega = \omega_0$. Substituting it into (32) we obtain the following expression for the energy losses

$$\frac{d\mathcal{E}}{dz} = \frac{4\pi e^4 n}{mv^2}\int_0^\infty dqq^3\left\{\left|\widetilde{Q}_f\right|^2 + 2Q_c\,\mathrm{Re}\widetilde{Q}_f\exp\left\{-i\frac{\omega_0 z}{2v^2\gamma^2} - i\frac{q^2 z}{2\omega_0}\right\} + Q_c^2\right\}. \quad (33)$$

The term in (33) containing Q_c^2, defines the contribution to the energy losses associated with Coulomb interaction of the moving particle with the electron subsystem of the plate. On the upper limit in this term the integral over q

diverges and it should be restricted by some maximum value q_0 defined from the condition of applicability of the used here model of the atomic electron subsystem as a set of harmonic oscillators. The latter is still valid if $R^{-1} \geq q_0 > \omega_0$, where R is the radius of screening of the atomic potential. As a result we come to the following formula for restricted energy losses, for which $q < q_0$:

$$\frac{d\mathcal{E}}{dz} = \frac{4\pi e^4 n}{mv^2}\left\{\int_0^{q_0} dq q^3 Q_c^2 + \int_0^{\infty} dq q^3\left[\widetilde{Q}_f^2 + 2\widetilde{Q}_f Q_c \cos\left(\frac{\omega_0 z}{2v^2\gamma^2} + \frac{q^2 z}{2\omega_0}\right)\right]\right\}. \tag{34}$$

Let us consider several limiting cases of the formula (34). For $z \to 0$, as can be easily verified,

$$\frac{d\mathcal{E}}{dz} = \frac{4\pi e^4 n}{mv^2}\left(\ln\frac{q_0}{\omega_p} - \frac{1}{2}\right).$$

This formula shows that the energy losses in the plate situated close to the border of the substance, from which the electron emerges, are defined by Fermi density effect like in substance, in which the particle moved before the collision with the plate.

In the region of distances z, which satisfy the conditions $\omega_0 / \omega_p^2 << z << \gamma^2 / \omega_0$, the energy losses in the plate are defined by the formula

$$\frac{d\mathcal{E}}{dz} = \frac{4\pi e^4 n}{mv^2}\left(\ln\frac{q_0\omega_p z}{2\omega_0} - \frac{1}{2} + G\right),$$

where $G = 0.577$ is the Euler's constant. Let us note that in the considered range of distances the logarithmic increase of the energy losses with the increase of z takes place. This effect is caused by the reconstruction of the field around the electron after its emergence from substance.

On sufficiently large distances from the point of the particle emission from substance $z > 2\gamma^2 / \omega_0$, according to (34),

$$\frac{d\mathcal{E}}{dz} = \frac{4\pi e^4 n}{mv^2}\left\{\left(\ln\frac{q_0 v\gamma}{\omega_0} - \frac{1}{2}\right) + \left(\ln\frac{\omega_p v\gamma}{\omega_0} - 1\right)\right\}. \tag{35}$$

The energy losses in this case do not depend on z. The first term in (35) corresponds to ionization produced by the particle's own Coulomb field and coincides with the corresponding result of Bethe and Bloch for contribution to energy losses associated with large impact parameters. Fermi density effect is not presented in ionization energy losses in the considered case of thin plate. The second term in (35) defines the contribution to ionization by the packet of free (radiated) waves. In the considered logarithmic approximation in (35) the interference term does not contribute to energy losses.

Let us draw attention to the fact that in the present paper we considered the energy transmission to bound electrons in the atoms of medium, for which $\omega_0 >> \omega_p$. The used here method is not applicable for description of energy transmission to free particles ($\omega_0 \to 0$), to which the paper [39] is devoted.

Thus with the increase of the distance in vacuum between the point of electron emission from substance and the plate there occurs the logarithmic increase of ionization energy losses per unit path from the value defined by Fermi density effect to the value defined by corresponding result of Bethe and Bloch complimented by contribution to ionization of radiation field. The distances z, within which such reconstruction of energy losses takes place, by the order of magnitude are defined by relation

$$z \approx 2\gamma^2 / \omega_0 .$$

In the case of ultra high energy particles these distances can reach macroscopic size. In particular, for electrons with the energy of the order of 100 Gev the length $z \approx 2\gamma^2 / \omega_0$ can reach several tens of meters and consequently the predicted effect of reconstruction of ionization energy losses can be observed at energies achievable on CERN accelerators.

CONCLUSION

The behavior of localized high-energy electromagnetic wave packets, which take place in processes of radiation by ultra relativistic electrons has been considered. It is shown that with the increase of the energy the stabilization of characteristics of motion of such packets takes place, which consists in substantial decrease of the speed of their dispersion. Essential here is the fact that at high energies the lengths on which the reconstruction of the form of such packets into packets of diverging waves takes place can reach macroscopic size, which can exceed the size of experimental facility.

Such situation takes place, for example, after ultra relativistic electron emission from substance into vacuum. It is shown that as a result of such emission a localized packet of free electromagnetic waves appears and transforms into a packet of diverging waves of transition radiation on large distance from the emission point. The transformation of a certain Fourier-harmonic of the packet field takes place within the coherence length of the radiation process, which substantially exceeds the length of the considered wave of radiation and can be macroscopic. On small distances from substance-vacuum interface the interference between the considered packet and the electron's own Coulomb field leads to suppression of low frequency Fourier-components in the field around the electron in the same way as inside the substance from which the electron was emitted. The electron with such field is known as 'half-bare' electron. It is shown that such state of electron should manifest itself, for example, in the particle ionization energy losses during further interaction of the electron with a plate situated in the direction of the electron motion on distances from the substance less than $2\gamma^2 / I$. In this case the effects in ionization energy losses associated with reconstruction of the field around the electron may be observed on distances, which considerably exceed the distances, on which the effect of Garibian takes place. It is shown that for small distance between the plate and the point of the electron emission from substance the particle ionization energy losses in the plate are defined by the formula, which takes into account the Fermi density effect. Then with the increase of this distance the ionization energy losses in the plate logarithmically increase, reaching the maximum value on distances, on which the separation of the Fourier-components of the particle's own field and the field of radiated waves takes place. In the case of ultra high energies of the particle these distances may be macroscopic, which opens new possibilities for investigation of manifestation of the electron in 'half-bare' state with suppressed low-frequency components of the surrounding field.

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