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PURE ELECTRON PLASMA STRUCTURES FORMATION IN AN EXTERNAL CONSTANT MAGNETIC FIELD

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Behavior of the pure electron plasma column rotating in an external constant magnetic field described in the kinetic approach is investigated. The initial stage of the axially asymmetric electrostatic oscillations instability development is considered. It is shown that the exponential increase of the electrostatic potential leads to the redistribution of the beam's density due to the ponderomotive force existence. The electrons motion under the influence of ponderomotive forces is modeled numerically. The simulation results are presented graphically in the form of particles set distribution in the beam's cross section. Qualitative estimates of the beam coupling parameter, which indicates the possibility of electron plasma cooling process are submitted.
KEY WORDS: electron plasma, backward reflection, ponderomotive force, pure electron plasma structures, beams cooling

ФОРМУВАННЯ ЗГУСТКІВ ЕЛЕКТРОННОЇ ПЛАЗМИ У ЗОВНІШНЬОМУ ПОСТОЙНОМУ МАГНИТНОМУ ПОЛІ

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Розглянуто початкову стадію розвитку нестійкості аксіально-несиметричних електростатичних коливань суцільного пучка електронної плазми, який обертається в зовнішньому магнітному полі. Показано, що експоненціальне наростання потенціалу призводить до перерозподілу щільності пучка під дією пондеромоторної сили . Рух електронів під дією пондеромоторної сили було промодельовано чисельно. Результати моделювання представлені графічно у вигляді розподілу часток у поперечному перерізі пучка. Наведено якісну оцінку коефіцієнту зв'язку пучка, яка вказує на можливість охолодження електронної плазми, тобто збільшення потенційної енергії частинок в порівнянні з їх кінетичною енергією.
КЛЮЧОВІ СЛОВА: електронна плазма, зворотне відбиття, пондеромоторна сила, електронні згустки, охолодження пучків

ФОРМИРОВАНИЕ СГУСТКОВ ЭЛЕКТРОННОЙ ПЛАЗМЫ ВО ВНЕШНЕМ ПОСТОЯННОМ МАГНИТНОМ ПОЛЕ

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Рассмотрена начальная стадия развития неустойчивости аксиально-несимметричных электростатических колебаний сплошного пучка электронной плазмы, который вращается во внешнем магнитном поле. Показано, что экспоненциальное нарастание потенциала приводит к перераспределению плотности пучка под действием пондеромоторной силы. Движение электронов под действием пондеромоторной силы было промоделировано численно. Результаты моделирования представлены графически в виде распределения частиц в поперечном сечении пучка. Приведена качественная оценка коэффициента связи пучка, которая указывает на возможность охлаждения электронной плазмы, то есть увеличения потенциальной энергии частиц по сравнению с их кинетической.
КЛЮЧЕВЫЕ СЛОВА: электронная плазма, обратное отражение, пондеромоторная сила, электронные сгустки, охлаждение пучков

Electrostatic oscillations of the electron plasma column rotating in a cylindrical trap confined by the external longitudinal magnetic and radial self-consistent electric fields are assumed to be stable. Such conclusions were made on the basis of the hydrodynamic (macroscopic) approach, while the kinetic theory (microscopic description) considered the electron plasma of complex shapes excluding the particle-wall interaction or idealizing it by application of the specular reflection as a boundary condition [1,2]. In general the diffuse reflection of electrons from a conductive surface, which is observed in experiments due to imperfection of the confinement camera or anisotropy of the medium [3-5] and is supposed to contribute to enhanced plasma heating [6] was neglected.

A special case of diffuse reflection is a "backward reflection" [7-9] during which the radial and azimuthal components of the velocity of particle collided with the wall of the trap change their signs to the contrary in a contrast to the "specular" reflection when only the sign of the radial velocity changes. A kinetic theory of the rotating cylindrical electron plasma considering a backward reflection of particles from the walls of the trap as the boundary condition was proposed in [10]. It is shown that the azimuthally asymmetric electrostatic oscillations in such system may be unstable.

Instability development in the Penning-Malmberg or Paul traps described in [10] can lead not only to ejection of particles on the chamber walls, but also to a particles' density redistribution and formation of the ordered structures such as bunches, Coulomb crystals and vortexes. The formation of such structures is possible in electron beams under the diocotron instability development [11,12] or under the influence of high-frequency ponderomotive force [13], the dynamics of similar electron bunches is considered in [14,15].

The change in electrons density profile is closely related to the probability of trapped particles cooling, since the potential energy of the particles which have formed bunches may greatly exceed their kinetic energy. Experimental and theoretical evaluation of the coupling parameter Γ (ratio of the potential energy of the beam's particles to their kinetic energy) given in [16,17] clearly indicate the cryogenic temperatures of electron bunches in the Penning-Malmberg traps which, therefore, are promising not only for the accumulation of particles but also for the electrons cooling and subsequent use for technological purposes [18].

The aim of this paper is to investigate the initial stage of pure electron plasma column instability development and subsequent ordered structures formation by modeling the growing ponderomotive force action on the electrons and estimating the electron plasma's coupling parameter.

PROBLEM FORMULATION

Let's consider the cylindrical waveguide in which the electron plasma beam performs an azimuthal rotation in self-consistent radial electric and external longitudinal magnetic fields [10]. The electron density's dependence on the transverse coordinate r is described by the following form:

$$n_e = \begin{cases} n_{oe}, & r \le R_0 \\ 0\ , & r > R_0, \end{cases}$$

where R_0 - radius of the waveguide, and n_{0e} - unperturbed beam's density. There exists a possibility of an unstable axially asymmetric ($l \ge 1$) electrostatic fluctuations arising which may lead to a redistribution of the electrons' density profile. The growth rate of those unstable oscillations is defined as $\operatorname{Im}\omega$ from the dispersion equation [10]:

$$\left(\xi_n^l\right)^2 = -\frac{6}{\pi^2}\frac{\Omega_e^2}{v_{te}^3}\frac{R_0\Omega_*}{l}\sum_{p=-\infty}^{\infty}(-1)^p\frac{\Omega_*}{\omega+p\Omega_*}\int_0^{\infty}e^{-x^2+2k_rx}\cos\left(2k_lx\right)dx, \tag{1}$$

where $\xi_n^l = \dfrac{\lambda_n^l}{R_0}$ - n-th root of the Bessel function of l-th order ($J_l\left(R_0\xi_n^l\right)=0$), $\Omega_* = \pm\omega_e\sqrt{1-\dfrac{2\Omega_e^2}{\omega_e^2}}$, $\Omega_e = \left(\dfrac{4\pi e^2 n_0}{m_e}\right)^{\frac{1}{2}}$ and $\omega_e = \dfrac{eH_0}{m_e c}$ – plasma and cyclotron frequencies respectively; $k_l = \dfrac{l v_{te}}{R_0\Omega_*}$, $k_r = \dfrac{\xi_n^l v_{te}}{\Omega_*}$, l - azimuthal wave number, $v_{te} = \left(\dfrac{T_e}{m_e}\right)^{\frac{1}{2}}$ - the electron thermal velocity, H_0 - the external magnetic field, m_e and e - electron's mass and charge respectively, c - the speed of light in vacuum.

Analysis of the dispersion equation (1) predicts the instability of the perturbations of the electrostatic potential in the form:

$$\phi_{l,n}\left(r,l,t\right) = \Phi_{0l}J_0\left(\xi_n^l r\right)e^{-it\operatorname{Re}\omega+t\operatorname{Im}\omega}\cos l\phi, \tag{2}$$

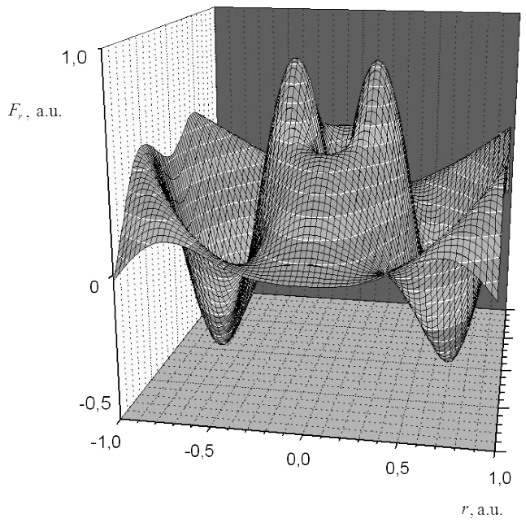
where Φ_{0l} – the amplitude of electrostatic potential perturbation.

Solution (2) describes the formation of the electrostatic potential's humps and pits in the electron plasma rotating in the laboratory frame with $l\Omega$ frequency. At low densities of the plasma $\left(\Omega_e << \omega_e\right)$ the potential's oscillations frequency (2) is large enough ($\Omega_* >> \Omega$, where $\Omega = \dfrac{\omega_e}{2}\left[1\pm\sqrt{1-\dfrac{2\Omega_e^2}{\omega_e^2}}\right]$) therefore it's easier to describe the behavior of the electron beam introducing the ponderomotive force. Based on the definition, the ponderomotive force is a force that acts on a unit volume of a substance in the presence of oscillating and non-uniform electromagnetic fields due to the fact that the material is electrically charged, conductive, polarized and magnetized [19]. In the case of the azimuthal flow of electrons in the presence of potential's perturbations (2), this force is given by:

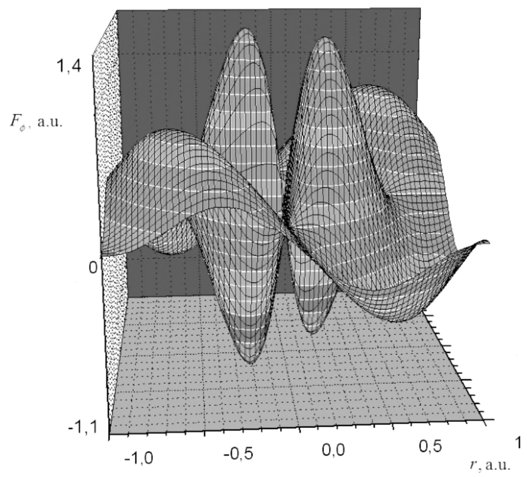
$$F_r = -\frac{e^2}{4m\omega^2}\frac{\partial}{\partial r}\left|\nabla\phi_l\right|^2$$
$$F_\phi = -\frac{e^2}{4m\omega^2}\frac{1}{r}\frac{\partial}{\partial\phi}\left|\nabla\phi_l\right|^2, \tag{3}$$

where F_r and F_ϕ - radial and azimuthal components of the ponderomotive force, which in the case of RF fields is also called the Miller force. Ponderomotive force's action on the electron beam rotating azimuthally leads to a redistribution of the particles.

Obviously, under the instability development the unperturbed stepped profile of the electrons' distribution function will change significantly and the equations (1) and (2) will not strictly apply. Nevertheless, they can be used to describe the initial stage of the electron beam's density redistribution ($\mathrm{Im}\,\omega << \mathrm{Re}\,\omega$).



a)



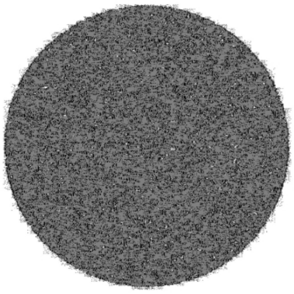
b)

Fig. 1. Miller (ponderomotive) force calculated for the electron beam l=1 radial (a) and azimuthal (b) components

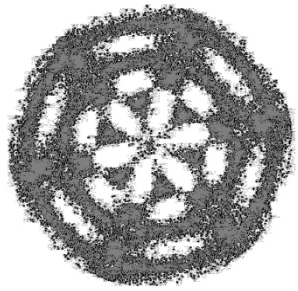
SIMULATION RESULTS

Fig. 1 shows the topology of the Miller (ponderomotive) force acting on a rotating electron beam *(l*=1). As seen from Fig. 1, in a cylindrical electron beam rotating azimuthally in the external magnetic field with the boundary condition of particles backward reflection the electrons are "pushed" off the areas where the Miller force is negative and accumulated in the areas where Miller force is positive.

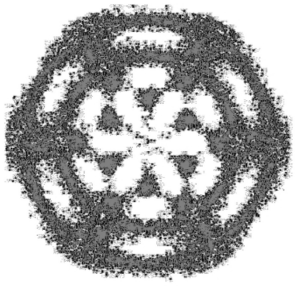
The following method of numerical simulation of the ordered structures' dynamics is applied: at the initial time a set of particles (N>10^5) is randomly distributed over the cross section of the confinement chamber so that their density is not dependent on the radius. Runge-Kutta method of the fourth order is used to calculate the motion equations and to determine coordinates and velocities of the particles under the influence of ponderomotive force. The results of numerical modeling for a selected group of electrons are shown in Fig. 2.



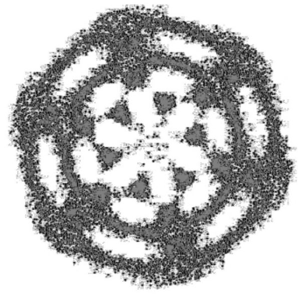
a)



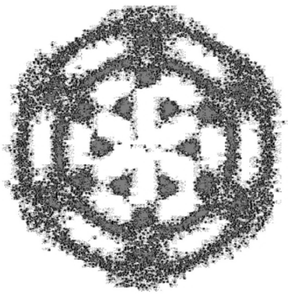
b)



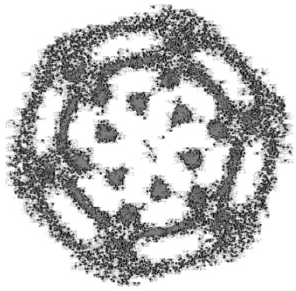
c)



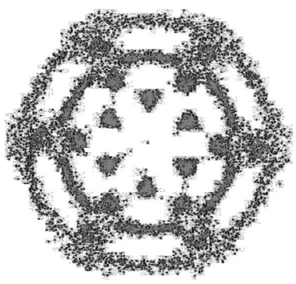
d)



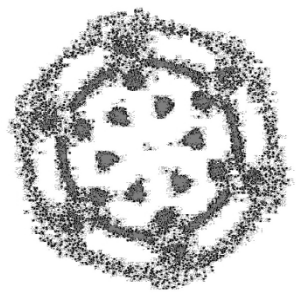
e)



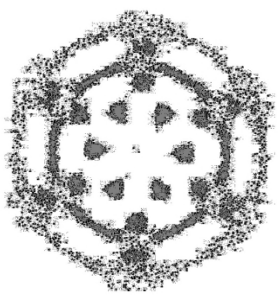
f)



g)



h)



i)

Fig. 2. Redistribution of the electrons set in the beam (*l*=3) cross section under the action of Miller force and the ordered structures formation. Each figure from a) to i) corresponds to 1000 rotations of the beam, assuming $\frac{\mathrm{Im}(\omega)}{\mathrm{Re}(\omega)} \approx 10^{-5}$.

According to [20,21], the electron plasma state can be described by the correlation parameter Γ i.e. the ratio of the plasma particle's kinetic energy to its potential energy ($\Gamma = \frac{E_{pot}}{E_{kin}}$ where E_{kin} and E_{pot} – kinetic and potential energies of the beams particle respectively). For the electron plasma discussed above, the evaluation of the correlation parameter can be performed as follows:

$$\Gamma(t) \propto \frac{2e\Phi_{0l}\left|J_0\left(\xi_n^l r\right)e^{-i\operatorname{Re}\omega t}\cos l\phi\right|}{m_e v_e^2}e^{\gamma t}, \tag{4}$$

where γ - the growth rate of the instability (2).

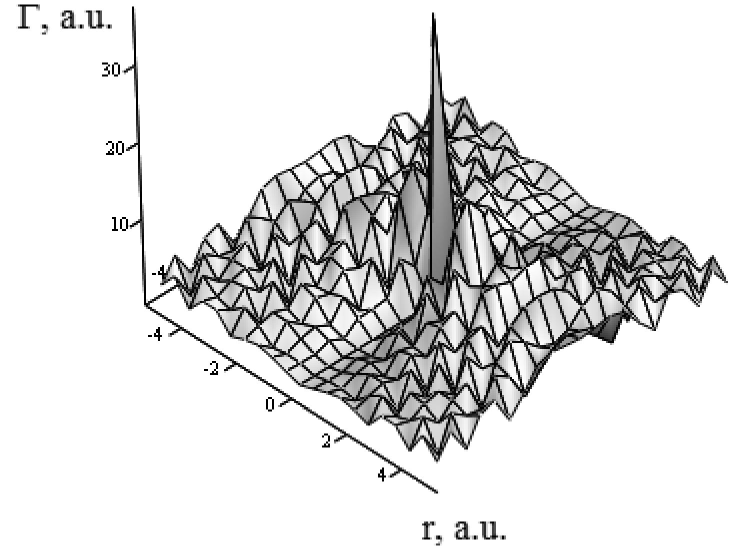


Fig. 3. Estimated value of the coupling parameter Γ in the cross section of the electron beam (l=2) after 1000 of rotations.

Fig.3 presents the estimate of the correlation parameter in the cross section of the confinement chamber after 10^3 of beam's (l=2) rotations. According to Fig.3 the instability development resulting a redistribution of the beam's density may also lead to the beam's cooling because the potential energy of the electrons located closer to the axis of the cylindrical trap greatly exceeds their kinetic energy. Thus the phase transition from gas (plasma) into the liquid state becomes possible. Assuming that the equations (2-4) remain valid for a longer time, further cooling the beam to the cryogenic temperatures (about 100K) and its transition to the state electron crystal [21] may also take place.

CONCLUSIONS

Slowly growing instability leads to a redistribution of the beam's particles under the action of the ponderomotive force and can cause the formation of rotating electron bunches. Under the weak distortion conditions of the initial electron plasma's density profile the formation of electron bunches rotating along with the plasma is possible. The possibility of electron beam cooling and its subsequent transition from the gaseous (plasma) to the liquid state until the formation of electronic crystals [11-15] may occur.

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