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WAVES OF MAGNETIZED PLASMA-FILLED WAVEGUIDE WITH ZERO VALUE OF EITHER OF TWO TRANSVERSE WAVENUMBERS

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This paper describes the electromagnetic properties of the waves in a magnetized plasma-filled waveguide with one transverse wavenumber being equal to zero. It was found, that despite zero denominators of the field expressions for such waves, the validity of these expressions is not violated. As a result it was shown that the wave frequency and field change continuously when one of its transverse wavenumbers crosses zero. It was also found, that the point of such crossing changes with the sign change of the wave azimuthal index.
KEY WORDS: plasma-filled waveguide, magnetized plasma, dispersion equation, eigenfrequencies, eigenfields

ВОЛНЫ МАГНИТОАКТИВНОГО ПЛАЗМЕННОГО ВОЛНОВОДА С НУЛЕВЫМ ЗНАЧЕНИЕМ ОДНОГО ИЗ ДВУХ ПОПЕРЕЧНЫХ ВОЛНОВЫХ ЧИСЕЛ

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В работе рассмотрены электромагнитные свойства волн в магнитоактивном плазменном волноводе, у которых одно из их поперечных волновых чисел равно нулю. Найдено, что, несмотря на обращение в нуль знаменателей выражений для поперечных полей таких волн, применимость этих выражений не нарушается. В результате показано, что непрерывными являются изменения поля и частоты волны плазменного волновода при переходе одного из ее поперечных волновых чисел через нулевое значение. Установлено, что точка такого перехода изменяется с изменением знака азимутального индекса волны.
КЛЮЧЕВЫЕ СЛОВА: плазменный волновод, магнитоактивная плазма, дисперсионное уравнение, собственный частоты, собственные поля

ХВИЛІ МАГНІТОАКТИВНОГО ПЛАЗМОВОГО ХВИЛЕВОДУ З НУЛЬОВИМ ЗНАЧЕННЯМ ОДНОГО ІЗ ДВОХ ПОПЕРЕЧНИХ ХВИЛЬОВИХ ЧИСЕЛ

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В роботі було розглянуто електромагнітні властивості хвиль у магнітоактивному плазмовому хвилеводі, у яких одне з поперечних хвильових чисел дорівнює нулю. Було знайдено, що, незважаючи на звернення в нуль знаменників виразів для поперечних полів таких хвиль, застосовність цих виразів не порушується. У підсумку показано, що безперервними є зміни поля та частоти хвилі плазмового хвилеводу при переході одного з її поперечних хвильових чисел через нульове значення. Встановлено, що точка такого переходу змінюється зі зміною знака азимутального індексу хвилі.
КЛЮЧОВІ СЛОВА: плазмовий хвилевід, магнітоактивна плазма, дисперсійне рівняння, власні частоти, власні поля

It is known [1] that microwave devices and particle accelerators with plasma-filled electrodynamic structures have several advantages as compared to their vacuum counterparts. However, if the structure is placed in an external magnetic field, a plasma filling can substantially complicate its dispersion properties and the spatial field distribution.

In the smooth empty waveguide waves of two types can propagate independently. A separation of the waves into the transverse electric (TE) and transverse magnetic (TM) types is due to the differences in the topography of their fields. Besides, the type of wave (and also its transverse indices) can be determined, if the value of the transverse wavenumber for this wave is known.

When the waveguide is filled with magnetized plasma, the waveguide modes are hybrid. Each of them is characterized by two values - k_1 and k_2. Similar to the vacuum case, these values are also often called the transverse wavenumbers. In the general case, the separation of the hybrid waves into special types is quite arbitrary. Traditionally, they are separated into HE and EH waves depending on their cutoff frequencies. For these frequencies fields of HE waves become transverse electric and those of EH waves - transverse magnetic.

Of special interest is the case of the hybrid waves of a plasma-filled waveguide, for which one of the transverse wavenumbers (k_1 or k_2) is zero. It is not difficult to show that these waves satisfy the dispersion relation of the right-polarized or left-polarized waves in unbounded magnetized plasma with immobile ions. In this case, the denominators in the expressions [2] for the transverse field components are equal to zero, and the direct numerical calculation of the field distribution as well as a numerical solution of the dispersion equation become impossible. For the first time E.G. Alexov has addressed this issue in [3]. This paper studies the case of $k_i = 0$ for a waveguide partially filled with a magnetized plasma.

A conclusion about the singular nature of the fields can only be obtained by considering their form in the neighborhood of the singular point. An analytical treatment is required to do this rigorously. Despite the fact that such treatment was not presented in [3], the case of $k_i = 0$ was attributed to one with infinite values of the transverse fields. Thus the well-known expressions [2] for the waveguide fields were declared to be invalid in this case. It is worth noting that the validity of these expressions does not have any other known restrictions by now. In [3] the field expressions were redefined. For the waves with $k_i = 0$, an additional relation between the longitudinal components of the electric and magnetic fields was introduced. This relation allowed one to eliminate the zeros in the denominators of the field expressions.

For the waveguide completely filled with magnetized plasma, the case of $k_i = 0$ was studied in [4]. This paper also contains the statement that the field expressions [2] are singular in this case. New expressions for the longitudinal field components of the waves with $k_i = 0$ were obtained. It was shown, that they contain a quasi-static part. This feature differentiates these waves from other waves in a plasma-filled waveguide. With these longitudinal field components the nonsingular expressions for the transverse fields and also the dispersion relation in the case of $k_i = 0$ were determined. However, it remains unclear how these expressions are related to the results in [2]. Moreover, the domain of applicability of the expressions in [2] is not completely understood by now, since the statement that they are singular in the case of $k_i = 0$ is also used in [4] without proof. This situation has motivated the present study of waves with $k_i = 0$.

The main aim of the study is the investigation on continuity of electromagnetic properties for waveguide modes in the point $k_i = 0$. This is quite important for their potential application in powerful electronics.

EIGENFREQUENCIES AND EIGENFIELDS OF THE WAVEGUIDE FILLED WITH MAGNETIZED PLASMA

Let us consider a metallic cylindrical waveguide completely filled with plasma. The waveguide is placed in an external magnetic field of finite magnitude B_0. The direction of the magnetic field coincides with the waveguide axis. The plasma is assumed to be homogeneous, cold and collisionless with immobile ions.

The derivation scheme for the dispersion equation of such a waveguide is well known [2]. Assuming that in cylindrical coordinates $\{r,\ \varphi, z\}$, the wave quantities have the form $A(\mathbf{r},t) = A(r)\exp(i\omega t - ik_z z - il\varphi)$, the expressions for the field components can easily be obtained from Maxwell's equations:

$$E_z(r) = \sum_{i=1}^{2} A_i J_l(k_i r), \tag{1}$$

$$E_\varphi(r) = -\frac{1}{\varepsilon_2 k_z \zeta^4}\sum_{i=1}^{2} A_i\left[\alpha_i k_i J_l'(k_i r) - \beta_i \frac{\varepsilon_2 l}{r} J_l(k_i r)\right], \tag{2}$$

where ω is the wave frequency, k_z and l are the axial and azimuthal wavenumbers, $2\varepsilon_1 k_i^2 = k_0^2 - (-1)^i\, sign(\omega - \omega_H)\sqrt{k_0^4 - 4\varepsilon_3\varepsilon_1\zeta^4}$, $k_0^2 = -\chi^2(\varepsilon_1 + \varepsilon_3 + \varepsilon_2 b)$, $\chi^2 = k_z^2 - \varepsilon_1 k^2$, $b = \varepsilon_2 k^2/\chi^2$, $\zeta^4 = \chi^4 - \varepsilon_2^2 k^4 = \chi^4(1-b)(1+b)$, $\alpha_i = (k_z k \varepsilon_2)^2 + \chi^2\eta_i$, $\beta_i = (k_z^2\chi^2 + k^2\eta_i)$, $\eta_i = (\varepsilon_3\chi^2 + \varepsilon_1 k_i^2)$, $\varepsilon_1 = 1 - \omega_p^2/(\omega^2 - \omega_H^2)$, $\varepsilon_2 = -\omega_p^2\omega_H/\left[\omega(\omega^2 - \omega_H^2)\right]$, $\varepsilon_3 = 1 - \omega_p^2/\omega^2$, $\omega_p = (4\pi e^2 n_e/m_e)^{1/2}$ is the electron plasma frequency, $\omega_H = eB_0/m_e c$ is electron cyclotron frequency, e, m_e, n_e are the charge, mass, and density of the plasma electrons, $k = \omega/c$ is the wavevector in free space, A_1 and A_2 are the amplitudes of the waveguide eigenfields. The relation between these amplitudes can be found from one of the following boundary conditions at the perfectly conducting waveguide surface:

$$E_z\big|_{r=R} = 0,\ E_\varphi\big|_{r=R} = 0. \tag{3}$$

With this relation and also with the second boundary condition, the dispersion equation for a waveguide completely filled with a magnetized plasma can be obtained:

$$D(\omega,k_z)\equiv 1/\zeta^4\left\{\alpha_j\Phi_l\left(k_jR\right)J_l\left(k_iR\right)-\alpha_i\Phi_l\left(k_iR\right)J_l\left(k_jR\right)\right\}=0\,, \tag{4}$$

where $\Phi_l\left(k_iR\right)=k_iRJ_l'\left(k_iR\right)-lbJ_l\left(k_iR\right)$, indices i, j are non-coincident and can take the either value 1 or 2. The dispersion equation (4) (see also [5]) can be reduced to the standard form [2-4] using the identity $\left(\alpha_1-\alpha_2\right)=\chi^2/k^2\left(\beta_1-\beta_2\right)=\varepsilon_1\chi^2\left(k_1^2-k_2^2\right)$.

It should be noted that a factor $1/\zeta^4$ in the dispersion equation is usually assumed to be insignificant and is hence omitted. Obviously this is true only if $\zeta^4\neq 0$. For the same reason, a product ε_2k_z is assumed below to be nonzero and is not introduced in equation (4). Condition $\zeta^4=0$ is fulfilled for those solutions of the dispersion equation (4) that also satisfy the dispersion equation for the right-polarized ($b=1$) or left-polarized ($b=-1$) waves in the unbounded magnetized plasma.

If there is a singularity $\zeta^4=0$ in (2), it can be formally removed by multiplying the amplitudes A_1 and A_2 by a factor proportional to ζ^4 [3]. Obviously such multiplication of both amplitudes is consistent with the waveguide problem, since the dispersion equation uniquely determines only their ratio A_1/A_2. However, such multiplication also reduces the longitudinal field components [3], and the dispersion equation to identical zeros when $\zeta^4=0$.Therefore, such formal removal of the singularity does not allow to define the frequencies and fields for waveguide modes satisfying the relation $\zeta^4=0$.

WAVES OF MAGNETIZED PLASMA-FILLED WAVEGUIDE WITH $k_i=0$

Let us investigate the validity of equations (2) and (4) when $\zeta^4=0$. As follows from the form of k_i, one of the transverse wavenumbers is equal to zero for those waveguide waves whose frequencies $\omega\left(k_z\right)$ satisfy the relation $\zeta^4=0$ or $\omega=\omega_p$ ($\varepsilon_3=0$) (see, Fig. 1). Without taking into account the case that $\omega=\omega_p$, conditions $k_i=0$ and $\zeta^4=0$ are assumed below to be equivalent. If condition $\zeta^4=0$ is fulfilled, the denominators of functions $E_\varphi\left(r\right)$ and $D(\omega,k_z)$ in equations (2) and (4), respectively, become zero. Hence these equations cannot directly be used in the numerical calculations. However, a conclusion about the validity of these equations can only be drawn if their form at small $\left|\zeta^4\right|$ is known.

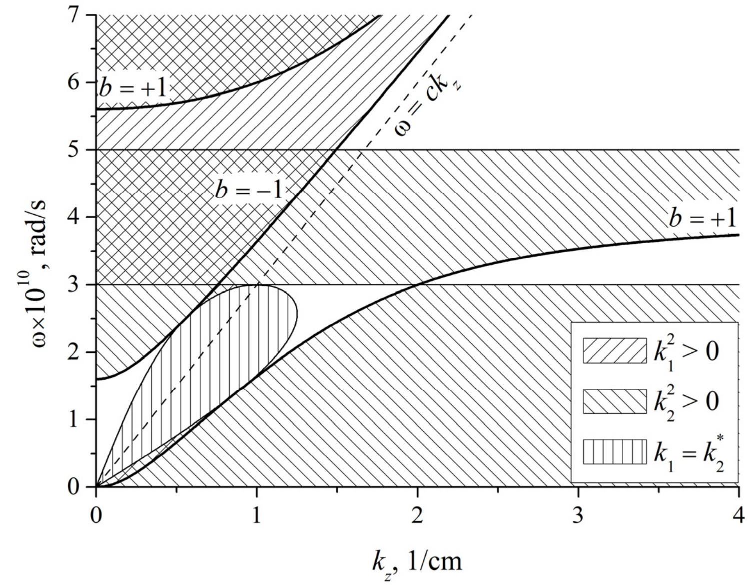

Fig. 1. Domains of real, imaginary, and complex values of $k_{1,2}$, and also the solutions of the dispersion equations for right-polarized ($b=1$) and left-polarized ($b=-1$) waves in unbounded magnetized plasma ($\omega_p=3\times10^{10}$ rad/s, $\omega_H=4\times10^{10}$ rad/s)

If $\left|\zeta^4/\chi^4\right|<<1$, some functions present in (1), (2), and (4) can be simplified to read

$$k_i^2 = \frac{\varepsilon_3\zeta^4}{k_0^2},\ k_j^2 = \frac{k_0^2}{\varepsilon_1} - \frac{\varepsilon_3\zeta^4}{k_0^2},$$

$$\alpha_i = -\chi^2 k_0^2 - \varepsilon_1\left(1 - \frac{\varepsilon_3\chi^2}{k_0^2}\right)\zeta^4,\ \beta_i = -k^2 k_0^2 + \varepsilon_1\left(\frac{1}{\varepsilon_1} + \frac{\varepsilon_3 k^2}{k_0^2}\right)\zeta^4, \tag{5}$$

$$\alpha_j \equiv \alpha_{j0}\zeta^4 = -\varepsilon_1\left(1 + \frac{\varepsilon_3\chi^2}{k_0^2}\right)\zeta^4,\ \beta_j \equiv \beta_{j0}\zeta^4 = \varepsilon_1\left(\frac{1}{\varepsilon_1} - \frac{\varepsilon_3 k^2}{k_0^2}\right)\zeta^4.$$

The derivation of (5) implies that k_0^2 is not small. As is evident from (1), (2), k_0^2 is small only in a close neighborhood of frequencies satisfing the relation $\varepsilon_1 + \varepsilon_3 = \mp\varepsilon_2$, when $b \approx \pm 1$. With the exception of these regions condition $\left|\zeta^4/\chi^4\right| << 1$ leads to $k_i^2 << 1$.

In the case that $k_i^2 << 1$, for $l \ge 0$ one has

$$J_l\left(k_i R\right) \approx \frac{1}{l!}\left(\frac{k_i R}{2}\right)^l, \tag{6}$$

$$\frac{\left(k_i R\right) J_l'\left(k_i R\right)}{J_l\left(k_i R\right)} \approx l - \frac{2}{(l+1)}\left(\frac{k_i R}{2}\right)^2. \tag{7}$$

Considering that

$$\Phi_l\left(k_j R\right) \approx \left(\mp k_j R\right) J_{l\pm1}\left(k_j R\right), \tag{8}$$

for $b \approx \pm 1$ and using (5)-(7), for waves with $l \ge 0$ we reduce the fields (1), (2) in the case of $\left|\zeta^4/\chi^4\right| << 1$ to the form

$$E_z(r) = \frac{A_i}{l!}\left(\frac{k_i}{2}\right)^l r^l + A_j J_l\left(k_j r\right), \tag{9}$$

$$E_\varphi(r) = \frac{-1}{\varepsilon_2 k_z r}\left\{\frac{A_i}{(1+b)l!}\left(\frac{k_i}{2}\right)^l r^l l\left[\frac{\beta_i}{\chi^2} + (1+b)\left(\frac{\varepsilon_3\chi^2 R^2}{2l(l+1)} - \frac{k_z^2}{k^2}\right)\right] + A_j\left[\alpha_{j0} k_j r J_l'\left(k_j r\right) - \beta_{j0}\varepsilon_2 l J_l\left(k_j r\right)\right]\right\}, \tag{10}$$

and the dispersion equation (4) to the form

$$D(\omega,k_z) \equiv \frac{\left(\zeta^4\right)^{l/2}}{1+b} D_{\pm1}(\omega,k_z) = \frac{\left(\zeta^4\right)^{l/2}}{1+b}\left\{\pm\alpha_{j0} k_j R J_{l\mp1}\left(k_j R\right)(1+b) - \frac{l}{2}\left[\frac{2k_0^2}{\chi^2} - \frac{\varepsilon_3\chi^2 R^2}{l(l+1)}(1+b)\right] J_l\left(k_j R\right)\right\} = 0, \tag{11}$$

or

$$D(\omega,k_z) = \begin{cases} \delta^{l/2} D_{+1}(\omega,k_z) = 0, & |b-1| = |\delta| << 1 \\ \delta^{l/2-1} D_{-1}(\omega,k_z) = 0, & |b+1| = |\delta| << 1 \end{cases}, \tag{12}$$

where $D_{\pm1}(\omega,k_z)$ are the functions that do not contain any singular points. As is evident from (12), their zeros are the solutions of the dispersion equation (4), when $b = \pm 1 + \delta$ and δ is small but nonzero.

Let us consider the case of $b = 1$. As is evident from (10), (12), in this case functions E_φ and $D(\omega,k_z)$ do not have singularities. In addition, for axisymmetric waves ($l = 0$) it is an easy task to calculate the frequencies $\omega(k_z)$ satisfying the condition $b = 1$ ($k_i = 0$). They are discrete zeros of function $D_{+1}(\omega,k_z)$.

However, when $l > 0$ the dispersion equation (12) is satisfied for all ω related to k_z by the equation $b = 1$. At the same time, a number of such ω and k_z reduce the tangential fields (9) and (10) on the waveguide wall to zero only if $A_j = 0$. Such solutions of the dispersion equation are unphysical and correspond to zero fields. They should be excluded from our consideration.

This can easily be done considering that each solution of the dispersion equation (4) allows to determine fields (1) and (2) only with an arbitrary amplitude A_1 (or A_2). Therefore, one of two amplitudes can be redefined as

$$A_i' = \frac{A_i}{l!}\left(\frac{k_i}{2}\right)^l, \tag{13}$$

without violating the validity of (1), (2), and (4).

Substituting (13) into (9) and (10) and using the boundary conditions (3) we find the physical solutions of the dispersion equation for $b=1$. They are zeros of the function $D_{+1}(\omega,k_z)$. The fields associated with these solutions satisfy the boundary conditions on the waveguide wall and have nonzero distributions.

This conclusion can also be obtained without use of (13). For this purpose one has to eliminate the constant A_i from (9) and (10) with one of the boundary conditions (3). Using then the second boundary condition, one obtains an equation of the form $D_{+1}(\omega,k_z)=0$. This equation determines the dispersion properties of the waves, for which $b=1$ ($k_i=0$). Their fields have an arbitrary amplitude A_j.

With the new amplitude A_i', the fields (9) and (10) take the same form as those in [4]. Thus (13) establishes a relation between the field representations in [4] and [2] when $b=1$.

Let us now consider the case of $b=-1$. Since a change $b\to -b$ does not modify equation (4) for $l=0$, the dispersion equation of axisymmetric waves in this case will have the same form $D_{+1}(\omega,k_z)=0$ as before. The dispersion properties of the other waves with $l>0$ and $b=-1$ are determined by the second of equations (12).

As is clear from (12), this equation has a singularity $\delta=0$ ($\zeta^4=0$) when $l=1$. The same singularity is present in expression (10). This singularity can be eliminated as before by redefining of the amplitude A_i

$$A_i'=\frac{2}{(1+b)}\frac{A_i}{l!}\left(\frac{k_i}{2}\right)^l. \tag{14}$$

Obviously such form of relation (14) allows one also to use it in the case of $b=1$ when it reduces to the form (13). With boundary conditions (3) and substitution of (14) into (9), (10) we find the solutions of the dispersion equation for waves with $l>0$ when $b=1$. They are zeros of the function $D_{-1}(\omega,k_z)$. The fields related to these solutions have non-zero distributions and satisfy the boundary conditions.

Hence the assumption made in [3, 4] about the singular nature of the transverse fields and the dispersion equation of a plasma-filled waveguide [2] is valid only for waves with $l=1$ and $b=-1$. In other cases such singularity is not occurring, if $\zeta^4=0$. Moreover this singularity can be eliminated by using the redefinition of the amplitude (14) which is not in contradiction to the eigenfrequency problem for the plasma-filled waveguide.

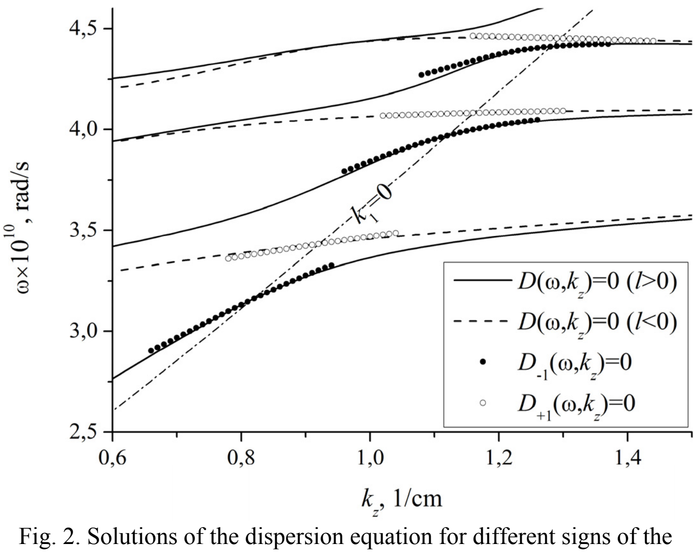

Fig. 2. Solutions of the dispersion equation for different signs of the azimuthal index and also zeros of functions $D_{\pm1}(\omega,k_z)$ in the neighborhood of $k_1=0$ ($\omega_p=3\times10^{10}$ rad/s, $\omega_H=4\times10^{10}$ rad/s, $|l|=1$).

The relation (14) allows one to define the transition from waves with finite transverse wavenumbers to those with $k_i=0$. During such transition the electromagnetic properties of the plasma-filled waveguide change continuously. Thus we have shown that the field expressions (1), (2) and the dispersion relation (4) presented in [2] remain also valid in the case that $k_i=0$ ($\zeta^4=0$).

It can be seen from the dispersion equation (4), that both the change of the form $b\to -b$ ($\varepsilon_2\to-\varepsilon_2$) and that of the form $l\to -l$ in this equation are equivalent. Therefore, solutions of the dispersion equation for $l<0$ are zeros of the function $D_{-1}(\omega,k_z)$ when $b=1$ and those of function $D_{+1}(\omega,k_z)$ when $b=-1$. The following changes $\varepsilon_2\to-\varepsilon_2$ and $l\to|l|$ should be used in $D_{\pm1}(\omega,k_z)$ to find these solutions.

The change of the form $\varepsilon_2 \to -\varepsilon_2$ does not affect the curve $k_i = 0$ in the (ω, k_z)-plane. As a result the solutions of the dispersion equation (4) that belong to this curve are different for waves with $l > 0$ and $l < 0$ (Fig. 2). This result has not been mentioned before.

CONCLUSIONS

The paper studies the electromagnetic properties of waves in a waveguide completely filled with a magnetized plasma with the transverse wavenumber k_i ($i = 1$ or 2) equal to zero. These waves satisfy the dispersion relations for right-polarized ($b = 1$) and left-polarized ($b = -1$) waves in unbounded magnetized plasma.

For all ω related to k_z by the equations $b = 1$ or $b = -1$, a division by zero of the well-known [2] field expressions and the dispersion equation of a plasma-filled waveguide occurs. However, their validity is not always violated when $b = \pm 1$. It was shown that this fact depends on the value of the azimuthal index l. For example, for the axially symmetric waves ($l = 0$) the fields and dispersion equation tend to the limits without singularity as $b \to \pm 1$. For $b = \pm 1$ the singularity is also absent for all l satisfying the condition $sign(l) = \mp 1$. However, under this condition the dispersion equation becomes zero for all ω and k_z related to each other by the relation $b = \pm 1$. At the same time, not all such $\omega(k_z)$ are physical solutions of the dispersion equation.

Physical solutions are those which satisfy the boundary conditions and have non-zero field distributions. Such solutions of the dispersion relation can be obtained with a redefinition of one of two amplitudes for the waveguide eigenfields. As a result the fields and the dispersion equation can be reduced to a form feasible for direct numerical analysis when $b = \pm 1$. This form coincides with that of the expressions obtained in [4], if $sign(l) = \mp 1$. Furthermore it was shown that they are the limits of the original expressions [2] for the fields and the dispersion relation as $b \to \pm 1$ in the case of $sign(l) = \mp 1$. When $sign(l) = \pm 1$ these original expressions have other asymptotes as $b \to \pm 1$. These asymptotes contain a singularity in the case of $|l| = 1$. However, it has been shown that this singularity can be eliminated. For this purpose one has again to redefine one of two amplitudes of the waveguide fields.

It should be noted that such redefinition does not affect the validity of the known [2] expressions for the fields and the dispersion relation, since the fields of the plasma-filled waveguide have either of two amplitudes of arbitrary value. Thus we have shown that the expressions for the fields and the dispersion equation [2] remain valid for $b = \pm 1$ ($k_i = 0$). As a result the field and frequency of a wave in the plasma-filled waveguide change continuously when one of its transverse wavenumbers crosses zero.

The change of the form $b \to -b$ in the dispersion equation is equivalent to the change $l \to -l$ and does not affect the curve $k_i = 0$ in the (ω, k_z)-plane. Therefore the solutions of the dispersion equation of magnetized plasma-filled waveguide that belong to this curve are different for waves which differ in signs of the azimuthal index. This result has not been mentioned before.

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