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## INFLUENCE OF THE SAFETY FACTOR VARIATION ON EXCITATION OF RESONANT MAGNETIC PERTURBATION IN TOKAMAK ROTATING EDGE PLASMAS

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Recently the possibility of resonant excitation of pressure perturbation by external helical magnetic perturbations near the rotating plasma edge was shown taking into consideration the finite plasma conductivity. In present paper the influence of the small safety factor variation on the pressure perturbation and on the plasma current response is studied. This phenomenon may explain the existence of the small window of safety factor values where ELMs were completely eliminated. The possibility to control the plasma current response to penetration of external helical resonant magnetic perturbations into the edge plasmas is shown. The investigation is carried out in the frame of one-fluid MHD.

**KEYWORDS:** resonant magnetic perturbations; edge localized modes; plasma rotation; plasma conductivity; tokamak.

### ВЛИЯНИЕ ИЗМЕНЕНИЙ ЗАПАСА УСТОЙЧИВОСТИ ТОКАМАКА НА ВОЗБУЖДЕНИЕ РЕЗОНАНСНЫХ МАГНИТНЫХ ВОЗМУЩЕНИЙ ВБЛИЗИ КРАЯ ВРАЩАЮЩЕЙСЯ ПЛАЗМЫ

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Ранее была показана возможность резонансного возбуждения возмущений давления у края плазмы внешними винтовыми возмущениями магнитного поля. Вращение плазмы и учёт её конечной проводимости играют ключевую роль при изучении этого явления. В настоящей работе исследовано влияние небольших изменений запаса устойчивости на возмущения давления и тока-отклика в плазме. Это явление может объяснить существование небольшого окна значений запаса устойчивости, при которых ELMы полностью подавлены. Показана возможность управления током-откликом плазмы на проникновение внешних винтовых резонансных магнитных возмущений. Исследование проведено в рамках одножидкостной МГД.

**КЛЮЧЕВЫЕ СЛОВА:** резонансные магнитные возмущения; вращение плазмы; проводимость плазмы; токамак.

### ВПЛИВ ЗМІНИ ЗАПАСУ СТІЙКОСТІ ТОКАМАКА НА ЗБУДЖЕННЯ РЕЗОНАНСНИХ МАГНІТНИХ ЗБУРЕНЬ БІЛЯ КРАЮ ОБЕРТОВОЇ ПЛАЗМИ

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Раніше було показано можливість резонансного збудження збурень тиску біля краю плазми зовнішніми гвинтовими збуреннями магнітного поля. Обертання плазми та врахування її скінченної провідності відіграють ключову роль при вивченні цього явища. У цій роботі досліджено вплив невеликих змін запаса стійкості на збурення тиску та струм-відгук у плазмі. Це явище може пояснити існування невеликого вікна значень запаса стійкості, при яких ELMи повністю пригнічуються. Показано можливість керування струмом-відгуком плазми на проникнення зовнішніх гвинтових резонансних магнітних збурень. Дослідження проведено у рамках однорідної МГД.

**КЛЮЧОВІ СЛОВА:** резонансні магнітні збурення; обертання плазми; провідність плазми; токамак.

Control of Edge Localized Modes (ELMs) is a critical issue of the present day large tokamaks and future tokamak-reactor ITER operation [1, 2]. Experiments at the DIII-D tokamak (major radius  $R=1.68\text{m}$ , minor plasma radius  $a_{\text{pl}}=0.61\text{m}$ ) have shown that ELMs can be suppressed by small external low frequency helical resonant magnetic perturbations (RMP) [3]. In DIII-D ELMs suppression takes place, when safety factor on the 95% normalized flux surface  $q_{95}$  is inside a narrow range  $3.5 < q_{95} < 3.9$  ( $\Delta q_{95} \lesssim 0.3$ ) [4]. Small changes of the safety factor were the result of slow plasma current increasing [4] in these experiments.

It was observed in experiments, that the toroidal rotation affects on ELM behavior [4, 5]. In DIII-D when the applied RMP level was just below that required for full ELM suppression, additional toroidal rotation with certain velocity lead to full ELM suppression [4] (a counter-NBI torque was applied to a co-rotating plasma).

Previously [6] a possibility of the pressure perturbation resonant excitation by external helical magnetic perturbations near

the plasma edge has been shown in the frame of one-fluid MHD, when the plasma rotation and the plasma response (conductivity) has being taken into account. A perfect shielding of external helical magnetic perturbation is missing in this case.

In the present paper the influence of these resonant pressure perturbations on the plasma current response to penetration of external helical magnetic field into the rotating edge plasma is investigated under the safety factor changing. Considered plasma parameters are closed to DIII-D experiments [3,4].

The aim of this paper is to propose an explanation of the existence of the safety factor “window”  $\Delta q_{95} \lesssim 0.3$  where ELMs full suppression takes place in DIII-D experiments.

### BASIC EQUATIONS

We consider a current carrying toroidal plasma with nested equilibrium circular magnetic surfaces ( $\rho_0$  is the radius of the magnetic surfaces,  $\omega_0$  is the poloidal angle in the cross-section  $\zeta = const$ ,  $\zeta$  is the toroidal angle). Each magnetic surface is shifted with respect to the magnetic axis ( $\xi$  is the shift,  $R$  is the radius of the magnetic axis). Near the plasma edge the equilibrium toroidal contravariant component of the magnetic field,  $B_0^\zeta = \Phi' / (2\pi\sqrt{g})$ , is large in comparison with the poloidal one,  $B_0^\theta = \chi' / (2\pi\sqrt{g})$ ,  $\Phi'$  and  $\chi'$  are the radial derivatives of toroidal and poloidal fluxes, respectively  $q(a) = \Phi' / \chi'$  is the safety factor,  $S = aq' / q$  is the shear. The known expressions for metric tensor are used [7].

On each equilibrium magnetic surface [7] we introduce a straight magnetic field line coordinate system ( $a, \theta, \zeta$ )  $\rho_0 = a$ ,  $\omega_0 = \theta + \lambda(a)\sin\theta$  ( $\xi'(a)$  is the radial derivative of Shafranov's shift)

$$\lambda(a) = -\xi'(a) - a/R, \tag{1}$$

$$\xi'(a) = \frac{1}{aR} \left( \frac{\chi'(a)}{2\pi R} \right)^{-2} \int_0^a \left[ 16\pi p_0(b) + \left( \frac{\chi'(b)}{2\pi R} \right)^2 \right] b db. \tag{2}$$

Assuming periodicity in both  $\theta$  and  $\zeta$ , we take the perturbations in the form

$$X(a, \theta, \zeta, t) = \sum_{m,n} X_{mn}(a) \exp[i(m\theta - n\zeta - \omega t)], \tag{3}$$

where  $m$  are poloidal and  $n$  toroidal mode numbers, respectively,  $\omega$  is the frequency of the external perturbation.

Assuming that  $\zeta$ - contravariant component of the magnetic perturbation  $B^\zeta \approx 0$ , for perturbations with  $m \gg 1$ ,  $nq \gg 1$  from the one-fluid MHD equations in a linear approximation in  $1/R$  the next equations were found [6] ( $B_{0\zeta}(a) = \Phi' / 2\pi a$ ,  $B_{0\omega_0}(a) = \chi' / 2\pi R$ ):

$$F_m(a) \left[ i(a^2 B_m^\theta)' + mB_m^a \right] + \frac{4\pi SqR}{B_{0\zeta}^2(a)} p_0'(B_{m-1}^a + B_{m+1}^a) + \frac{4\pi iqR}{B_{0\zeta}^2(a)} p_0'(B_{m-1}^\theta - B_{m+1}^\theta) + \frac{4\pi aR}{c} \frac{B_m^a}{B_{0\zeta}(a)} j_{0\zeta}' - \frac{8\pi im}{B_{0\zeta}(a)} \frac{a}{R} \left( \mu^2 - 1 + \frac{ap_0'}{B_{0\omega_0}^2(a)} - \frac{R}{a} S\xi' \right) p_m - \frac{4\pi i}{B_{0\zeta}(a)} (ap_{m-1}' - ap_{m+1}') + \frac{4\pi i}{B_{0\zeta}(a)} [(m-1)p_{m-1} + (m+1)p_{m+1}] = 0, \tag{4}$$

$$p_m = -\frac{ip_0' V_m^a}{\omega_{im}} = \frac{ip_0' R}{F_m(a)} \frac{1}{B_{0\zeta}} \left[ B_m^a + \frac{i}{\omega_{im}} \frac{c^2 m}{4\pi\sigma(a)} \frac{1}{a^2} [i(a^2 B_m^\theta)' + mB_m^a] \right], \tag{5}$$

$$\omega_{im} B_m^a = -F_m(a) \frac{B_{0\zeta}}{R} V_m^a - \frac{ic^2 m}{4\pi\sigma a^2} [i(a^2 B_m^\theta)' + mB_m^a], \tag{6}$$

$$(aB_m^a)' + imaB_m^\theta = 0, \tag{7}$$

where

$$\omega_m = \omega - \frac{B_{0\zeta}}{B_0} \left[ \frac{F_m(a)}{R} V_{0\parallel} + \frac{m}{a} \left( c \frac{p_{0i}'}{en_0 B_0} - c \frac{E_{0a}}{B_0} \right) \right], \tag{8}$$

$$F_m(a) = m\mu(a) - n, \quad \mu = 1/q. \tag{9}$$

Unlike [6], in present paper we put the sound velocity  $c_s$  is equal to 0 ( $c_s=0$ ). Equilibrium parameters are denoted by the subscript 0,  $p_0$  is the plasma and  $p_{0i}$  ions plasma pressures, respectively,  $j_{0\zeta}$  is the equilibrium current density,  $\sigma$  is the plasma conductivity. All poloidal harmonics of perturbations  $B_m^a$ ,  $B_m^\theta$  and  $p_m$  have the same toroidal mode number  $n$ . In our consideration all poloidal harmonic amplitudes of perturbations have finite values. The number of poloidal harmonics with finite values of amplitudes depends on the antenna spectrum (external perturbation). We took into account the equilibrium poloidal plasma rotation due to the existence of an equilibrium radial electric field  $E_{0a}$ , the ion diamagnetic drift and the parallel with respect to equilibrium magnetic field plasma rotation with a velocity  $V_{0\parallel}$ . The

value of  $F_m(a)$  is equal to zero inside the plasma, when  $q(a_{res}) = m/n$ .

Near the plasma edge the inequality  $S\xi' \gg 1$  ( $S \sim 4$ ) takes place, the term with parameter  $S\xi'$  is the main term in Eq.(4). From Eqs. (4)-(7) we get in this case ( $\omega=0$ ,  $a_N = a/a_{pl}$ )

$$\frac{1}{a_N} \frac{d}{da_N} \left( a_N \frac{d}{da_N} (a_N B_m^a) \right) - \frac{m^2}{a_N^2} (a_N B_m^a) - \frac{m}{a_N^2} Q_m(a_N) (a_N B_m^a) = 0, \quad (10)$$

where

$$Q_m(a_N) = \frac{K_m(a_N) A_m(a_N) \left( m K_m(a_N) F_m^2(a_N) + i \frac{B_{0\zeta}}{B_0} A_m(a_N) \frac{c}{4\pi\sigma(a_N)a} \right)}{\left( m K_m(a_N) F_m^2(a_N) \right)^2 + \left( A_m(a_N) \frac{c}{4\pi\sigma(a_N)a} \right)^2}, \quad (11)$$

$$K_m(a_N) = F_m(a_N) \frac{a V_{0||}}{R mc} + \frac{1}{B_0} \left( \frac{1}{p_{0i}} \frac{dp_{0i}}{da_N} \frac{T_{0i}(a_N)}{ea_{pl}} - E_{0a}(a_N) \right), \quad (12)$$

$$A_m(a_N) = \frac{8\pi}{B_{0\zeta}^2} a_N \frac{dp_0}{da_N} m^2 (\mu^2 - 1 - \frac{R}{a} S\xi'). \quad (13)$$

The last term in Eq.(10) describes the plasma response on penetration of external perturbation. Because of  $\mathbf{J} = \frac{c}{4\pi} \text{rot}\mathbf{B}$  ( $\mathbf{J}$  is the current density) and contravariant component of  $\text{rot}\mathbf{B}$

$$(\text{rot}\mathbf{B})^\zeta \approx -\frac{i}{aR} [i(a^2 B_m^\theta)' + m B_m^a] = \frac{i}{maR} \left[ \frac{d}{da} \left( a \frac{d}{da} (a B_m^a) \right) - \frac{m^2}{a} (a B_m^a) \right] \quad (14)$$

it is clear that the parameter  $Q_m$  is characteristic of the plasma current response on penetration of external perturbation.

The radial derivatives of plasma pressures  $p_0$  and  $p_{0i}$  near the plasma edge play the important role in Eq. (10). The behaviors of the equilibrium pressure gradients and equilibrium radial electric field  $E_{0a}$  are shown in Figs. 1,2 for typical DIII-D experimental conditions ([3,4,8]).

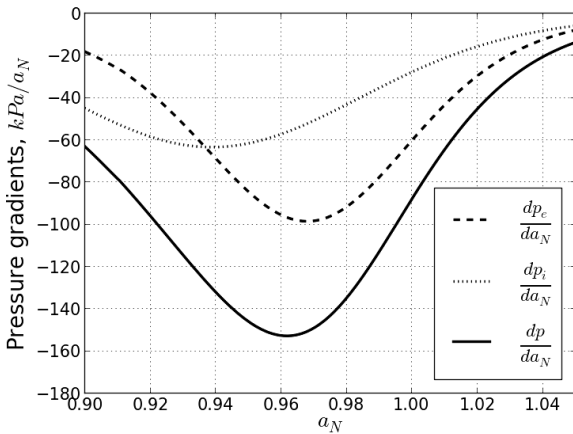


Fig.1. Equilibrium pressure gradients (in  $kPa/a_N$ ).

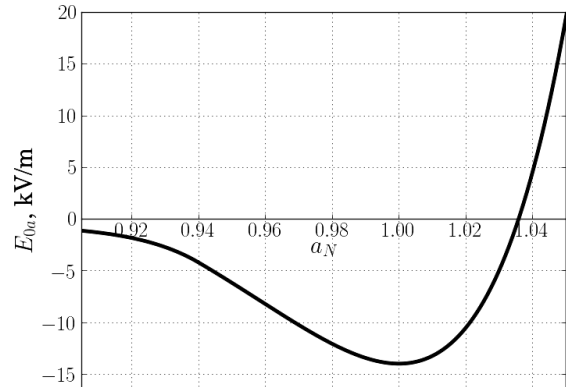


Fig.2. Equilibrium radial electric field.

## RESULTS AND DISCUSSIONS

Poloidal modes  $m = - (9 - 14)$  and toroidal mode  $n = 3$  are considered, as in DIII-D. For simplicity the calculations for case  $m = -11$  are presented only. The profile  $q(a_N) = b + 3.6a_N^{5.6}$  approximates near plasma edge the DIII-D experimental conditions. The parameter  $b$  is changed from 1 to 0.78 values to simulate the changes of  $q_{95}$  during the DIII-D experiments [4].

From Eqs. (4), (5) the pressure perturbation is presented in the next form:

$$p_m(a_N) = a_N \frac{dp_0}{da_N} \frac{R}{a} \frac{m K_m(a_N) F_m(a_N) \left( im \frac{B_{0\zeta}}{B_0} K_m(a_N) F_m^2(a_N) - A_m(a_N) \frac{c}{4\pi\sigma(a_N)a} \right) \frac{B_m^a}{B_0}}{\left( m K_m(a_N) F_m^2(a_N) \right)^2 + \left( A_m(a_N) \frac{c}{4\pi\sigma(a_N)a} \right)^2}. \quad (15)$$

The resonant excitation of pressure perturbations by external low frequency helical magnetic field near the plasma edge is possible when  $F_m(a_N) \approx 0$  or  $K_m(a_N) \approx 0$ . The case  $K_m(a_N) \approx 0$  occurs when plasma is rotated and finite plasma conductivity is taken into account. The resonant  $K_m(a_N) \approx 0$  takes place in edge plasma if

$$\frac{1}{p_{0i}} \frac{dp_{0i}}{da_N} \frac{T_{0i}(a_N)}{ea_{pl}} = E_{0a}(a_N). \quad (16)$$

The position of  $K_m(a_N) = 0$  resonance depends on direction of plasma rotation with a velocity  $V_{||}$ . The position of resonance  $K_m(a_N) = 0$  does not depend on  $m$  practically,  $m = -(9 - 14)$ .

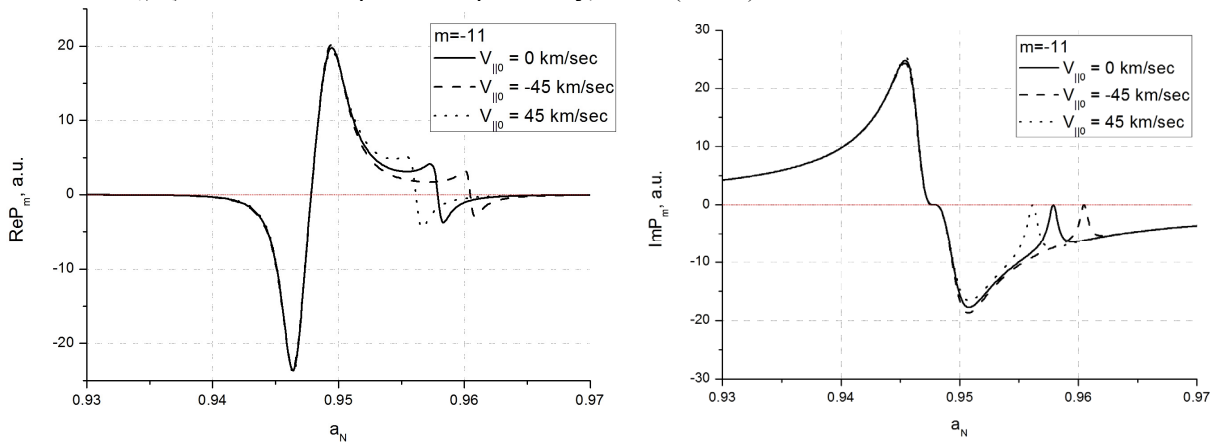


Fig.3. The pressure perturbation resonant excitation ( $q(a_N) = 1 + 3.6a_N^{5.6}$ ).

But position of  $F_m(a_N) = 0$  resonance depends on  $m$  strongly. The possibility of the resonant excitation of pressure perturbation (due to the plasma rotation and finite conductivity) by external helical magnetic perturbations near the plasma edge has been shown in [6]. It may affect the excitation of ballooning and peeling modes because of edge plasma pressure change.

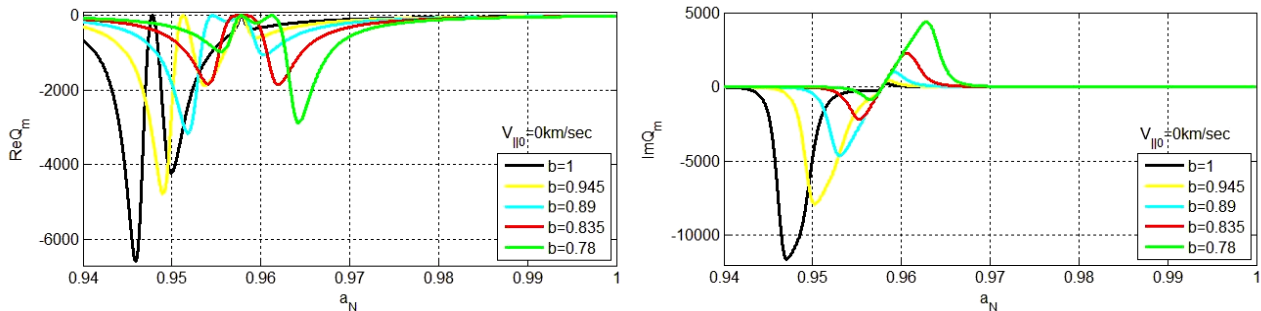


Fig.4. Dependence of radial profiles of  $ReQ_m$  and  $ImQ_m$  on parameter  $b$  values for case  $V_{||} = 0$  ( $m=-11$ ).

Radial profiles of  $ReQ_m$  and  $ImQ_m$  are shown ( $m = -11$ ) in Figs. 4-6. Here  $B_{0c} > 0$ . Small changes in the value of parameter  $b$  affect on profiles  $Q_m(a_N)$  strongly. With reduction of parameter  $b$  the position of resonance  $F_m(a_N) = 0$  moves to position of resonance  $K_m(a_N) = 0$ . For example, they coincide for case  $V_{||} = 0$  under  $b=0.8369$ . Amplitudes of  $ReQ_m$  and  $ImQ_m$  are reduced strongly, when resonance  $F_m(a_N) = 0$  moves to resonance  $K_m(a_N) = 0$ . And profiles of  $ReQ_m$  and  $ImQ_m$  become symmetric with respect to resonance position.

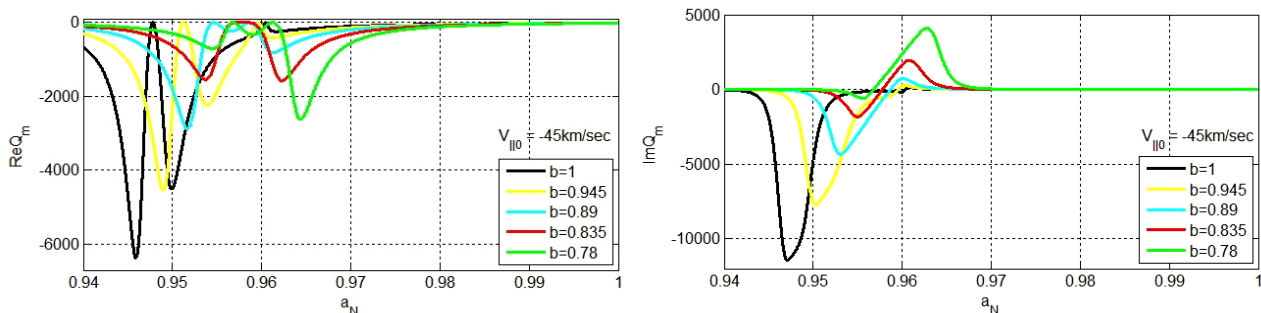


Fig.5. Dependence of radial profiles of  $ReQ_m$  and  $ImQ_m$  on parameter  $b$  values for case  $V_{||} = -45 \text{ km/s}$  ( $m=-11$ ).

Small changes in the value of parameter  $b$  correspond to small changes of the safety factor  $q$  ( $\Delta q_{95} \lesssim 0.3$ ). Hence, the control of plasma current response on penetration of external perturbation is possible as result of the safety factor variation at the edge of rotating tokamak plasmas due to the slow change of the plasma current.

The influence of direction of plasma rotation with velocity  $V_{0||}$  on the plasma current response is shown in Figs.5,6. In case of  $V_{0||} = -45 \text{ km/s}$  the amplitudes of the plasma current response  $ReQ_m$  and  $ImQ_m$  are smaller but in case  $V_{0||} = 45 \text{ km/s}$  are larger in comparison with case of  $V_{0||} = 0$ .

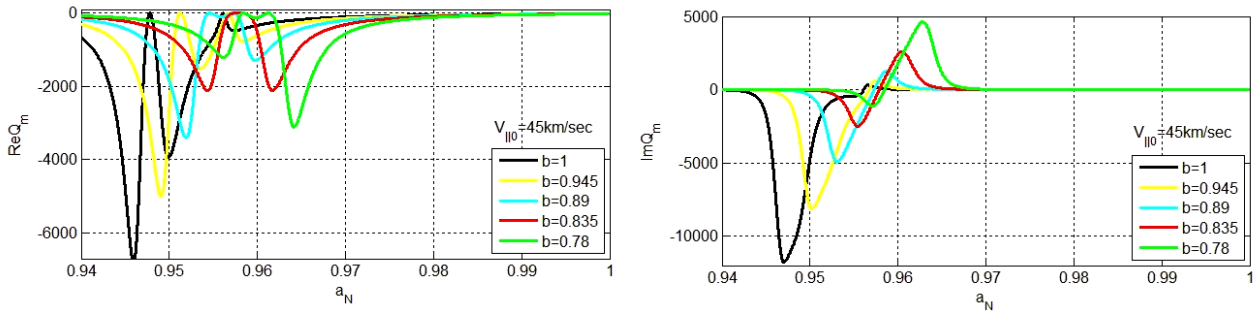


Fig.6. Dependence of radial profiles of  $ReQ_m$  and  $ImQ_m$  on parameter  $b$  values for case  $V_{0||} = 45 \text{ km/s}$  ( $m=-11$ ).

### CONCLUSIONS

The influence of the small safety factor variation on the resonant excitation of the pressure perturbation is studied. This influence leads to change of the plasma current response to penetration of external helical magnetic field into the rotating plasma with finite conductivity. This phenomenon occurs if position of the rational magnetic surface ( $q(a_{res}) = m/n$ ) is closed to the radial coordinate where poloidal velocity of plasma rotation  $cE_{0a}(a_N)/B_0$  is equal to ion drift velocity  $(cT_{0i}(a_N)/eB_0 p_{0i} a_{pl})(dp_{0i}/da_N)$ . Effect of the plasma rotation with velocity  $V_{0||}$  is studied also.

In the paper the typical DIII-D experimental data was used. Obtained results may be applied to control the plasma stability during experiments in tokamaks JET, DIII-D, TEXTOR and future ITER operation.

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