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INFLUENCE OF THE RELATIVISTIC ELECTRON BEAM SPECTRAL CHARACTERISTICS ON THE INSTABILITY DEVELOPMENT PROCESS AT NEAR-LIMITING CURRENT

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The paper presents the results of the numerical research of the influence of spectral characteristics of the relativistic electron beam, restricted by a spatial charge, on the instability development at near-limiting current. The beam system model is described permitting to calculate with minor errors the process characteristics for both narrow and wide electron energy spread in the beam. Instability development phases are defined and their duration is calculated. The dependences of instability development time and virtual cathode steady-state operation on the relativistic electron beam spectrum width are determined.

KEY WORDS: instability development, electron beam, virtual cathode, spectral characteristics, computer simulation

ВПЛИВ СПЕКТРАЛЬНИХ ХАРАКТЕРИСТИК РЕЛЯТИВІСТСЬКИХ ЕЛЕКТРОННИХ ПУЧКІВ НА ПРОЦЕС РОЗВИТКУ НЕСТІЙКОСТІ ПРИ СТРУМІ БЛИЗЬКОМУ ДО ГРАНИЧНОГО

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Представлені результати чисельного дослідження впливу спектральних характеристик релятивістського електронного пучка на розвиток нестійкості при струмі , близьким до граничного , обмеженого просторовим зарядом. Описана модель пучкової системи , що дозволяє з малими похибками розраховувати характеристики процесів , як для малого , так і для великого розкиду енергії електронів в пучку. Виділено фази розвитку нестійкості , визначено їх тривалості. Встановлено залежності часу розвитку нестійкості і характеру усталеного режиму віртуального катода від ширини спектра релятивістського електронного пучка.

КЛЮЧОВІ СЛОВА: розвиток нестійкості, електронний пучок, віртуальний катод, спектральні характеристики, комп'ютерне моделювання

ВЛИЯНИЕ СПЕКТРАЛЬНЫХ ХАРАКТЕРИСТИК РЕЛЯТИВИСТСКОГО ЭЛЕКТРОННОГО ПУЧКА НА ПРОЦЕСС РАЗВИТИЯ НЕУСТОЙЧИВОСТИ ПРИ ТОКЕ БЛИЗКОМ К ПРЕДЕЛЬНОМУ

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Представлены результаты численного исследования влияния спектральных характеристик релятивистского электронного пучка на развитие неустойчивости при токе, близком к предельному, ограниченному пространственным зарядом. Описана модель пучковой системы, позволяющая с малыми погрешностями рассчитывать характеристики процессов, как для малых, так и для больших разбросов энергии электронов в пучке. Выделены фазы развития неустойчивости, определены их длительности. Установлены зависимости времени развития неустойчивости и характера установившегося режима виртуального катода от ширины спектра релятивистского электронного пучка.

КЛЮЧЕВЫЕ СЛОВА: развитие неустойчивости, электронный пучок, виртуальный катод, спектральные характеристики, компьютерное моделирование

It is well-known that at injection of the current close to some critical value (limit, space-charge limited or, so-called, supercritical current) into the diode gap the Bursian instability arises in the drift gap. The virtual cathode is formed during an instability development [1], which oscillates in the space and reflects electrons backward to the injection plane. The subject of our study is a scenario of the relativistic electron beam instability onset if the beam current is near the limit or slightly exceeds it. Also, the influence of relativistic electron beam spectral characteristics on the instability development, mode and virtual cathode operation parameters are researched. The research on the scenario of instability development in the wide high-current electron beam requires the use of models possessing minor numerical errors for simulation of beams without energy spread, on the one hand, and permitting the adequate simulation of beams with a quite wide energy spread on the other hand. Frequently one uses different models for simulation of the beam without energy spread and that with a wide energy spread, however, the question arises: which of these models is more adequate?

The aim of this paper is to create a one-dimensional model of the beam system permitting to calculate with minor errors the process characteristics for both the narrow and wide electron energy spread in the beam. This model is the first step to creating a tool for the research on beams in a wide range of electron energies – a three-dimensional model of the beam. The paper is also aimed at numerical research on the electron beam spectrum width influence on the instability development process.

SIMULATION MODEL

As a simulation object we consider the drift area restricted, in the direction of beam propagation, by the grounded electron-transparent planes. In the transverse direction the drift area is not restricted (Fig.1). The electric field is self-consistent, potential one. A continuous injection of 1 MeV electrons into the system is provided. Initial particle pulses obey to the Gauss distribution. The investigated values were relative pulse spread parameter dp / p_0 from 0 to $5 \cdot 10^{-2}$, where p_0 is the average value of the initial particle pulse, dp is the pulse spread.

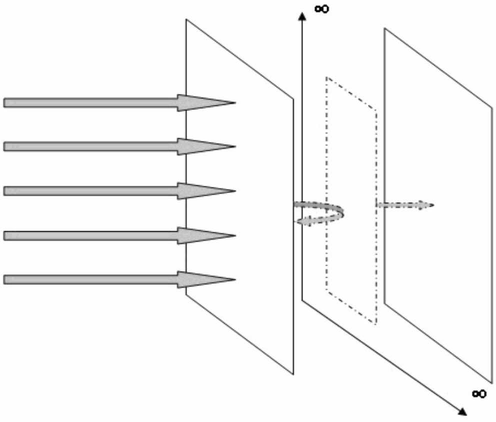


Fig.1. The drift area with an injected beam

The system is described by the equations for the self-consistent potential φ and electric field intensity E

$$\Delta\varphi = -4\pi\rho, \varphi|_B = 0 \tag{1}$$

$$E = -grad\ \varphi , \tag{2}$$

and by the electron motion equations

$$\frac{dp}{dt} = qE\ ,\ p|_{x=0} = f(p_0)\ , \tag{3}$$

where $f(p_0)$ is the statistic pulse distribution function, p is the particle pulse and q is its charge. The equation for x particle coordinates, with taking into account the relativistic correction, has the following form

$$\frac{dx}{dt} = \frac{pc}{\sqrt{p^2 + m^2c^2}}\ , \tag{4}$$

where m is the particle mass, c is the velocity of light. So, we have a problem of wide electron beam propagation of *1d1v* dimension.

For simulation of similar systems as a basic model one widely uses a discrete PIC model for one-dimensional electron plasma, described in the book by R. Hockney "Computer Simulation Using Particles" [2]. Below given are the equations of the discrete model modified for the given problem.

Charge distribution:

$$\rho_p = \frac{1}{H}\sum_{i=1}^{N_P} q_i W(x_i - x_p)\ , \tag{5}$$

where

$$W(x) = \begin{cases} 1 & at\ |x| \le h/2 \\ 0 & if\ not \end{cases}. \tag{6}$$

Field equations:

$$\frac{\varphi_{p+1} - 2\varphi_p + \varphi_{p-1}}{H^2} = -\frac{\rho_p}{\varepsilon_0}, \tag{7}$$

$$E_p = \frac{\varphi_{p-1} - \varphi_{p+1}}{2H}. \tag{8}$$

Force interpolation:

$$E_i^n = E(x_i^n) = N_s \sum_{p=0}^{Ng-1} W(x_i^n - x_p) E_p^n. \tag{9}$$

Motion equations:

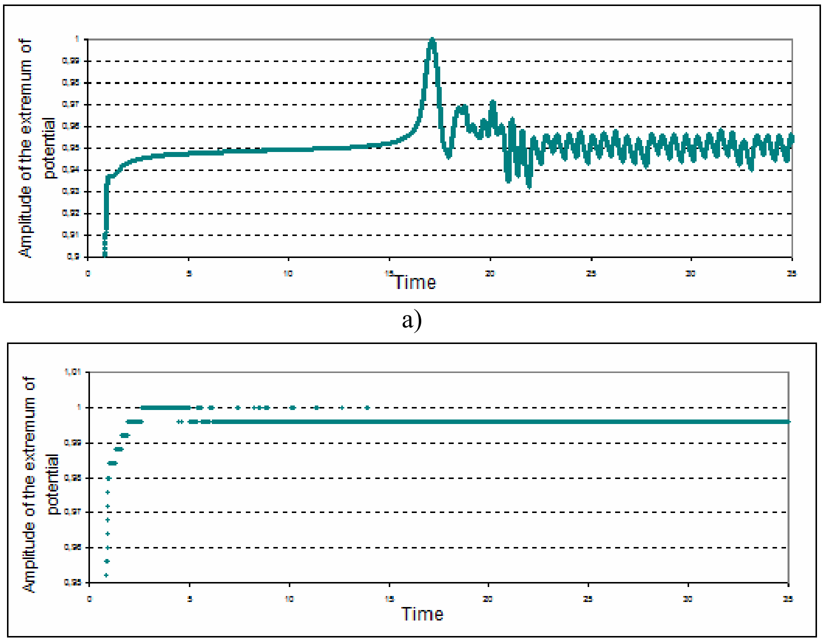
$$\frac{p_i^{n+1/2} - p_i^{n-1/2}}{dt} = qE(x_i^n), \tag{10}$$

$$\frac{x_i^{n+1} - x_i^n}{dt} = \frac{p_i^{n+1/2} c}{\sqrt{(p_i^{n+1/2})^2 + N_s^2 m_e^2 c^2}}, \tag{11}$$

where N_s is the coefficient for finite-size particles. To avoid significant numerical errors, caused by the charge discrimination and spatial resolution, an average number of particles for 1 mesh point, not less than 250 have been provided in the model. A comprehensive description of the computation model is given in [3]. As a result of the above modifications we succeeded in obtaining the model capable to describe the beam with both near and wide energy spreads.

RESULTS AND DISCUSSION

During of investigations the amplitude of the potential extremum was observed. Fig.2 presents the amplitude of the potential extremum for the beam with an energy spread of 0.01% (a) and 1% (b). Also we observed the fluxes of passing particles (denoted by “+” in Fig.3) and reflected particles (denoted by “-“in Fig.3) with an energy spread of 0.01% (a) and 1% (b).



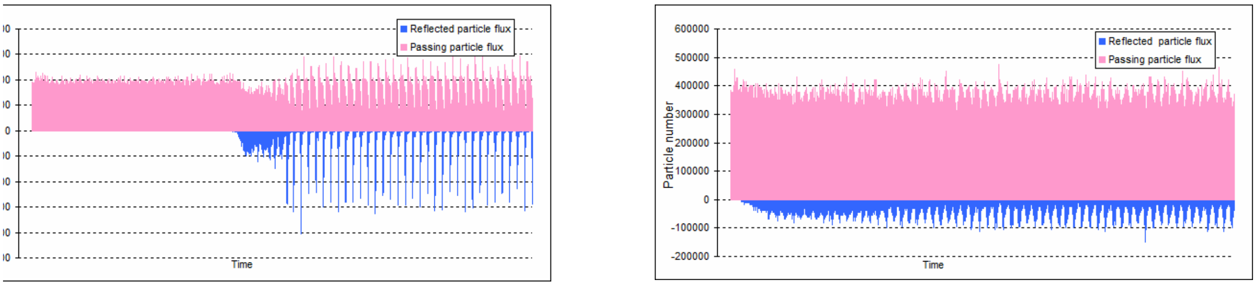
a)

b)

Fig.2. The amplitude of the potential extremum
a) energy spread of 0.01%, b) - 1%

The amplitudes of the extremum are shown in the relative units, for a unit time taken was the time of electron transit through the drift gap with an initial pulse p_0. From Fig.2a,3a it is seen that the time of instability development and virtual cathode formation exceeds significantly the time of electron transit through the drift gap. Fig.2b,3a show that the energy spread increases when the instability development time decreases appreciably. However, if the current density decreases the instability development time increases, but in this case the energy spread decreases to 0.01% and there is no instability development.

Below in Fig.4 the plots of the particle density in different instants of time before the appearance of the first reflected particles (from 15 to 17 transit times) are shown. From Fig.4 one can see that the instability development begins when the particle density profiles are changing so that their point of extremum displaces towards the beam injection plane and the amplitude is slowly increasing.



a) b)

Fig.3. Fluxes of passing and reflected particles
a) energy spread of 0.01%, b) - 1%

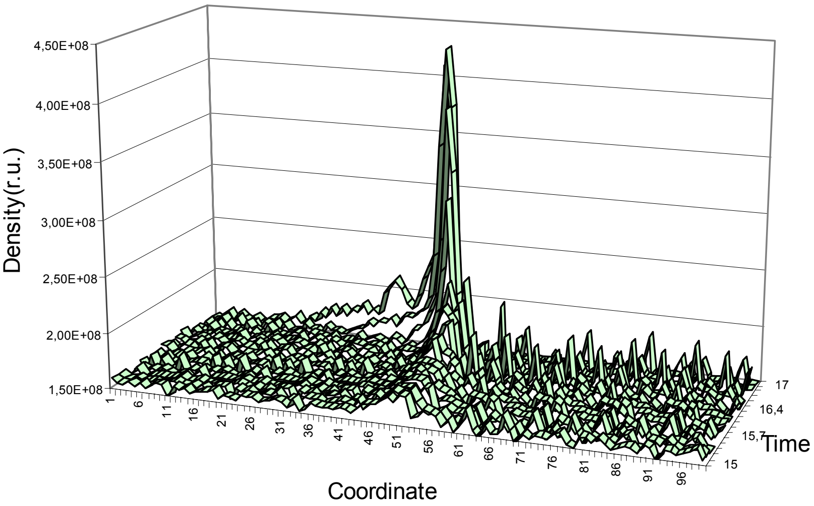


Fig.4. Particle density profiles in different instants of time from 15 to 17 transit times for the beam with an energy spread of 0.01%.

This situation leads to the beam instability development with the current values lower than these obtained by the analytic solution [4]. Particular attention should be given to the presence of a long preliminary stage of instability development.

The above analysis of figures has revealed four main phases of electron beam instability development. Initial phase is characterized by the slow smooth rise of the potential extremum and slow increase of the charge density (in Fig.2a from 2 to 15 time units). Linear stage is characterized by the exponential growth of the potential extremum and charge density, as well as, by the slight displacement of the coordinate of the potential extremum and charge density towards the system origin (in Fig.2a from 15 to 17 time units). Transient phase is characterized by the chaotic oscillations of the potential extremum and discharge density; there arises a reflected particle flux (in Fig.2a from 17 to 22 unit times). Nonlinear stage is the region of existence of the virtual cathode with steady-state oscillations of the potential extremum; there are a passing particle flux and a reflected particle flux (in Fig.2a from 2 to 22 time units).

CONCLUSIONS

The research has shown that the relativistic electron beam spectral characteristics exert influence on the instability development and virtual cathode parameters when the current is neat the limiting value. The process of potential instability development in the electron beam is characterized by the times significantly exceeding the transit time of the system and strongly depends on the energy spread. The behavior of virtual cathode steady-state oscillations also depends on the energy spread.

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