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MANIFESTATION OF NUCLEAR CLUSTERIZATION IN COULOMB SUMS

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The new experimental Coulomb sums values of ${}^6\text{Li}$ and ${}^7\text{Li}$ nuclei have been obtained at momentum transfers range $q = 0.750 \div 1.000 \text{ fm}^{-1}$ and $q = 0.750 \div 1.125 \text{ fm}^{-1}$, respectively, extending significantly the earlier reported momentum transfers range of Coulomb sums for these nuclei. The dependence of the ${}^6\text{Li}$ and ${}^7\text{Li}$ Coulomb sums on the momentum transferred is shown to differ substantially from same dependences for all the other investigated nuclei. It is suggested that the observed feature of the ${}^6\text{Li}$ and ${}^7\text{Li}$ Coulomb sums is related to a strong clusterization of these nuclei. The parameter q_p corresponding to the value of the momentum transferred, at which the Coulomb sum ceases to grow and remains constant for larger momentum transfers, is introduced. The values of the parameter q_p for the lithium isotopes nuclei were obtained ($q_p({}^7\text{Li}) = 1.20 \pm 0.10 \text{ fm}^{-1}$ and $q_p({}^6\text{Li}) = 1.35 \pm 0.10 \text{ fm}^{-1}$), which are much lower than q_p of ${}^4\text{He}$, ${}^{12}\text{C}$, ${}^{40}\text{Ca}$, ${}^{48}\text{Ca}$ and ${}^{56}\text{Fe}$ nuclei ($q_p \approx 2 \text{ fm}^{-1}$). The graph represented the q_p values of the nuclei as a function of the isolation parameter x , which characterizing the degree of nuclear clusterization, is constructed. The obtained graph shows the explicit proportionality of the q_p dependence from the parameter x and predicts the q_p value for the Coulomb sum of the ${}^9\text{Be}$ nucleus not yet measured.

KEY WORDS: Coulomb sum, lithium isotopes, light and medium nuclei, nuclear clusterization, cluster isolation parameter x

ПРОЯВ КЛАСТЕРИЗАЦІЇ ЯДЕР В КУЛОНІВСЬКИХ СУМАХ

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Отримано нові експериментальні значення кулонівських сум ядер ${}^6\text{Li}$ та ${}^7\text{Li}$ в діапазоні переданих імпульсів $q = 0,750 \div 1,000 \text{ fm}^{-1}$ та $q = 0,750 \div 1,125 \text{ fm}^{-1}$, відповідно, які суттєво розширяють діапазон по q раніше опублікованих кулонівських сум цих ядер. Показано, що залежність кулонівських сум від переданого імпульсу ізотопів літію істотно відрізняється від таких же залежностей для всіх інших ядер. Висловлено припущення про те, що виявлена особливість кулонівських сум ядер ${}^6\text{Li}$ та ${}^7\text{Li}$ пов'язана з сильною кластеризацією цих ядер. Введено параметр q_p , що відповідає значенню переданого імпульсу, при якому кулонівська сума перестає рости, і залишається незмінною при більших переданих імпульсах. Отримано значення параметру q_p для ядер ізотопів літію ($q_p({}^7\text{Li}) = 1,20 \pm 0,10 \text{ fm}^{-1}$ та $q_p({}^6\text{Li}) = 1,35 \pm 0,10 \text{ fm}^{-1}$), які значно нижчі, ніж q_p для ядер ${}^4\text{He}$, ${}^{12}\text{C}$, ${}^{40}\text{Ca}$, ${}^{48}\text{Ca}$ та ${}^{56}\text{Fe}$ ($q_p \approx 2 \text{ fm}^{-1}$). Побудовано графік, на якому значення q_p низки ядер наведені як функція параметру відособленості x , що характеризує ступінь їх кластеризації. Цей графік показує явну пропорційність між величиною q_p та параметром x передбачає значення q_p для ще не вимірюваної кулонівської суми ядра ${}^9\text{Be}$.

КЛЮЧОВІ СЛОВА: кулонівська сума, ізотопи літію, легкі та середні ядра, кластеризація ядра, параметр відособленості x

ПРОЯВЛЕНИЕ КЛАСТЕРИЗАЦИИ ЯДЕР В КУЛОНОВСКИХ СУММАХ

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Получены новые экспериментальные значения кулоновских сумм ядер ${}^6\text{Li}$ и ${}^7\text{Li}$ в диапазоне переданных импульсов $q = 0,750 \div 1,000 \text{ fm}^{-1}$ и $q = 0,750 \div 1,125 \text{ fm}^{-1}$, соответственно, которые существенно расширяют диапазон по q ранее опубликованных кулоновских сумм этих ядер. Показано, что зависимость кулоновских сумм от переданного импульса ядер ${}^6\text{Li}$ и ${}^7\text{Li}$ существенно отличается от таких зависимостей для всех других ядер. Высказано предположение о том, что обнаруженная особенность кулоновских сумм ядер ${}^6\text{Li}$ и ${}^7\text{Li}$ связана с сильной кластеризацией этих ядер. Введен параметр q_p , соответствующий значению переданного импульса, при котором кулоновская сумма перестает расти и остается постоянной при больших переданных импульсах. Получены значения параметра q_p для ядер изотопов лития ($q_p({}^7\text{Li}) = 1,20 \pm 0,10 \text{ fm}^{-1}$ и $q_p({}^6\text{Li}) = 1,35 \pm 0,10 \text{ fm}^{-1}$), которые значительно ниже, чем q_p ядер ${}^4\text{He}$, ${}^{12}\text{C}$, ${}^{40}\text{Ca}$, ${}^{48}\text{Ca}$ и ${}^{56}\text{Fe}$ ($q_p \approx 2 \text{ fm}^{-1}$). Построен график, на котором значения q_p ряда ядер представлены как функция параметра обособленности x характеризующего степень их кластеризации. Этот график показывает явную пропорциональность величины q_p от параметра x и предсказывает значение q_p для еще не измеренной кулоновской суммы ядра ${}^9\text{Be}$.

КЛЮЧЕВЫЕ СЛОВА: кулоновская сумма, изотопы лития, легкие и средние ядра, кластеризация ядра, параметр обособленности x

In the double-differential cross-section for electron scattering by the nucleus, ($d^2\sigma/dQd\omega$), the contributions from the electron-nucleus interaction may be separated by means of longitudinal and transverse components of the electromagnetic field. Accordingly, these contributions are called the longitudinal and transverse response functions $R_L(q,\omega)$ and $R_T(q,\omega)$, respectively. According to [1], the double-differential cross-section is related to the response functions by the equation

$$\frac{d^2\sigma}{d\Omega d\omega}(\theta, E_0, \omega) = \sigma_M(\theta, E_0) \left\{ \frac{Q^4}{q^4} R_L(q, \omega) + \left[\frac{1}{2} \frac{Q^2}{q^2} + \tan^2 \frac{\theta}{2} \right] R_T(q, \omega) \right\}, \quad (1)$$

where ω , q , $Q = (q^2 - \omega^2)^{1/2}$ are, respectively, the energy, 3-momentum, 4-momentum transferred to the nucleus by the incident electron of initial energy E_0 and scattered by the angle θ ; $\sigma_M(E_0, \theta) = e^4 \cos^2(\theta/2) / [4 E_0^2 \sin^4(\theta/2)]$ is the Mott cross-section; e is the electron charge.

In the treatment of the experimental data, one must take into account the influence of the nuclear electrostatic field on the incident electron. For this purpose, the correction ΔE_0 is introduced into the definition of the 3-momentum transfer $q = \{4(E_0 + \Delta E_0)[(E_0 + \Delta E_0) - \omega] \sin^2(\theta/2) + \omega^2\}^{1/2}$. The correction ΔE_0 is given by $k(3/2)Ze^2/R$, where R is the radius of the equivalent homogeneous distribution. According to [2], for electrons scattered by light nuclei to the continuum region the coefficient k is equal to 0.8.

The experimental data on the longitudinal functions $R_L(q, \omega)$ are generally represented as Coulomb sum

$$S_L(q) = \int_{\omega_{el}^+}^{\infty} \frac{R_L(q, \omega)}{\left[\tilde{G}_E(Q^2) \right]^2} d\omega, \quad (2)$$

where $\left[\tilde{G}_E(Q^2) \right]^2 = Z\eta \left(\left[G_E^p(Q^2) \right]^2 + \frac{N}{Z} \left[G_E^n(Q^2) \right]^2 \right)$. Here, ω_{el}^+ , being the lower limit of integral (2), corresponds to the energy transfer of the elastic electron scattering peak, and the superscript “+” excludes the contribution of this peak to the integral; N and Z denote the number of neutrons and protons in the nucleus, respectively; $\eta = [1+Q^2/(4M^2)] \times [1+Q^2/(2M^2)]^{-1}$ is the correction for the relativistic effect of nucleon motion in the nucleus; M is the proton mass; G_E^p and G_E^n are the charge form factors of the proton and the neutron, respectively.

For all the nuclei studied, the behavior of $S_L(q)$ with variations in the momentum transfer is similar in its character. With an increase in q , the $S_L(q)$ increases until at a certain momentum transfer value denoted as q_p , the $S_L(q)$ takes on constant values forming the function $S_L(q)$ plateau. For almost all previously studied nuclei we have $q_p \approx 2 \text{ fm}^{-1}$. By way of illustration, Fig. 1 shows the experimental $S_L(q)$ values for the ${}^4\text{He}$ nucleus [3, 4, 5].

The authors of papers [6, 7] have determined $S_L(q)$ values for the ${}^6\text{Li}$ nucleus, and have found that the behavior of the function differs from the usual one (Fig. 1). It can be seen that the $S_L(q)$ function reaches the plateau at $q_p \approx 1.4 \text{ fm}^{-1}$, this being much earlier in q than in the case with ${}^4\text{He}$ and other nuclei. In the ${}^7\text{Li}$ case, in the measurement range $q = 1.250 \div 1.625 \text{ fm}^{-1}$ [8], the function $S_L(q)$ is constant within the experimental error and is equal to unity. It means that if the $S_L(q)$ value is lower at certain momentum transfers, then it will reach the plateau range at $q_p \leq 1.3 \text{ fm}^{-1}$. Thus, the data of [8] do not specify q_p for the ${}^7\text{Li}$ nucleus, but restrict the upper value of this quantity. The authors of works [7, 8] have put forward the hypothesis that a comparatively low q_p value in the ${}^6,7\text{Li}$ case may be due to the Coulomb sum manifestation of clusterization peculiar to the nuclei under discussion.

However, on a more rigorous approach to the problem of relationship between the q_p value and nuclear clusterization it should be noted that this hypothesis is actually based only on the experimental q_p value of the ${}^6\text{Li}$ nucleus. As regards the q_p value of ${}^7\text{Li}$, from the data of [8] it follows that it is not higher than that of ${}^6\text{Li}$, and it is not improbable that it may be substantially lower. The last version would be in contrast with the proposed hypothesis, because if the q_p value is related to the clusterization (and the nuclei ${}^6\text{Li}$ and ${}^7\text{Li}$ are close in the degree of clusterization), then the q_p values of these nuclei should also be little different from each other.

The aim of the present article is checking the hypothesis for the relationship between the nuclear clusterization and the momentum transfer value q_p . For the purpose it is necessary: a) to determine the q_p value for the ${}^7\text{Li}$ nucleus; b) to define more exactly the q_p value for the ${}^6\text{Li}$ nucleus; c) to obtain the q_p values for the previously investigated nuclei.

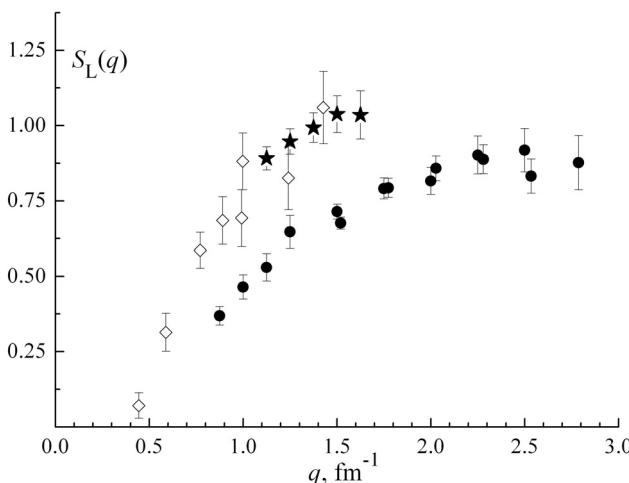


Fig. 1. Coulomb sums of ${}^4\text{He}$ and ${}^6\text{Li}$ nuclei. Full circles – ${}^4\text{He}$ [3, 4, 5], diamonds – ${}^6\text{Li}$ [6], full asterisks – ${}^6\text{Li}$ [7].

THE EXPERIMENT AND TREATMENT OF THE MEASURED DATA

The measurements, from which the present $S_L(q)$ values were determined, were carried out at the experimental facility SP-95 with the use of the electron beam from the NSC KIPT electron linear accelerator LUE-300. The electron beam of monochromaticity between 0.4% and 0.6%, and of energies ranging from 104 to 259 MeV, was incident on the ${}^6\text{Li}$ (or ${}^7\text{Li}$) target, the isotopic enrichment of which in the nuclide of interest was determined to be 90.5% (or 93.8%),

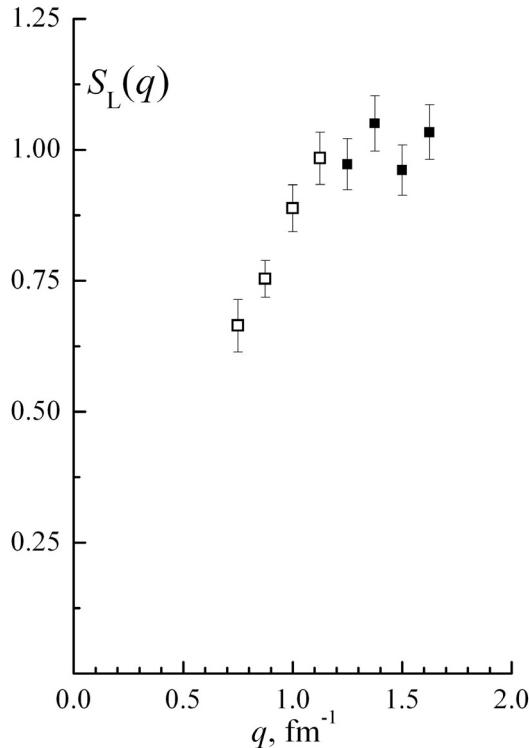


Fig. 2. Coulomb sum of ${}^7\text{Li}$. Full squares – ${}^7\text{Li}$ [8]; open squares - ${}^7\text{Li}$ (present data).

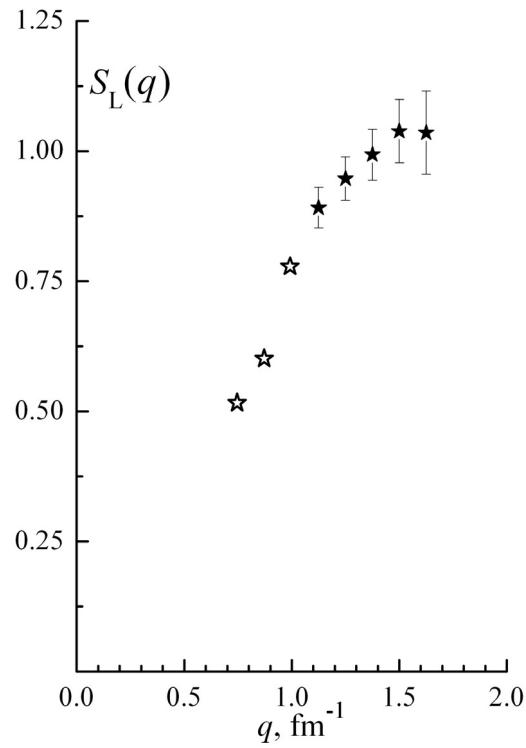


Fig. 3. Coulomb sum of ${}^6\text{Li}$. Full circles– ${}^6\text{Li}$ [7]; open circles - ${}^6\text{Li}$ (present data).

respectively. The measurements were performed at electron scattering angles from 34.2° up to 160° . For momentum analysis of scattered electrons we have used the spectrometer that had the second-order double focusing in vertical and horizontal planes [9]. Electrons in the focal plane of the spectrometer were registered by the 8-channel scintillation Cherenkov counter [10]. The description of the facility has been given in several publications [7,8,11,12].

The experiment was designed so that the response functions at several constant 3-momentum transfer values with q ranging from 0.750 to 1.625 fm^{-1} , and also, the Coulomb sums corresponding to these functions, could be obtained from the measurements. It should be mentioned that the most complicated and labor-consuming stage in these experiments is the processing of the measurement results for yielding the response functions and the Coulomb sums. Taking into account the long duration of the processing, the work was planned so as to obtain first the data measured at the highest q values, and then to process the data corresponding to lower momentum transfers. One of the advantages of this approach was the point that if the processing of a part of the experimental data yielded the physical data of prime interest, they could be discussed and submitted for publication at once, without waiting for the final processing of the whole body of initial measured data.

At the previous stage of measured data processing, we have obtained in this way four $S_L(q)$ values for ${}^7\text{Li}$ at $q = 1.250 \dots 1.625 \text{ fm}^{-1}$ [8], and five $S_L(q)$ values for ${}^6\text{Li}$ at $q = 1.125 \dots 1.625 \text{ fm}^{-1}$ [7].

By the present time, in addition to the above-given values, we have obtained $S_L(q)$ values for ${}^7\text{Li}$ at $q = 0.750 \dots 1.125 \text{ fm}^{-1}$ (Fig. 2), and preliminary $S_L(q)$ values for ${}^6\text{Li}$ at $q = 0.750 \dots 1.000 \text{ fm}^{-1}$ (Fig. 3).

NUCLEAR CLUSTERIZATION AND THE COULOMB SUM

To analyze the relationship between the momentum transfer q_p and the nuclear clusterization, the q_p value determination must be formalized using a certain simple procedure, which will be applied to the experimental $S_L(q)$ values of the nuclei under consideration. We define q_p as the momentum transfer that corresponds to the point of intersection of two straight lines, one of which (horizontal) approximates the $S_L(q)$ values on the plateau of $S_L(q)$ as a function of q , and the other line approximates the $S_L(q)$ values before reaching the plateau formation, starting from $S_L \approx 2/3 \times S_{L,p}$, where $S_{L,p}$ is the $S_L(q)$ value on the plateau. The given definition of the momentum transfer q_p is exemplified by the $S_L(q)$ for the ${}^4\text{He}$ nucleus (Fig. 4).

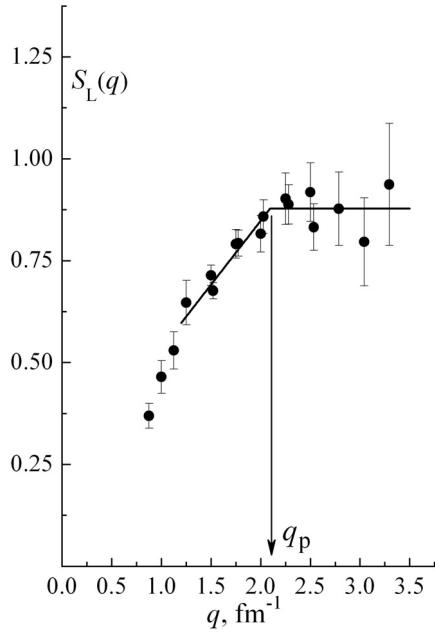


Fig. 4. Coulomb sum of ${}^4\text{He}$. Full circles – ${}^4\text{He}$ [3,4,5]; horizontal and inclined lines – data fitting; the intersection of the lines determines the q_p value.

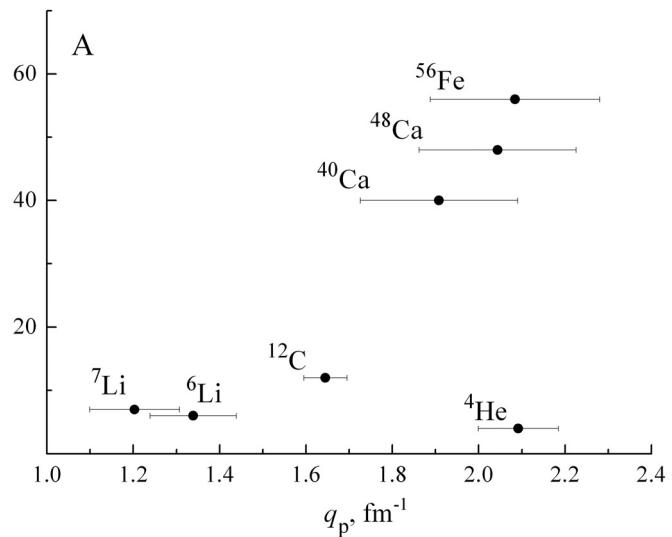


Fig. 5. Momentum transfers q_p for different nuclei with atomic mass A.

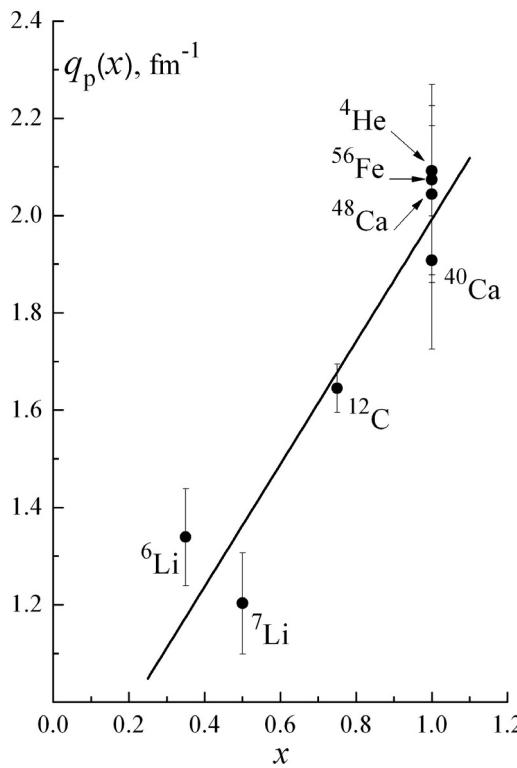


Fig. 6. Momentum transfer q_p versus the isolation parameter x for different nuclei. The straight line represents the data fitting by the linear dependence.

We apply this definition of q_p to all the nuclei having the atomic mass $A \geq 4$, for which a sufficient amount of experimental $S_L(q)$ data is available known. These are the data of the present work and of our previous works on the nuclei ${}^6\text{Li}$ [6, 7, 8], ${}^4\text{He}$ [3] and ${}^{12}\text{C}$ [13]. Besides, from [5], we have used the experimental $S_L(q)$ data obtained at the Saclay and Bates Laboratories for ${}^4\text{He}$, ${}^{12}\text{C}$, ${}^{40}\text{Ca}$, ${}^{48}\text{Ca}$, ${}^{56}\text{Fe}$. The momentum transfers q_p derived from these data are shown in Fig. 5. It can be seen that the q_p values of the nuclei ${}^4\text{He}$, ${}^{40}\text{Ca}$, ${}^{48}\text{Ca}$, ${}^{56}\text{Fe}$ are grouped at $q_p = (1.9 \dots 2.1) \text{ fm}^{-1}$, and in the case of ${}^6\text{Li}$ and ${}^7\text{Li}$ – at $q_p = (1.20 \dots 1.35) \text{ fm}^{-1}$. For the ${}^{12}\text{C}$ nucleus we have $q_p = 1.65 \text{ fm}^{-1}$. The momentum q_p grouping of the nuclei, observed in Fig. 5, corresponds to their distribution over the isolation parameter x ^{1/}. The first-group nuclei are not clusterized, whereas the second-group nuclei are strongly clusterized. For example, for the ${}^6\text{Li}$ nucleus, the parameter x varies between 0.3 and 0.4 [6,14,15], while for ${}^7\text{Li}$ we have $x = 0.5$ [14]. With this approach, we arrive at understanding of the intermediate value (between the two groups) $q_p = 1.65 \text{ fm}^{-1}$ of the ${}^{12}\text{C}$ nucleus, which is clusterized substantially less than the nuclei of the lithium isotopes. For the ${}^{12}\text{C}$ nucleus, the parameter x ranges from 0.7 to 0.8 [14].

Let us consider the momentum q_p as a function of the parameter x . For this purpose we put the isolation parameter of the nuclei ${}^4\text{He}$, ${}^{40}\text{Ca}$, ${}^{48}\text{Ca}$, ${}^{56}\text{Fe}$ to be equal to 1.0. As is obvious from Fig. 6, the x dependence of q_p is close to linear, this being in agreement with the result

^{1/} The isolation parameter of the nuclear cluster “ x ” defines the degree to which the clusters are formed within the nucleus [14]. The x value varies from $x = 1$ (shell model, e.g., ${}^4\text{He}$) to $x = 0$ (limiting case of the nuclear clusterization model).

of fitting the straight line to all the data with the least χ^2 value. Note that the observed dependence displays a high sensitivity of q_p to the x value. After refinement of $q_p(x)^2$, this feature of the function considered might be used for determination of x from the q_p value. However, because of the laborious procedure of obtaining experimental Coulomb sums, this method would be hardly applicable in practice.

RESULTS AND CONCLUSIONS

The results of the present work can be summarized as follows.

- A. Experimental $S_L(q)$ values of the nuclei ${}^6\text{Li}$ and ${}^7\text{Li}$ have been obtained at momentum transfers $q = 0.750 \dots 1.000 \text{ fm}^{-1}$ and $q = 0.750 \dots 1.125 \text{ fm}^{-1}$, respectively. This has essentially extended the range of the measured $S_L(q)$ towards q values lower than those investigated in [7, 8].
- B. Using the $S_L(q)$ data of the present work and of works [6-8], the momentum transfer q_p has been determined for the ${}^7\text{Li}$ nucleus ($q_p = 1.20 \pm 0.10 \text{ fm}^{-1}$), and has been redetermined more exactly for the ${}^6\text{Li}$ nucleus ($q_p = 1.35 \pm 0.10 \text{ fm}^{-1}$). The analysis of the available literature data on the Coulomb sums for ${}^4\text{He}$, ${}^{12}\text{C}$, ${}^{40}\text{Ca}$, ${}^{48}\text{Ca}$ and ${}^{56}\text{Fe}$ has yielded the q_p values for the mentioned nuclei (Fig. 5).
- C. The momentum transfers q_p of nuclear lithium isotopes have been found to be much lower than those in the case of other nuclei.

The comparison of the present experimental data with the data obtained elsewhere for a number of nuclei has demonstrated the validity of the hypothesis as to the effect of nuclear clusterization on the Coulomb sum of the nucleus.

The effect manifests itself in the observable proportionality of the momentum transfer q_p to the isolation parameter x , which characterizes the degree of nuclear clusterization. The hypothesis under discussion is also supported by the Coulomb sum measurement data for the ${}^9\text{Be}$ nucleus at $q = 0.8 \dots 1.7 \text{ fm}^{-1}$, from which the momentum q_p of this nucleus can be derived. Since the parameter $x = 0.6$ [14], related to the ${}^9\text{Be}$ nucleus, lies between the x values of the nuclei of lithium isotopes and the ones of ${}^{12}\text{C}$, then, according to the proposed hypothesis, its momentum transfer q_p should also lie between the q_p values of the mentioned nuclei, i.e., in the range from 1.3 fm^{-1} to 1.6 fm^{-1} .

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^{2/} In particular, checking of x values for the nuclei of lithium isotopes.