UDC 621.37:621.391

THE DIGITAL METHODS FOR DETECTION OF SELECTIVE SPECTRAL ANALYSIS OF COMPLEX SIGNALS

S. Veklych, S. Rassomakhin

V. N. Karazin Kharkiv National University, Svobody sq.,4, Kharkov, 61022, Ukraine
s.veklych@gmail.com, rassomakhin@karazin.ua

Reviewer: Vyacheslav Kharchenko, Doctor of Sciences (Engineering), Full Prof., Academicians of the Academy of Applied Radioelectronics Sciences, N.Ye. Zhukovskiy National Aerospace University – Kharkiv Aviation Institute (KhAI), 17 Chkalov St., Kharkiv, 61070, Ukraine.
v_s_kharchenko@ukr.net

Received on May 2017

Abstract. *The shortcomings of application of fast Fourier transformation algorithm at detection of separate tones of a signal are considered. The relevance of application of algebraic methods at demodulation of signals in modern information transmission systems is emphasized. The Goertzel algorithm of selective spectrum analysis is considered, the method of the linear algebraic processing of complex signal structures in case of detection of separate tones in a signal range is offered. The analysis of efficiency of the known and proposed method of selective spectrum analysis is carried out. It is concluded that the application of the method of linear algebraic processing of complex signal structures will allow us to calculate the parameters of the signal spectrum of only the needed nomenclature of frequencies by solving the system of linear algebraic equations without the use of fast Fourier transformation.*

Keywords*: fast Fourier transform (FFT), filter with infinite impulse response (IIR-filters), system of linear algebraic equations (SLAE), digital signal processing.*

1 Introduction

In the modern conditions characterized by complexity of the tasks solved by radio systems and a variety of an interfering situation development, enough perfect systems it is possible only on the basis of modern methods of optimization. The common problem of synthesis of radio engineering systems can conditionally be subdivided into two private tasks: the choice of the "best" signals for achievement of the required result taking into account a real situation and optimum processing of the accepted signals. The traditional method of the primary identification of parameters and demodulation of controlled signals is currently their analysis on the basis of the fast Fourier transform (FFT) algorithm. Use the FFT algorithms for processing of OFDM signal assumes the existence of exact information on the signal. At the solution of problems of radio monitoring and demodulation these data are, as a rule, unknown [1].

The device FFT optimized on computing expenses on the basis of decimation algorithms on the frequency or time not always is preferable from the point of view of excess dimensionality of the task. For example, if the signal range on an interval of discretization of the channel consists of small number quadrature frequency components $f_1, f_2, \ldots, f_m >> 0$, then for its complete processing it is enough to calculate only the amplitude coefficients. If to use a FFT, then based on properties of a computing algorithm determination will be carried out for $2 \cdot T \cdot f_m >> m$ amplitudes, i.e. excessively excess problem will be solved [2]. In this regard, development of the software and hardware tools of digital signal processing oriented only on use FFT algorithms not always is justified. Development of theoretical and practical bases of use of the ordinary apparatus of linear algebra for optimization of computing expenses and increase in characteristics of accuracy of recognition and demodulation of complex signals is of interest. We will consider some algorithms and methods of signals processing that allow to find separate tones of a signal, without solving an excess problem.

2 The Goertzel algorithm for detection of separate harmonious components

The Goertzel algorithm is a procedure for calculating the discrete Fourier transform. It makes it possible to reduce the number of necessary multiplications, but to a very small multiplier. The complexity of this algorithm is equal to n^2 therefore it does not belong to FFT algorithms. Goertzel algorithm is useful when the Fourier transformation component is required to calculate small quantity, – as a rule, no more than $\log_2 n$ from n a component. As FFT algorithms calculate all components of transformation, in these cases it is necessary to discard unnecessary components [3].

Algorithm Goertzel allows to calculate the value of k-th bin of N dot DFT:

$$S_N(k) = \sum_{n=0}^{(N-1)} x(n) \cdot W_N^{kn}, \quad W_N^{kn} = \exp\left(-j\frac{2\pi}{N}nk\right) \tag{1}$$

It represents an IIR filter of the second order with two real coefficients in the feedback and one complex coefficient in the direct link of the filter. The structure of the Goertzel's digital filter is shown in Fig. 1.

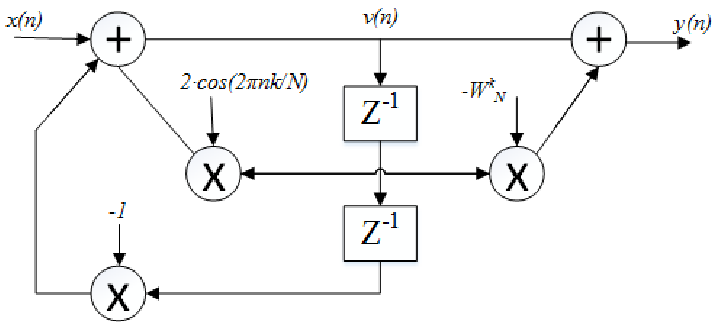

Fig. 1 – IIR filter implementation of the Goertzel algorithm

In order to obtain the indicated values of the k-th DFT coefficient in the Goertzel algorithm, only each $(N-1)$-th value of this coefficient is preserved, which provides one operation of complex multiplication in the filter straight chain and N real operations for calculating the intermediate results in the return circuit of the filter. Note that it is the refusal of receiving all output samples (therefore, their storage) in the straight filter circuit, which provides the Goertzel algorithm with saving in the number of computations in comparison with the definition of the nth DFT coefficient S, "*in the forehead*", according to the relation (1).

The filter's $y(n)$ output is equal to the DFT output frequency coefficient, $X(m)$, at the time index $n = N$, where the first time index value is $n = 0$. To be equivalent to the DFT, the frequency-domain index m must an integer in the range $0 \le m \le N-1$. The z-domain transfer function of the Goertzel algorithm is

$$H_G(z) = Y(z)/X(z) = \frac{1 - e^{-j2\pi m/N} \cdot z^{-1}}{1 - 2\cos(2\pi m/N) \cdot z^{-1} + z^{-2}}, \tag{2}$$

where $z^{-1} = e^{-j\omega t}$, and $z^{-2} = e^{-j2\omega t}$.

The differential equations of the filter Goertzel in a time domain are defined as follows:

1. The coefficients of return circuit of the filter are calculating:

$$v(n) = 2\cos(2\pi m/N) \cdot v(n-1) - v(n-2) + x(n). \tag{3}$$

2. The coefficients of straight filter chain are calculating:

$$y(n) = v(n) - W_N^k \cdot v(n-1). \tag{4}$$

Thus, the main advantages of the Goertzel algorithm over the standard radix-2 FFT for single tone detection consist in the following:

- N does not need to be an integer power of 2.
- The tone frequency can be any value between zero and sampling rate.
- The amount of filter coefficient storage is reduced.

- No storing a block of input data is needed before processing can begin (as with the FFT). Processing can begin with first input time sample.
- No data bit reversal is needed for Goertzel.
- If you implement the Goertzel algorithm M times to detect M tones, Goertzel is more efficient than FFT when $M < \log_2 N$.
- At computing an N -point $X(m)$ DFT bin value is that equation (3) need only be computed once after the arrival of the N-th input sample. Thus for real $x(n)$ inputs the filter requires $N+2$ real multiplies and $2N+1$ real adds to compute an N-point $X(m)$ [4].

One of disadvantages given an algorithm is that it does not allow to calculate a large number a component of Fourier coefficients. In tasks when it is necessary to calculate components of coefficients for several a component at once, this algorithm will not be effective. The use of FFT will also not allow to reduce the number of the calculated Fourier coefficients. In such cases it is proposed to use the method of linear algebraic processing of complex signal structures.

3 The method of linear algebraic processing of complex signal structures

Application of the Goertzel algorithm and the FFT method for calculating parameters of the signal spectrum is computationally cost and difficult to implement. To simplify the calculation of signals spectrum coefficients it is proposed to use the method of algebraic demodulation of complex signal structures. The idea of this method consists in statistical detection of amount of the observed fixed values of phases of harmonic oscillations on subcarrier frequencies.

For this the system of linear algebraic equations (SLAE) is composed:

$$A \cdot X = B, \tag{5}$$

where A - the matrix of amplitudes of the quadrature components on a modulation interval; B - the vector of the signal values in the digital representation in each sample of the modulation interval; X - the vector of required values of amplitude for a given modulation interval.

The dimension of the matrix A is defined by the number of samples N which are taken into account in the analysis of a signal on one interval of modulation and the number of considered quadrature of harmonics $(2 \cdot n_{f_{\max}})$. Depending on relation of vertical and horizontal the matrix's dimension the system (5) can be underdetermined $(N < 2 \cdot n_{f_{\max}})$, determined $(N = 2 \cdot n_{f_{\max}})$ and redefined $(N > 2 \cdot n_{f_{\max}})$. We will consider these cases in more detail.

The simplest case is when the system (5) can be strictly determined $(N = 2 \cdot n_{f_{\max}})$, so practically always SLAE is compatible and the solution of the system exists and the only. The number of the equations equals to the number of the required unknowns $(2 \cdot n_{f_{\max}})$ determining the amplitudes of quadrature signal's components. To solve the SLAE on the i-th modulation interval, it is necessary to select $(2 \cdot n_{f_{\max}})$ evenly located samples of the samples' array $P = \{p_0, p_1, ...\}$ beginning from the position where the full clock cycle of the signal begins to be observed. The square matrix of coefficients at unknown of SLAE is formed by the following rule:

$$\begin{aligned} &\mathbf{A}_1 = \left\| a_{i,j} \right\|, \quad i, j = 0, \ldots, \left(2 \cdot n_{f\,\max} - 1\right); \\ &a_{i,j} = \sin\left[2\pi(f_0 + q \cdot \Delta f) \cdot t_i\right], \\ &0 \le j \le n_{f\,\max} - 1; \\ &a_{i,j} = \cos\left[2\pi(f_0 + q \cdot \Delta f) \cdot t_i\right], \\ &n_f \le j \le 2 \cdot n_{f\,\max} - 1; \end{aligned} \quad , \tag{6}$$

where $q = 0,1,..., n_{f_{\max}}$.

The vector of the absolute terms is formed in the form a vector of signal's measurements on a duration of one modulation interval:

$$B_1 = \left\{ b_0, b_1, ..., b_{2n_{f\max}-1} \right\},\ b_i = p_i, \tag{7}$$

where $i = 0,...,2 \cdot n_{f_{\max}} - 1$.

The solution of the defined SLAE

$$A_1 \cdot X_1 = B_1 \quad \Rightarrow \quad X_1 = A_1^{-1} \cdot B_1 \tag{8}$$

gives the necessary estimate of the amplitudes' vector of the quadrature components $\mathbf{X}_1 = \left\{ x_0^1, \ldots, x_{(2 \cdot n_{f\max} - 1)}^1 \right\}$ corresponding to the permissible values of the subcarriers of the frequencies. The next case when the system (5) is underdetermined $(N < 2 \cdot n_{f_{\max}})$ while at the such systems either have an infinite number of solutions, or do not have a solution at all. The underdetermined SLAE can be solved by the method of the pseudo-inverse matrix of Moore-Penrose. According of the method of the pseudo-inverse matrix among the set of the solutions by underdetermined SLAE the normal solution is chosen – the solution with minimum norm among the solutions satisfying condition $\|X_1\| = \min\limits_{X_1}$. The normal solution exists and it is the only and is found by a formula:

$$X_1 = A_1^{+} \cdot B_1, \tag{9}$$

where A^{+} - the pseudo-inverse matrix of Moore-Penrose with dimension $2 \cdot n_{f_{\max}} \times 2 \cdot n_{f_{\max}}$.

The matrix A^{+} is defined by the equation:

$$A_1 \cdot A_1^{+} \cdot A_1 = A_1. \tag{10}$$

The solution (9) which is pertinent for writing down in the form of $X_1^{+} = A_1^{+} \cdot B_1$ gives a zero error $\|A_1 \cdot X_1^{+} - B_1\| = 0$, that is the solution is the pseudo-solution and among all pseudo-solution has as the normal solution, the minimum norm [5]. The most advantageous from the point of view of the maximum accounting of information on a signal is the case of the solution redefined SLAE $(N > 2 \cdot n_{f_{\max}})$. For formation redefined SLAE additional measurements of a signal from the sample containing P bigger quantity of the equations with the same number of unknown are used.

The degree of the redefined system is characterized by coefficient $\mu = W/2$ and describes asymmetry of the matrix dimensions $W \times 2$. This $W = \left\lfloor \frac{Fd}{V} \right\rfloor$, where Fd – a discretization frequency of the signal; V – a modulation rate; the sign $\lfloor \cdot \rfloor$ – means the rounding to the nearest smaller integer; the number 2 – means quantity used quadrature a component by means of which the signal, and, therefore, the number of unknown is set. The matrix A_2 and the vector B_2 are formed using the maximum number of measurements on a modulation interval T_p, determined by value $Num \approx T_p / t_d$:

$$\begin{aligned}
&\mathbf{A}_2 = \|a_{i,j}\|, \quad i = 0, \ldots, (Num - 1), \\
&j = 0...(2 \cdot n_{f\max} - 1); \\
&a_{i,j} = \sin\left[2\pi(f_0 + q \cdot \Delta f) \cdot t_i\right], \\
&0 \le j \le n_{f\max} - 1; \\
&a_{i,j} = \cos\left[2\pi(f_0 + q \cdot \Delta f) \cdot t_i\right], \\
&n_f \le j \le 2 \cdot n_{f\max} - 1;
\end{aligned} \tag{11}$$

$$\mathbf{B}_2 = \left\{ b_0, \ldots, b_{(Num-1)} \right\}, \quad b_v = q_v, \\ v = 0, \ldots, (Num-1). \tag{12}$$

The SLAE has the form:

$$\mathbf{A}_2 \cdot \mathbf{X}_2 = \mathbf{B}_2 \tag{13}$$

The system (13) has the set of solutions. In practice most often use criterion of the least squares, leading to a assessment of the form:

$$\mathbf{X}_2^* = (\mathbf{A}_2{}^T \cdot \mathbf{A}_2)^{-1} \mathbf{A}_2{}^T \cdot \mathbf{B}_2 . \tag{14}$$

The solution of system (14) is the approximate, but the result turns out more exact, than at the solution of strict system (8). The noise stability of the solution is achieved by averaging the effect of the interference with a number of signal measurements exceeding the minimum necessary. The calculation of the phase angle vector by solving system (14) by the method of algebraic processing of complex signal structures requires approximately the same number of operations like when using the FFT method. When the dimension of the matrix A_2 equal to $Num \times 2$ the number of operations required to solve the system (14) is approximately equal to $Num \cdot \log_2 Num$. The main feature of this method is that for computation of parameters of a range of signals of the selected nomenclature of frequencies, there is no need to calculate all parameters of a signals spectrum.

Finally, we can consider the important case for practice when the SLAE is weakly determined. A weakly determined system is a system described by the matrix A with a determinant not equal to zero $|A| \approx 0$, but the number of conditionality $|A^{-1}| \cdot |A|$ is very large. As some equations appearing in such system are represented by a linear combination of other equations actually the system is underdetermined $(N < 2 \cdot n_{f_{\max}})$. Depending on a concrete type of a vector of right-side part B or exists an infinite set of solutions or none exists. For the solution of such type of systems the method of regularization is used. This method is based on the use of additional a priori information on the solution, which can be both qualitative and quantitative. The concept of regularization reduces to the replacement of the SLAE solution of the form (5) by the problem of minimization of a Tikhonov functional:

$$\Omega(X, \lambda) = \left| A \cdot X - B \right|^2 + \lambda \cdot \left| X - x0 \right|^2 , \tag{15}$$

where λ – the small positive parameter of regularization; $x0$ – a priori estimate vector.

The problem of minimization of a Tikhonov functional can be reduced to solving another SLAE:

$$(A^T \cdot A + \lambda \cdot I) \cdot X = A^T \cdot B + \lambda \cdot x0 , \tag{16}$$

which at $\lambda \to 0$ passes into initial weakly determined system, and at big λ, being well defined, has the solution $x0$. Obviously, some intermediate value establishing a certain compromise between the acceptable conditionality and proximity to an initial task will be optimum [6]. The method of algebraic processing of complex signal structures allows carried out demodulation of signal by solving the SLAE without using FFT method. At demodulation of a signal by this method it is necessary that the SLAE was redefined since only the redefined SLAE allows to consider as much as possible information on a signal and gives the only solution of the system. Due to redefinition of SLAE the noise stability of this solution by averaging of action of noises at a large number of measurements of a signal is reached. The use of this method in case of demodulation of signals will allow to calculate parameters of a range of signals only of the necessary nomenclature of frequencies.

3 Conclusions

When processing complex signal structures for computation of parameters of a range of several tens tones in case of application of an algorithm of a FFT is solving a overly excess task. When using the Goertzel algorithm for the solution of this task which is implemented in the form of IIR filter of the second order the efficiency of the algorithm comes down to computing complexity of the FFT. To effectively solve this problem it is proposed to apply the method of algebraic processing of complex signal structures, which will allow us to calculate the parameters of the signal spectrum of only the needed nomenclature of frequencies by solving the SLAE without the use of FFT.

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Рецензент: В’ячеслав Харченко, д.т.н., проф., академік Академії наук прикладної радіоелектроніки, Національний аерокосмічний університет ім. М. Є. Жуковського, Харків, Україна. E-mail: v_s_kharchenko@ukr.net

Надійшло: Травень 2017.

Автори:
Сергій Веклич, аспірант, Харківський національний університет імені В.Н. Каразіна, Харків, Україна.
E-mail: s.veklych@gmail.com
Сергій Рассомахін, д.т.н., проф., зав.каф. БІСТ, Харківський національний університет імені В.Н. Каразіна, Харків, Україна.
E-mail: rassomakhin@karazin.ua

Порівняльний аналіз алгоритму Герцеля та способу алгебраїчної обробки складних сигнальних конструкцій при виявленні окремих тонів сигналу.

Анотація. Розглянуто недоліки застосування алгоритму швидкого перетворення Фур'є при виявленні окремих тонів сигналу. Підкреслюється актуальність застосування алгебраїчних методів при демодуляції сигналів в сучасних системах передачі інформації. Розглянуто алгоритм селективного спектрального аналізу, запропонований спосіб лінійної алгебраїчної обробки складних сигнальних конструкцій при виявленні окремих тонів в спектрі сигналу. Зроблено аналіз ефективності відомого і запропонованого способу селективного спектрального аналізу. Зроблено висновок, що застосування способу лінійної алгебраїчної обробки складних сигнальних конструкцій дозволить обчислювати параметри спектру сигналів тільки потрібної номенклатури, шляхом вирішення СЛАР без використання швидкого перетворення Фур’є.

Ключові слова: ШПФ, НІХ-фільтри, СЛАР, цифрова обробка сигналів.

Рецензент: Вячеслав Харченко, д.т.н., проф., академик Академии наук прикладной радиоэлектроники, Национальный аэрокосмический университет им. М. Є. Жуковского, Харьков, Украина. E-mail: v_s_kharchenko@ukr.net

Поступила: Май 2017.

Авторы:
Сергей Веклич, аспирант, Харьковский национальный университет имени В.Н. Каразина, Харьков, Украина.
E-mail: s.veklych@gmail.com
Сергей Рассомахин, д.т.н., проф., зав. каф. БИСТ, Харьковский национальный университет имени В.Н. Каразина, Харьков, Украина.
E-mail: rassomakhin@karazin.ua

Сравнительный анализ алгоритма Герцеля и способа алгебраической обработки сложных сигнальных конструкций при определении отдельных тонов сигнала.

Аннотация. Рассмотрены недостатки применения алгоритма быстрого преобразования Фурье при обнаружении отдельных тонов сигнала. Подчеркивается актуальность применения алгебраических методов при демодуляции сигналов в современных системах передачи информации. Рассмотрен алгоритм селективного спектрального анализа, предложен способ линейной алгебраической обработки сложных сигнальных конструкций при обнаружении отдельных тонов в спектре сигнала. Произведен анализ эффективности известного и предложенного способа селективного спектрального анализа. Сделан вывод, что применение способа линейной алгебраической обработки сложных сигнальных конструкций позволит вычислять параметры спектра сигналов только необходимой номенклатуры, путем решения СЛАУ без использования быстрого преобразования Фурье.

Ключевые слова: БПФ, БИХ-фильтры, СЛАУ, цифровая обработка сигналов.