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ESTIMATE OF NOISE-IMMUNITY FOR INDIVISIBLE CODES

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Abstract. *Considered the problem of finding general criteria to assess the effectiveness of indivisible codes.*

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Since recently, the problems connected with the telecommunication, in particular with binary encoding, transmission, and decoding of the digital information, have acquired a special importance. The latter phenomenon is well comprehensible because of the wide use of personal computers in the telecommunication processes.

As the flows of digital information transmitted steadily grow, the probability of errors caused by the channel noise also increases. The classical approach to removing the influence of noise consists in adding redundancy to the encoded messages [1]. The codes with redundancy are usually subdivided into two classes: divisible and indivisible ones [2]. It is well-known that for the divisible codes, the code distance serves as an estimate of their efficiency [3]. Unfortunately, this is not the case for the indivisible codes. Therefore, the problem of finding a general criterion for estimation of the efficiency of the indivisible codes with redundancy arises naturally.

One of the most important tasks for a telecommunication system consists in transmission of the maximal amount of information for a certain (fixed) period of time with the error probability p_e subject to the constraint $p_e \leq p < 1$. In order to obtain an estimate of a code's noise-immunity, we determine a quotient of detected erroneous combinations as follows:

$$D = 1 - \frac{M}{N}; \tag{1}$$

here M is the number of allowed (i.e. regular) code words, whereas N is the total number of possible combinations in the code.

The potential noise-immunity of a code is determined by expression (1). It is usually assumed that the information source generates the code words with equal probabilities, as well as the probabilities of them being transformed into both the regular and forbidden code words also coincide. Such an assumption however ignores the real properties of both the telecommunication channel and the information sources. Indeed, the latter most often generates the code words with unequal probabilities. Moreover, the code words are transformed into regular and forbidden ones with different probabilities, too.

In this paper, we take the above features into account by considering separately the probabilities for ***i-th*** code word: *(a)* to be generated as $P_i \geq 0$; *(b)* to be transformed into a forbidden word as p_i^f; *(c)* the same into a regular code word as p_i^r; and *(d)* to be transmitted correctly as p_i^i.

It is evident that

$$\sum_{i=1}^{M} P_i = 1, \tag{2}$$

and $p_i^f + p_i^r + p_i^i = 1.$ (3)

Now calculate the quotient of detected errors as

$$Z = \sum_{i=1}^{M} P_i p_i^f = \sum_{i=1}^{M} \sum_{j=M+1}^{N} P_i p_{ij}^f \; ; \tag{4}$$

here p_{ij}^f is the probability of transmission of *i*-th regular code word into *j*-th forbidden word. Similarly, the quotient of undetectable errors is equal to

$$V = \sum_{i=1}^{M} P_i p_i^r = \sum_{i=1}^{M} \sum_{j=1,\, j\neq i}^{M} P_i p_{ij}^r \; ; \tag{5}$$

here p_{ij}^r is the probability for *i*-th regular code word to be transformed into *j*-th allowed one. Therefore, the summary quotient of errors equals

$$W = Z + V \tag{6}$$

whereas the quotient of correct transmissions is

$$\Pi = \sum_{i=1}^{M} P_i p_i^i \, . \tag{7}$$

Now making use of (1) - (7) it is readily verified that

$$Z + V + \Pi = 1, \tag{8}$$

which means that Z, V, Π are equal to probabilities of transmission of the regular words into forbidden, other regular ones, and itself, respectively.

In particular case when $p_{ij}^f = p_{ij}^r = p_i^i = 1/N, \forall i, j$, , it is easy to verify that the values Z, V, Π do not depend upon the characteristics of the information source. Conversely, if $P_i = 1/M$ for each *i*, the values Z, V, II depend only upon the probabilities p_i^f, p_i^r, p_i^i.

One of the most important features of a code is the probability of transmission of a regular code word to some other regular word, i.e. the probability of undetectable error

$$p_e = V = 1 - Z - \Pi, \tag{9}$$

which increases when Z decreases. Therefore, in order to diminish p_e one needs either to maximize Z or to minimize V. The problem can be solved by a reasonable encoding of the symbols generated by the information source. An optimal encoding algorithm is proposed.

Example 1. We need transmitting three symbols A, B, C generated with probabilities P_1 = 0,6; P_2 = 0,3; P_3 = 0,1, respectively. They all are encoded by binary words. The channel is subject to some amount of noise which causes the following errors: 0 can be transformed to 1 with probability q_{01} = 0,1 whereas 1 can be transmitted to 1 with than of q_{10} = 0,2. The problem of developing a good coding system arises.

First choose the binary words 000, 001 and 011 to encode the symbols A, B, and C, resp. Therefore, the combinations 010, 100, 101, 110, and 111 are forbidden ones. At last, *M* = 3, *N* = 8.

Probabilities of correct transmissions of the regular words are readily calculated and equal resp.: p_1^1 = 0,729 for A, p_2^2 = 0,648 for B and p_3^3 = 0,576 for C.

From (7), we obtain the probability of the correct transmission as Π = 0,6894. In addition, (4) implies Z = 0,1684, whereas from (5) we have V = 0,1422.

In order to minimize V, we have to exchange the encoding rule as follows: A is encoded by 000, B – by 011 and C – by 001. Then it is obtained straightforward that $V_* = V_{min}$ = 0,1314; Z^* = 0,675; Π_* = 0,1936.

Otherwise, in order to maximize Z, we need encode A by 011, B by 000 and C by 001. In this case, $Z^{**} = Z_{max}$ = 0,2125; V_{**} = 0,1584; Π = 0,6291.

As it is illustrated by the above example, for an unsymmetrical channel (i.e. if $q_{01} \neq q_{10}$), the minimal p_e corresponds to the V_{min}, but not to the Z_{max}.

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Оцінка завадостійкості неподільних кодів
Анотація. Розглядається проблема знаходження загальних критеріїв для оцінки ефективності неподільних кодів.

Ключові слова: завадостійкість, неподільний код, канал.

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Оценка помехоустойчивости неделимых кодов
Аннотация. Рассматривается проблема нахождения общих критериев для оценки эффективности неделимых кодов.

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