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# MATHEMATICAL MODEL FOR THE FINGERPRINT MINUTIAE DISTORTION

Sergey Rassomakhin[1], Alexandr Kuznetsov[1], Vladimir Shlokin[1], Anna Uvarova[2]

[1] V. N. Karazin Kharkiv National University, 6 Svobody Sq., Kharkiv, 61022, Ukraine
rassomakhin@karazin.ua, kuznetsov@karazin.ua, vshlokin@ukr.net,

[2] Yuzhnoye State Design Office, Dnipro, 3 Krivorozhskaya St., 49008, Ukraine
annet.uvarova@gmail.com

**Reviewer**: Serhii Toliupa, Doctor of Sciences (Engineering), Full Prof., Taras Shevchenko National University of Kiev, 81 Lomonosova St., Kyiv, 03189, Ukraine.
tolupa@i.ua

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***Abstract**. This paper involves the research of biometric fingerprint images, minutiae and the mathematical probabilistic model of their distortion. The suggested model is based on heuristic analysis of the fingerprint scanning results with account for the nature of the potential errors. She allows to model a typical minutiae behavior in the biometric fingerprint images. The most typical distortion types were modeled, including the displacement of the fingerprint`s geometrical center, fingerprint rotation, minutiae deletion, as well as the distance changes between minutiae pairs.*

***Keywords:** biometric authentication; fingerprint images; minutia.*

## 1 Introduction

Biometric authentication is being widely used in the current information systems [1-21]. The most developed methods in this area include face identification [1-4], retina identification [5-8], identification using fingerprint ridges [9-16] etc. At the same time, the problem of minutiae distortion modeling stands out as one of the most complex in the field of fingerprint authentication, the main biometric authentication method [15,16]. This paper involves examination of biometric fingerprint images [9-16], as well as the development of mathematical probabilistic model for the minutiae distortion. To analyze the distribution of characteristics and the errors that occur during fingerprint processing we use the database [17]. We also use SourceAFIS package [18,19] as the main tool for fingerprint processing.

The results of analysis made allowed us to identify the following main factors that lead to differences in several portraits of the same fingerprint: - geometric center displacements caused by a change in the position of the object in the scan field; - rotation of images arising for the same reasons; - "erasure" or the appearance of "false" minutiae due to incorrect settings of the scanner algorithm or the entry of foreign objects in the scanning field; - drifting of the relative location of minutiae due to errors in the recognition algorithm.

Let us consider in detail each type of distortion, we will investigate the most significant factors for possible modeling of fingerprint minutiae distortions.

## 2 The modeling of geometric center displacement error

These errors occur due to inaccuracy of the location of the scan object relative to the center of the scan field. To describe the distribution of such errors, it is advisable to use the unimodal trapezoidal centered probability density function (PDF) of the form:

$$\Phi(d_y),\Phi(d_x)=\begin{cases}12.5\cdot d_x+3.75 & if\ \ -0.3\le d_x<-0.1;\\ 2.5 & if\ \ -0.1\le d_x\le 0.1;\\ -12.5d_x+3.75 & if\ \ 0.1<d_x\le 0.3;\\ 0 & if\ \ |d_x|>0.3.\end{cases} \quad (1)$$

 

We use the inverse function method.

If $z$:*unif* $[0,1]$, then to obtain a random variable $d_x$ with distribution (1) it is necessary to use the transformation:

$$d_x(z)=\begin{cases}\sqrt{0.16\cdot z}-0.3 & if \quad 0\le z<0.25;\\ 0.4\cdot z-0.2 & if \quad 0.25\le z\le 0.75;\\ -\sqrt{0.16\cdot(1-z)}+0.3 & if \quad 0.75<z\le 1.\end{cases} \qquad (2)$$

It is fair to assume that displacement errors having the PDF (2) act independently on the coordinates $X, Y$ of the unit fingerprint portrait.

The correctness of the transformation (2) is illustrated by a histogram of the computational experiment results in Fig. 1.

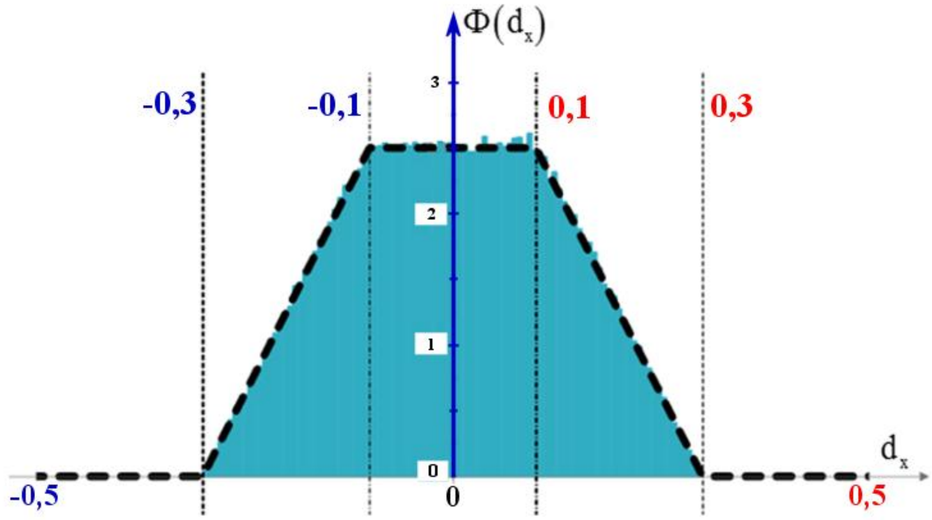


Fig. 1 – Result of statistical tests of the random coordinate error generator

### 3 The modeling of minutiae erasure and false appearance minutiae errors

The distribution of erasure errors can be parametrized by the value $p_E$ - the erasure probability of a single individual characteristic point on the fingerprint portrait. Assuming the independence of erasure errors, their distribution can be described by the usual discrete binomial distribution:

$$P_E(k)=\sum_{i=0}^{k}\binom{i}{N}\cdot p_E{}^{i}\cdot(1-p_E)^{N-i}, \qquad (3)$$

where $P_E(k)$ - the probability of erasure is not more than $k$ minutiae of the portrait; $N$ - the number of minutiae detected, determined by the distribution

$$Q(N_i)=\binom{i}{30}\cdot\left(\frac{1}{2}\right)^{30}, i\in[0,30], N_i\in[15,45],$$

where $Q(N_i)$ is the probability that the number of minutiae of the portrait (*taking into account minutiae masked outside the unit square*) will be a value $N_i$.

The size of the parameter $p_E$ for various methods of fingerprint processing, in our opinion, can be located within. $0<p_E\le 0.1$. To simulate the erasure process after obtaining the portrait model (Table 1), using a random generator which is uniformly distributed in a unit number interval, the vector $Z=\{z_1,z_2,\cdots,z_N\}$ is generated, the elements of which $z_i$:*unif* $[0,1]$, $i=1\ldots N$.

On the basis of the vector $Z$, an erasure vector $E=\{e_1,e_2,\cdots,e_N\}$ is calculated whose elements have a binary value and are obtained by a functional transformation of the vector coordinates $Z$:

$$e_i=\left\lfloor \frac{z_i}{1-p_E} \right\rfloor,\quad e_i=[0,1],\ i=1\ldots N. \tag{4}$$

Next, the rows in the portrait table (Table 1), which have numbers that correspond to the ordinal numbers of the unit vector $E$ elements, are removed from the table and, accordingly, from the portrait of minuses.

We base the probabilistic description of the errors associated with the false minutiae appearance on Poisson distribution:

$$\Pr(K)=\frac{\lambda_A{}^K}{K!}e^{-\lambda_A} \tag{5}$$

where $\Pr(K)$ - probability of $K$ false minutiae appearance on the portrait sample; $\lambda_A$ - an empirically determined mathematical expectation of the false minutiae number.

Table. 1 – Portrait matrix

| № | X | Y | φ | № | X | Y | φ |
|---|---|---|---|---|---|---|---|
| **1** | 0.21 | -0.07 | 0.6 | **15** | 0.01 | 0.39 | 0.15 |
| **2** | 0.28 | 0.18 | 0.58 | **16** | -0.05 | -0.55 | 0.08 |
| **3** | 0.12 | -0 | 0.49 | **17** | 0.51 | 0.5 | 0.64 |
| **4** | 0.19 | 0.31 | 0.74 | **18** | -0.25 | -0.34 | 0.55 |
| **5** | 0.07 | -0.68 | 0.62 | **19** | 0.23 | -0.41 | 0.41 |
| **6** | 0.05 | -0.12 | 0.8 | **20** | 0.13 | -0.41 | 0.47 |
| **7** | -0.25 | 0.33 | 0.58 | **21** | 0.14 | 0.08 | 0.15 |
| **8** | -0.01 | 0.42 | 0.91 | **22** | 0.06 | 0.23 | 0.74 |
| **9** | 0.17 | 0.15 | 0.73 | **23** | -0.05 | 0.17 | 0.83 |
| **10** | 0.12 | 0.23 | 0.67 | **24** | 0.69 | 0.31 | 0.87 |
| **11** | 0.3 | 0.54 | 0.32 | **25** | 0.1 | 0.7 | 0.3 |
| **12** | 0.64 | -0.14 | 0.31 | **26** | -0.33 | -0.3 | 0.13 |
| **13** | 0.13 | 0.44 | 0.11 | **27** | -0.17 | -0.11 | 0.78 |
| **14** | 0.09 | -0.05 | 0.85 | **28** | 0.15 | 0.08 | 0.61 |

Approximation of a random variable - the number of false minutiae in a portrait subject to the Poisson distribution (5) is achieved using a random generator $unif[0,1]$ and a conventional binomial distribution as follows. Based on the statistical processing of a sufficient number of portraits, the mathematical expectation of the number of false minutiae in one portrait $\lambda_A$ is determined empirically. As a rule, $0.1\le\lambda_A\le0.5$ (in this case, minutiae additions tend to occur less likely than erasures). A vector is generated with elements uniformly distributed in the unit interval $z_i\sim unif[0,1]$, $i=1\ldots M$. Then, on the basis of a transformation similar to (4), we get an addition vector $A=\{a_1,a_2,\cdots,a_M\}$ of binary elements obtained according to:

$$a_i=\left\lfloor \frac{z_i}{1-\frac{\lambda_A}{M}} \right\rfloor,\quad a_i=[0,1],\ i=1\ldots M. \tag{6}$$

The number of unit elements in the vector is subject to the binomial law (3) with the parameter $P_E=\frac{\lambda_A}{M}$. Then the number of added false minutiae of the portrait is determined by the square of the length of the vector $A$:

$$K = |A|^2 = \sum_{i=1}^{M} a_i . \tag{7}$$

The approximation of the Poisson distribution (5) will be all the more accurate the larger the selected value $M$ . For an acceptable approximation of the Poisson distribution given $\lambda_A \ll 1$, it suffices to require following inequality:

$$M \geq \lambda^{-1} . \tag{8}$$

The simulation of the appearance of false minutiae is made on the basis of the value obtained as a result of the computational experiment, determined by (7): table 1 adds $K$ rows (when $K = 0$ rows are not added). The generation of values for the added rows is the same as for existing points in the table - using $z \sim unif\,[0,1]$. This does not exclude the case when additional points will be outside the unit square and will be disguised on the original portrait.

## 4 The modeling of image rotation errors

To model rotation errors, we accept the following conventions for the Cartesian coordinate system of the unit square of the fingerprint portrait.

Portrait image rotation around the geometric center of the unit square with coordinates $[0,0]$ is conveniently modeled by rotating the coordinate axes on the plane by a specified angle $\alpha$ (Fig. 2).

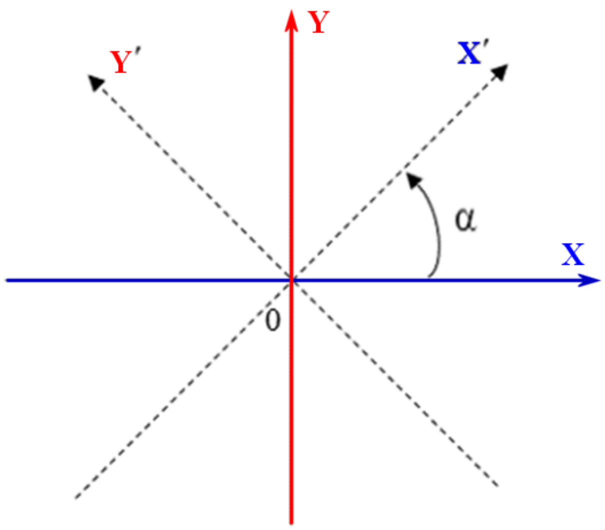

Fig. 2 – Rotation illustration

The zero value of the rotation angle corresponds to the axis $0X$ ; the positive direction of the angle is the counterclockwise movement. It should be borne in mind that turning the axes by an angle α is equivalent to rotating the original image (*in* $X\,0\,Y$ *coordinates*) in the opposite direction by an angle $-\alpha$ (in $X'0Y'$ coordinates). It is known that the relationship between the coordinates of an arbitrary point $\begin{pmatrix} X \\ Y \end{pmatrix}$ in the original coordinate system $X\,0\,Y$ and the coordinates of the new point $\begin{pmatrix} X' \\ Y' \end{pmatrix}$ in the system $X'0Y'$ deployed at the angle α is given in the matrix form by the following expression:

$$\begin{pmatrix} X' \\ Y' \end{pmatrix} = U(\alpha) \cdot \begin{pmatrix} X \\ Y \end{pmatrix},$$

where

$$U(\alpha) = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}. \tag{9}$$

Using the transformation (9) allows you to define an algorithm for modeling the image rotation. Let $G$ – be the portrait matrix (Table 1) of the size $(N \times 3)$. We divide it into submatrices

$G = (XY \| \varphi)$, where $XY$ is a submatrix of the size $(N \times 2)$, containing $N$ rows with a pair of coordinates $X$ and $Y$ for each of the $N$ points of the original portrait; $\varphi$ – is a vector column containing the angle values for $N$ corresponding points normalized in the interval $[0,1]$.

The rotation of portrait minutiae by an angle (given in radians) is achieved by transforming the submatrix: $XY$:

$$XY' = U(-\alpha) \cdot (XY)^T, \tag{10}$$

where $(XY)^T$ – transposed submatrix $XY$.

The change in the vector $\varphi$ associated with the rotation of the portrait is determined by the formula:

$$\varphi' = \left(\varphi - \frac{\alpha}{2 \cdot \pi}\right) \mathrm{mod}1 - \left\lfloor \left(\varphi - \frac{\alpha}{2 \cdot \pi}\right) \mathrm{mod}1 \right\rfloor, \tag{11}$$

where the operation $(\text{"arg"})\,\mathrm{mod}1$ extracts the fractional part of "arg", with respect to the sign.

The matrix of the portrait rotated by the angle $\alpha$ is determined by combining the resulting submatrix and the vector:

$$G' = \left((XY')^T \| \varphi'\right). \tag{12}$$

Rotation errors are determined by the distribution of the random rotation angle α. Empirical considerations allow us to limit the range of possible values within a right angle

$$\alpha \in \left[-\frac{\pi}{4}, +\frac{\pi}{4}\right], \tag{13}$$

and to show the requirements of unimodality and centering to the PDF. To simulate a random value α, we use a method based on the central limit theorem and allows us to approximate a truncated normal distribution by summing a limited number of centered random numbers uniformly distributed in a single range. We use the approximation of the normal distribution by summing the four linearly transformed random variables $z_i\!:\!unif\,[0,1]$, $i \in 1,\ldots,4$. The restriction (13) corresponds to the normalized values, then the realization of the random normalized rotation angle is obtained by a functional transformation of the form:

$$\alpha_H = \sum_{i=1}^{4} \left(\frac{z_i - 0.5}{16}\right). \tag{14}$$

The approximated normal PDF has the form:

$$f(\alpha_H) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\alpha_H{}^2}{2\sigma^2}\right), \ \sigma = \sqrt{\frac{1}{3 \cdot 256}}. \tag{15}$$

In Fig. 3 shows the form of the function (15) (dashed line) and the histogram of the probability distribution of the approximation (14) obtained with the number of tests equal to $10^5$.

As can be seen, the use of only four terms in the sum of expression (14) provides a good approximation of the normal PDF.

The absolute value of the random angle of rotation does not exceed the limits specified above $\pm 45^o$, and the RMS value is

$$\sigma = \sqrt{\frac{1}{3 \cdot 256}} \cdot 360^o = 0.036 \cdot 360^o \approx 13^o.$$

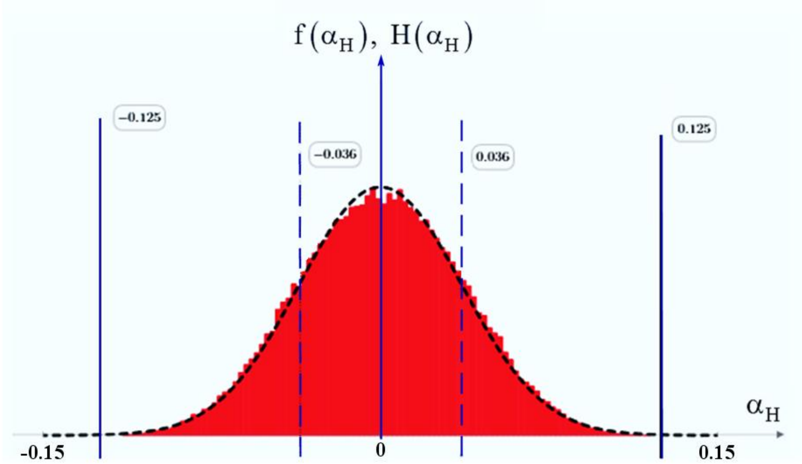


Fig. 3 – PDF and histogram of the approximation of the PDF of the normalized random rotation angle

Thus, as a result of the studies, this section based on the analysis of the experimental data of biometric fingerprint images developed an analytical probabilistic model for the formation and processing of minutiae that takes into account various quantitative and qualitative characteristics (*the density of minutiae distribution and linear displacements of the fingerprint center, the possibility of loss and / or appearance of new minutiae, the presence of angular errors, etc.*). The developed model allows us to generalize and formalize the process of minutiae generation, as well as to justify theoretical suggestions and recommendations for their processing in computerized access control systems.

## 5 Conclusions

Biometric authentication methods are being widely used in modern computerized access control systems. At the heart of their construction lies the processing of control points of various biometric images (*the iris of the eye, the outline of the face, the hand, papillary lines, etc.*). The analysis showed that the most widely used methods of processing fingerprint control points (minutiae), allowing reliably identifying a specific user and implementing various security services.

The proposed analytical probability model of minutiae distortions takes into account various quantitative and qualitative characteristics (*the density of minutiae distributions and linear displacements of the fingerprint center, the distribution of the minutiae angles, the possibility of loss and / or appearance of new minutiae, the presence of angular errors, etc*.). This allows us to generalize and formalize the process of minutiae generation, in order to be used in computerized access control systems and in other important applications [20-21].

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**Рецензент**: Сергій Толюпа, д.т.н., проф., Київський національний університет імені Т. Шевченка, Київ, 03189, Україна. E-mail: tolupa@i.ua

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**Автори:**

Сергій Рассомахін, д.т.н., зав. кафедри Безпеки інформаційних систем і технологій, Харківський національний університет ім. В. Н. Каразіна, майдан Свободи 6, Харків, 61022, Україна. E-mail: rassomakhin@karazin.ua

Олександр Кузнецов, д.т.н., проф., академік Академії наук прикладної радіоелектроніки, Харківський національний університет імені В.Н. Каразіна, майдан Свободи 6, Харків, 61022, Україна. E-mail: kuznetsov@karazin.ua

Володимир Шлокін, директор Інноваційного центру, Харківський національний університет ім. В.Н. Каразіна, майдан Свободи, 6, Харків, 61022, Україна. E-mail: vshlokin@ukr.net

Анна Уварова, провідний інженер, Конструкторське бюро «Південне» ім. М.К. Янгеля», вул. Криворізька 3, Дніпро, 49008, Україна. E-mail: annet.uvarova@gmail.com

**Математична модель спотворення відбитків пальців.**

**Анотація.** Дана стаття включає в себе дослідження біометричних зображень відбитків пальців, мінуцій та математичної імовірнісної моделі їх спотворення. Запропонована модель заснована на евристичному аналізі результатів сканування відбитків пальців з урахуванням характеру потенційних помилок. Вона дозволяє моделювати типову поведінку мінуцій в біометричних зображеннях відбитків пальців. Були змодельовані найбільш типові типи спотворень, включаючи зміщення геометричного центру відбитка пальця, обертання відбитка пальця, видалення дрібних деталей, а також зміни відстані між мінуціями.

**Ключові слова**: біометрична аутентифікація; зображення відбитків пальців; мінуції.

**Рецензент**: Сергей Толюпа, д.т.н., проф., Киевский национальный университет имени Т. Шевченко, Киев, 03189, Украина. E-mail: tolupa@i.ua

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**Авторы:**

Сергей Рассомахин, д.т.н., зав. кафедры Безопасности информационных систем и технологий, Харьковский национальный университет им. В. Н. Каразина, пл. Свободы 6, Харьков, 61022, Украина. E-mail: rassomakhin@karazin.ua

Александр Кузнецов, д.т.н., проф., Академик Академии наук прикладной радиоэлектроники, Харьковский национальный университет им. В. Н. Каразина, пл. Свободы 6, Харьков, 61022, Украина. E-mail: kuznetsov@karazin.ua

Владимир Шлокин, директор Инновационного центра, Харьковский национальный университет им. В. Н. Каразина, пл. Свободы, 6, Харьков, 61022, Украина. E-mail: vshlokin@ukr.net

Анна Уварова, ведущий инженер, Конструкторское бюро «Южное» им. М.К. Янгеля», ул. Криворожская 3, Днепр, 49008, Украина. E-mail: annet.uvarova@gmail.com

**Математическая модель искажения отпечатков пальцев.**

**Аннотация**. Данная статья включает в себя исследование биометрических изображений отпечатков пальцев, минуций и математической вероятностной модели их искажения. Предложенная модель основана на эвристическом анализе результатов сканирования отпечатков пальцев с учетом характера потенциальных ошибок. Она позволяет моделировать типичное поведение минуций в биометрических изображениях отпечатков пальцев. Были смоделированы наиболее характерные типы искажений, включая смещение геометрического центра отпечатка пальца, вращение отпечатка пальца, удаление мелких деталей, а также изменение расстояния между минуциями.

**Ключевые слова:** биометрическая аутентификация; изображения отпечатков пальцев; минуции.